Ex.:- Solve 1. $3(2+2i) - 2(2-3i) + (6+8i)$

2. If $z = 3+i$, find z^{-1}.

3. If $(3+4i)^2 - 2(x-iy) = x+iy$, find x and y.

Solv.:- 1. $6+6i-4+6i+6+8i = 8+20i$.

2. $z^{-1} = \dfrac{1}{3+i} \times \dfrac{3-i}{3-i}$

$= \dfrac{3-i}{9+1} = \dfrac{3}{10} - \dfrac{i}{10}$.

3. $(9+24i-16) - 2x + 2iy = x+iy$

بالمطابقة $-7-2x = x \Rightarrow -7 = 3x \Rightarrow x = \frac{-7}{3}$.

$24 + 2y = y \Rightarrow y = -24$.

H.W

1. let $z_1 = (a_1, b_1)$, $z_2 = (a_2, b_2)$, Find $z_1 z_2^{-1}$

2. $z_1 = 3+2i$, $z_2 = 6+8i$, find

$z_1 + z_2$, $z_1 z_2$.

3. $2z_1 + \frac{1}{3} z_2$.

Q) Find $i^{16}, i^{58}, i^{93}, i^{-13}$

بصورة عامة

$$i^{4n+r} = i^{r}$$

$i^{16} = i^{4(4)+0} = i^{0} = 1.$

$i^{58} = i^{4(14)+2} = i^{2} = -1.$

$i^{93} = i^{4(23)+1} = i.$

تقسم $\frac{93}{4}$

$i^{-13} = \frac{1}{i^{13}} = \frac{i^{4(4)+0}}{i^{13}} = \frac{i^{16}}{i^{13}} = i^{3} = -i$

$\frac{1}{i^{13}} = \frac{1}{i^{4(3)+1}} = \frac{1}{i} \cdot \frac{-i}{-i}$

$\frac{-i}{1} = -i$

Note:- $\sqrt{-a} = \sqrt{a}\sqrt{-1} = \sqrt{a}\, i, \ \forall a \geq 0.$

Ex.:- $Z = -5 = -5 + 0i.$

$Z = \sqrt{-100} = \sqrt{100}\, i = 0 + 10i.$

$Z = -1 - \sqrt{-3} = -1 - \sqrt{3}\, i$

$Z = \frac{1 + \sqrt{-16}}{4} = \frac{1 + 4i}{4} = \frac{1}{4} + i.$

Q) write the number $\frac{3-2i}{5+i}$ by format $a+ib$

$$\frac{3-2i}{5+i} = \frac{3-2i}{5+i} \cdot \frac{5-i}{5-i}$$

$$= \frac{13-13i}{26}$$

$$= \frac{13}{26} - \frac{13}{26} i$$

$$= \frac{1}{2} - \frac{1}{2} i.$$

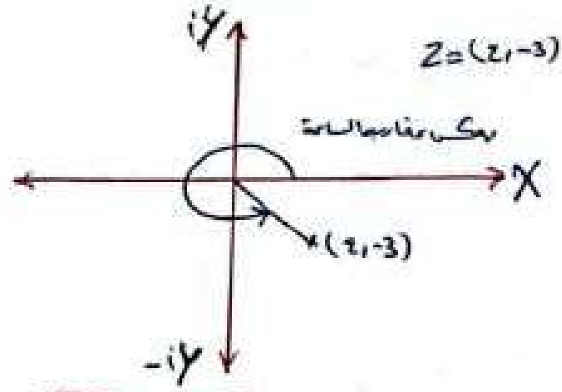
The Geometrical Representation for the complex number

التمثيل الهندسي

1. by the point $p(x,y)$ in xy plane.

2. by the vector $\vec{op}$ from $(0,0)$ to $p(x,y)$.

أي أن الأعداد الحقيقية تمثل
كالخط الأعداد الآن الأعداد
المعقدة تمثل بنقطتين في
المستوي الديكارتي يمثل العلاقة
بين $\mathbb{C}$, R^2 .



Q) prove that

1. $Im(Z_1+Z_2)=Im(Z_1)+Im(Z_2)$.
2. $Re(Z_1+Z_2)=Re(Z_1)+Re(Z_2)$.
3. $Im(iZ)=Re(Z)$.
4. $Re(iZ)=-Im(Z)$.

Proof:- let $Z_1=(a_1,b_1)=a_1+ib_1$, $Z_2=(a_2,b_2)=a_2+ib_2$.

$Re(Z_1)=a_1$, $Re(Z_2)=a_2$, $Im(Z_1)=b_1$, $Im(Z_2)=b_2$.

$Z_1+Z_2=a_1+a_2+i(b_1+b_2)$;-

$Im(Z_1+Z_2)=b_1+b_2=Im(Z_1)+Im(Z_2)$

$Re(Z_1+Z_2)=a_1+a_2=Re(Z_1)+Re(Z_2)$.

$Z_1=a_1+ib_1$

$\Rightarrow iZ_1=ia_1-b_1$, $Im(iZ_1)=a_1=Re(Z_1)$.

$Re(iZ_1)=-b_1=-Im(Z_1)$.

المقياس

Def.:- If $z \in \mathbb{C}$ and $z = a+ib$, then the (modules) absulute of z is defined by, $|z| = \sqrt{a^2+b^2}$ and $|z| = |-z|$.

Ex.:- $z_1 = 3+4i$, $z_2 = -2i$, $z_3 = 1$.

$|z_1| = \sqrt{9+16} = 5$.

$|z_2| = \sqrt{0+4} = 2$.

$|z_3| = \sqrt{1+0} = 1$.

Theorem:- let $z \in \mathbb{C}$ and $|z| \geq 0$, then $|z| = 0$ iff $z = 0$.

proof:- let $z = a+ib$ and suppose that $|z| = 0$

$\because |z| = \sqrt{a^2+b^2} = 0$ and is true when $a = 0 = b \Rightarrow z = 0$.

$\Leftarrow$ assume that $z = 0$, $z = (0,0) \Rightarrow a = 0 = b$ $\therefore |z| = 0$.

$\sqrt{0^2+0^2} = 0$

Ex.:- let $z = a+ib$, if $a = 3b$ and $|z| = \sqrt{10}$, then find a, b.

$$|z| = \sqrt{(3b)^2+b^2} = \sqrt{10} \Rightarrow \sqrt{9b^2+b^2} = \sqrt{10}$$

$$\Rightarrow \sqrt{10b^2} = \sqrt{10}$$

$$\Rightarrow \sqrt{10}\sqrt{b^2} = \sqrt{10} \Rightarrow b = 1$$

$$\therefore a = 3b\cancel{1} \Rightarrow a = 3$$

$$\therefore z = 3+i$$

proposition:- If $z_1, z_2 \in \mathbb{C}$, then مهمة

1. $|z_1 z_2| = |z_1||z_2|$.

2. $|z_1 + z_2| \leq |z_1| + |z_2|$. (schwarts inquality)

3. $|z_1| - |z_2| \leq |z_1 - z_2|$.

Proof(1):- let $z_1 = a_1 + ib_1$, $z_2 = a_2 + ib_2$

$$|z_1 z_2| = |(a_1+ib_1)(a_2+ib_2)| = |a_1a_2 - b_1b_2 + i(a_1b_2 + a_2b_1)|$$

$$= |(a_1a_2 - b_1b_2), (a_1b_2 + a_2b_1)|$$

$$= \sqrt{(a_1a_2 - b_1b_2)^2 + (a_1b_2 + a_2b_1)^2}$$ بتربيع الطرفين

$$|z_1 z_2|^2 = (a_1a_2 - b_1b_2)^2 + (a_1b_2 + a_2b_1)^2$$

$$= a_1^2a_2^2 - 2a_1a_2b_1b_2 + b_1^2b_2^2 + a_1^2b_2^2 + 2a_1b_2a_2b_1 + a_2^2b_1^2$$

$$= a_1^2(a_2^2 + b_2^2) + b_1^2(a_2^2 + b_2^2)$$

$$= (a_1^2 + b_1^2)(a_2^2 + b_2^2)$$ بأخذ الجذر التربيعي للطرفين

$$|z_1 z_2| = \sqrt{(a_1^2 + b_1^2)(a_2^2 + b_2^2)}$$

$$= \sqrt{(a_1^2 + b_1^2)}\sqrt{(a_2^2 + b_2^2)}$$

$$= |z_1||z_2|$$

$$\therefore |z_1 z_2| = |z_1||z_2|$$