② $(S \cup T) \circ R = (S \circ R) \cup (T \circ R)$.

Proof:- let $(x,z) \in (S \cup T) \circ R$

$\Rightarrow \exists y \in A \ni (x,y) \in R$ and $(y,z) \in S \cup T$

$\Rightarrow \exists y \in A \ni (x,y) \in R$ and $(y,z) \in S$ or $(y,z) \in T$

$\Rightarrow [(x,y) \in R$ and $(y,z) \in S]$ or $[(x,y) \in R$ and $(y,z) \in T]$

$\Rightarrow (x,z) \in S \circ R$ or $(x,z) \in T \circ R$

$\Rightarrow (x,z) \in (S \circ R) \cup (T \circ R)$.

$\Rightarrow (S \cup T) \circ R \in (S \circ R) \cup (T \circ R)$ ---- ①

$\Leftarrow$ H.w

③ $(S \cap T) \circ R = (S \circ R) \cap (T \circ R)$.

Proof:- let $(x,z) \in (S \cap T) \circ R$

$\Rightarrow \exists y \in A \ni (x,y) \in R$ and $(y,z) \in S \cap T$

$\Rightarrow \exists y \in A \ni (x,y) \in R$ and $(y,z) \in S$ and $(y,z) \in T$

$\Rightarrow [(x,y) \in R$ and $(y,z) \in S]$ and $[(x,y) \in R$ and $(y,z) \in T]$

$\Rightarrow (x,z) \in (S \circ R)$ and $(x,z) \in (T \circ R)$

$\Rightarrow (x,z) \in (S \circ R) \cap (T \circ R)$

$\Rightarrow (S \cap T) \circ R \subseteq (S \circ R) \cap (T \circ R)$ — ①

$\Leftarrow$ H.w

④ If $R \subseteq S$, then $T \circ R \subseteq T \circ S$, $(R \circ T \subseteq S \circ T)$.

Proof:- let $(x,z) \in T \circ R$

$\Rightarrow \exists y \in A \ni (x,y) \in R$ and $(y,z) \in T$

$\because R \subseteq S \Rightarrow (x,y) \in S$ and $(y,z) \in T$

$\Rightarrow (x,z) \in T \circ S$.

$\Rightarrow T \circ R \subseteq T \circ S$.

let $(x,z) \in R \circ T$

$\Rightarrow \exists y \in A \Rightarrow (x,y) \in T$ and $(y,z) \in R$

$\because R \subseteq S \Rightarrow (x,y) \in T$ and $(y,z) \in S$

$\Rightarrow (x,z) \in S \circ T$

$\Rightarrow R \circ T \subseteq S \circ T$.

⑤ $(S \circ R) \cap T = \emptyset$, (then) iff $(T \circ R^{-1}) \cap S = \emptyset$

Proof:- assume that $(T \circ R^{-1}) \cap S = \emptyset$

if $(S \circ R) \cap T \neq \emptyset$

let $(x,z) \in (S \circ R) \cap T$

$\Rightarrow (x,z) \in (S \circ R)$ and $(x,z) \in T$

$\Rightarrow \exists y \in A \ni (x,y) \in R$ and $(y,z) \in S$ and $(x,z) \in T$

$\Rightarrow \exists y \in A \ni (y,x) \in R^{-1}$ and $(y,z) \in S$ and $(x,z) \in T$

$\Rightarrow \exists y \in A \ni (y,x) \in R^{-1}$ and $(x,z) \in T$ and $(y,z) \in S$

$\Rightarrow (y,z) \in T \circ R^{-1}$ and $(y,z) \in S$

$\Rightarrow (y,z) \in (T \circ R^{-1}) \cap S$

$\Rightarrow (T \circ R^{-1}) \cap S \neq \emptyset$ and that C! $\therefore (S \circ R) \cap T = \emptyset$

$\Leftarrow$ suppose that $(S \circ R) \cap T = \emptyset$

if $(T \circ R^{-1}) \cap S \neq \emptyset$

let $(x,z) \in (T \circ R^{-1}) \cap S$

$\Rightarrow (x,z) \in T \circ R^{-1}$ and $(x,z) \in S$

$\Rightarrow \exists y \in A \ni (x,y) \in R^{-1}$ and $(y,z) \in T$ and $(x,z) \in S$

$\Rightarrow \exists y \in A \ni (y,x) \in R$ and $(y,z) \in T$ and $(x,z) \in S$

$\Rightarrow (y,x) \in R$ and $(x,z) \in S$ and $(y,z) \in T$

$\Rightarrow (y,z) \in (S \circ R)$ and $(y,z) \in T$

$\Rightarrow (y,z) \in (S \circ R) \cap T \neq \emptyset$ C!

$\therefore (T \circ R^{-1}) \cap S = \emptyset.$

6 $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$ and $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$. H.W

Proof:- let $(x,z) \in (S \circ R)^{-1}$

$\Rightarrow (z,x) \in (S \circ R)$

$\Rightarrow \exists y \in A \ni (z,y) \in R$ and $(y,x) \in S$

$\Rightarrow \exists y \in A \ni (z,y) \in R$ and $(x,y) \in S^{-1}$

$\Rightarrow (x,y) \in S^{-1}$ and $(y,z) \in R^{-1}$

$\Rightarrow (x,z) \in R^{-1} \circ S^{-1}$ $\quad \therefore (S \circ R)^{-1} \subseteq R^{-1} \circ S^{-1}$ —— ①

Now, let $(x,z) \in R^{-1} \circ S^{-1}$

$\Rightarrow \exists y \in A$ s.t $(x,y) \in S^{-1}$ and $(y,z) \in R^{-1}$

$\Rightarrow \exists y \in A$ s.t $(z,y) \in R$ and $(y,x) \in S$

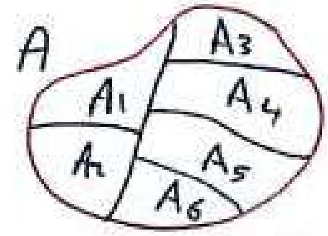
$\Rightarrow (z,x) \in S \circ R$

$\Rightarrow (x,z) \in (S \circ R)^{-1}$

$\therefore (R^{-1} \circ S^{-1}) \subseteq (S \circ R)^{-1}$ —— ②

from ① and ② we get $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$.

Def. :- Let A be a set by a partition (تجزئة) of A we mean a family $\{A_i\}_{i\in I}$ of nonempty subset of A with the following properties.



① $\forall i,j \in I$, $A_i \cap A_j = \emptyset$ or $A_i = A_j$

② $A = \bigcup_{i\in I} A_i$

$\{A_1, A_2, A_3, A_4, A_5, A_6\}$ is a partition of A.

Ex. :- Z_e, Z_o

① $Z_e \cap Z_o = \emptyset$

② $Z_e \cup Z_o = Z \Rightarrow Z_e, Z_o$ is partition of Z.

Ex. :- $A = \{a, b, c, d, e\}$

$A_1 = \{a, b\}$, $A_2 = \{c, d\}$, $A_3 = \{e\}$

$\Rightarrow A_1, A_2, A_3$ is a partition of A.

Ex. :- let $A = \{0, 5, 7, 9\}$

$P_1 = \{\{0, 5, 7\}, \{9\}\}$ is partition of A.

$P_2 = \{\{0, 9\}, \{7\}\}$ is not Partition of A.

(لعدم وجود الـ 5)

Ex.:- let $A = Z$ and ~~and~~ $P = \{Z_o, Z_e\}$ Partition of A.

Find A/R (quotient set of A by R).

بوجد صفوف التكافؤ

Sol.:- defined a relation $R = \{(a,b) : a - b = 2k, k \in Z\}$

1- R is equivalence relation

Q:- Let $R = \{(a,b) \in Z \times Z : a - b = 2k, k \in Z\}$ Find A/R.

$[0] = \{x \in Z, \exists k \in Z, x - 0 = 2k\}$

$= \{ ----, -4, -2, 0, 2, 4, ---\} = Z_e$

$[1] = \{x \in Z, \exists k \in Z, x - 1 = 2k\}$

$= \{x \in Z, \exists k \in Z, x = 2k + 1\}$

$= \{ ----, -3, -1, 1, 3, 5, ----\} = Z_o$

Note: A/R is the set of all equivalence classes of R in A.

$A/R = \{[0], [1]\} = \{Z_e, Z_o\}$.

* Functions * or Mappings

Defintion:- A function or (mapping) f from A to B ($f : A \longrightarrow B$) is a set of ordered pairs such that no two distinct Pairs have the some first component. Thus

$(x, y_1) \in f$ and $(x, y_2) \in f$ implies $y_1 = y_2$

$D_f = \{x : (x, y) \in f$ for some $y\}$

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$R_f = \{y : (x, y) \in f$ for some $x\}$