Ex Prove that $\frac{d}{dx}\tan x = \sec^2 x$

Solu

$$\frac{d \tan x}{dx} = \frac{d}{dx}\left[\frac{\sin x}{\cos x}\right]$$

$$= \frac{\cos x(\cos x) - \sin x(-\sin x)}{(\cos x)^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \sec^2 x$$

H.W.

Prove that :-

1. $\frac{d}{dx}\sec x = \sec x \tan x$

2. $\frac{d}{dx}\cot x = -\csc^2 x$

3. $\frac{d}{dx}\cos x = -\sin x$

Theorem

1. $\sin^2 x + \cos^2 x = 1$
2. $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$
3. $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$
4. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
5. $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
6. $\sin(2\alpha) = 2\sin\alpha\cos\alpha$
7. $\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$
8. $\sec^2\alpha = 1 + \tan^2\alpha$

Ex Find the derivitive y' for the following function

1. $y = x^2 \tan x$

$y' = x^2 \cdot \sec^2 x + \tan x \cdot 2x$

$= x^2 \sec^2 x + 2x \tan x$

2. $y = \sin(x^2 + 3)$

$y' = \cos(x^2 + 3) \cdot 2x$

$= 2x \cos(x^2 + 3)$

3. $y = 2\cos x - 3\sin x$

$y' = -2\sin x - 3\cos x$

4. $y = \dfrac{\sin x}{1 + \cos x}$

$$y' = \frac{(1+\cos x)(\cos x) - \sin x(-\sin x)}{(1+\cos x)^2}$$

$$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2} = \frac{\cos x + 1}{(1+\cos x)^2} = \frac{1}{1+\cos x}$$

5. $y = \sec x \tan x$

$y' = \sec x \cdot \sec^2 x + \sec x(\sec x \tan x)$

$= \sec^3 x + \sec^2 x \tan x$

6. $y = \dfrac{\cot x}{1+\csc x}$

$$y' = \frac{(1+\csc x)(-\csc^2 x) - \cot x(-\csc x \cdot \cot x)}{(1+\csc x)^2}$$

$$= \frac{-\csc^2 x - \csc^3 x + \cot^2 x \csc x}{(1+\csc x)^2}$$

$$= \frac{-\csc^2 x - \csc x\left[\underbrace{\csc^2 x}_{=1+\cot^2 x} - \cot^2 x\right]}{(1+\csc x)^2}$$

$$= \frac{-\csc^2 x - \csc x\left[1+\cot^2 x - \cot^2 x\right]}{(1+\csc x)^2}$$

$$= \frac{-\csc^2 x - \csc x}{(1+\csc x)^2} = \frac{-\csc x\left[\csc x + 1\right]}{(1+\csc x)^2}$$

$$= \frac{-\csc x}{(1+\csc x)}$$

Ex Find y'' for the following.

1. $y = \sec x$

$y' = \sec x \tan x$; $y'' = \sec x \cdot \sec^2 x + \tan x \sec x \tan x$

$= \sec^3 x + \tan^2 x \sec x$

$= \sec x\left[\sec^2 x + \tan^2 x\right]$

2. $y = x \cos x$

$y' = x(-\sin x) + \cos x(1) \Rightarrow y'' = -[x\cos x + \sin x(1) - \sin x]$

$y'' = -x\cos x - 2\sin x$

Ex Find $y''(\frac{\pi}{4})$ if $f(x) = \sec x$

$y = \sec x$

$y' = \sec x \tan x$

$y'' = \sec x \sec^2 x + \tan x \cdot \sec x \tan x$

$= \sec^3 x + \tan^2 x \sec x$

$y'(\frac{\pi}{4}) = \sec^3(\frac{\pi}{4}) + \tan^2(\frac{\pi}{4}) \sec(\frac{\pi}{4})$

$= \sec^3(45) + \tan^2(45) \sec(45)$

$= (\sqrt{2})^3 + (1)\sqrt{2} = 3\sqrt{2}$

Ex

Evaluate the following integrals:-

1. $\int \sin(x+y)\,dx = -\cos(x+y) + c$

2. $\int \cos 5x\,dx = \frac{1}{5}\sin 5x + c$

3. $\int (\frac{1}{x^2} + \sec^2 x)\,dx = \int \frac{1}{x^2}\,dx + \int \sec^2 x\,dx$

$= -\frac{1}{x} + \tan x + c$

4. $\int \sin^2 x\,dx = \int \frac{1-\cos 2x}{2}\,dx = \frac{1}{2}\int dx - \int \cos 2x\,dx$

$= \frac{1}{2}x - \frac{1}{4}\sin 2x + c$

5. $\int \sin^2 x \cos x \, dx = \frac{1}{3} \sin^3 x + c$

6. $\int \frac{\cos\sqrt{x}}{\sqrt{x}} \, dx = 2\sin\sqrt{x} + c$

7. $\int \cot^2 x \, dx = \int (\csc^2 x - 1) \, dx$

$= \int \csc^2 x \, dx - \int x = -\cot x - x + c$

8. $\int \cos^3 x \, dx = \int (1 - \sin^2 x) \cdot \cos x \, dx$

$= \int (\cos x - \sin^2 x \cos x) \, dx$

$= \sin x - \frac{1}{3} \sin^3 x + c$

9. $\int (\csc(\sin))^2 \cdot \cos x \, dx = \frac{1}{3} (\tan(\sin x))^3 + c$

10. $\int x \sec^2 x^2 \, dx = \frac{1}{2} \tan x^2 + c$

11. $\int \frac{\sin 2x}{(5 + \cos 2x)^3} \, dx = \int \sin 2x \cdot (5 + \cos 2x)^{-3} \, dx$

$= \frac{1}{4} \frac{1}{(5 + \cos 2x)^2} + c$

12. $\int \sec^3 2x \tan 2x \, dx$

$= \int \sec^2 x \, (\sec x \tan x) \, dx = \frac{1}{3} \sec^2 x + c$

13. $\int \sec x \, dx$

$$= \int \sec x \, \frac{\sec x + \tan x}{\sec x + \tan x} \, dx$$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} \, dx$$

$$= \ln|\sec x + \tan x| + c$$

Theorem:

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta}$$