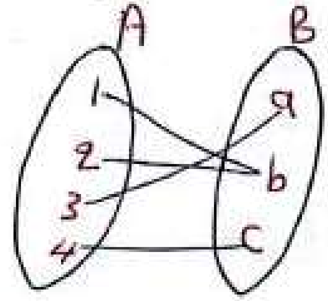
Ex.:- $f = \{(-1,0), (0,0), (1,2), (2,1)\}$ هذه علاقة

$D_f = \{-1, 0, 1, 2\}$, $R_f = \{0, 1, 2\}$

Ex.:- let $A = \{1,2,3,4\}$, $B = \{a, b, c\}$

$f = \{(1,b), (2,b), (3,a), (4,c)\}$



$\therefore f$ is a function

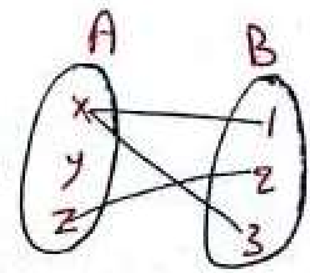
Ex.:- Let $A = \{x, y, z\}$, $B = \{1,2,3\}$

$f = \{(x,1), (x,3), (z,2)\}$ f ليس دالة

لان ① x لها صورتين

② y ليس لها صورة

$\therefore f$ is not function



Definition:- A function (or mapping) $f : A \to \boxed{B}$ is said to be surjective (onto) if it has the following property:-

$$\forall y \in B, \exists x \in A \text{ s.t } y = f(x)$$

y صورة لـ x

or $\quad Im f = B$

مجموعة الصور

هو المجال المقابل

Ex.:- $f: Z \longrightarrow N$ with $f(x) = x+3$ (29) $y = x+3$

$y = 0+3 = 3$

$y = 1+3 = 4$

$y = -2+3 = 1 \in N$

شاملة لان المجال المقابل هو المدى $= N$

f is a function for all $x \in \{x : x \geq -2\}$

Def.:- A function $f: A \to B$ is said to be injective (one-to-one) if it has the following property:-

$\forall x_1, x_2 \in A$ if $f(x_1) = f(x_2)$, then $x_1 = x_2$ دائماً نستخدم بالبراهين

or $\forall x_1, x_2 \in A$ if $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$. هذه بالأمثلة

Def.:- A function $f: A \to B$ is said to be bijective متقابلة if it is both surjective and injective.

Example:-

$3 \in N$

$\sqrt{3} \notin N$

1- Let $f: N \longrightarrow N$ such that $f(x) = x^2$, $\forall x \in N$

f is not onto $Im f \neq N$.

$y = x^2$

$\Rightarrow x = \mp\sqrt{y}$

f is one to one, $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$.

f is not bijective function.

2- Let $f: R \to R$ such that $f(x) = 2x^3 + 7$, $\forall x \in R$

f is onto $Im f = R$.

$y = 2x^3 + 7$

$y - 7 = 2x^3$

$x^3 = \frac{y-7}{2}$

$x = \sqrt[3]{\frac{y-7}{2}}$

f is one to one, $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$.

$1 \neq -1 \Rightarrow f(1) = 9 \neq f(-1)$

f is bijective function.

3- Let $f : \mathbb{R} \longrightarrow [-1,1]$ such that $f(x) = \cos x, \forall x \in \mathbb{R}$.

f is onto $\operatorname{Im} f = [-1,1]$.

f is not one to one.

f is not bijective function. $-1 \neq 1 \Rightarrow f(-1) = f(1)$

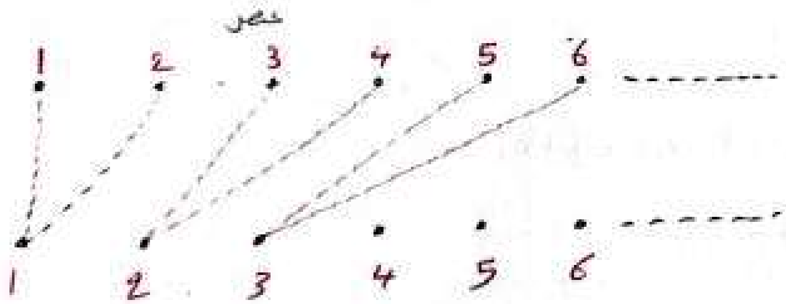
4. Let $f : \mathbb{N} \rightarrow \mathbb{N}$, $f(n) = 2n$, $\forall n \in \mathbb{N}$.

f is not onto.

f is one to one.

f is not bijective.

5- $f : \mathbb{N} \rightarrow \mathbb{N}$, $f(n) = \begin{cases} \dfrac{n+1}{2} & \text{if } n \text{ odd} \\ \dfrac{n}{2} & \text{if } n \text{ even} \end{cases}$



f is onto.

f is not one to one. 3 العنصر $\neq$ العنصر 4

لكن الصور

المتساوية

الي هي 2

f is not bijective.

Th. Let S and T are sets and $f: S\times T \to T\times S$, (30)

then f is bijective function.

Proof:- $f(s,t) = (t,s)$, $\forall\ s,t,(s,t) \in S\times T$

first we prove that f is well defined function:

let $(s_1,t_1),(s_2,t_2) \in S\times T$ such that

$$(s_1,t_1) = (s_2,t_2) \Rightarrow s_1 = s_2 \ , t_1 = t_2$$
$$\Rightarrow (t_1,s_1) = (t_2,s_2)$$
$$\Rightarrow f(s_1,t_1) = f(s_2,t_2)$$

$\therefore f$ is well-defined function.

to prove f is one to one

let $f(s_1,t_1) = f(s_2,t_2)$, $\forall\ (s_1,t_1),(s_2,t_2) \in S\times T$

$\Rightarrow (t_1,s_1) = (t_2,s_2)$.

$\Rightarrow t_1 = t_2 \ , s_1 = s_2$

$\Rightarrow (s_1,t_1) = (s_2,t_2)$

$\therefore f$ is 1-1.

$\forall\ (t,s) \in T\times S\ \exists (s,t) \in S\times T$ s.t $f(s,t) = (t,s)$

$\Rightarrow f$ is onto.

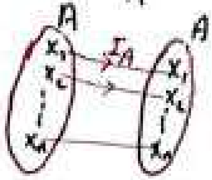
$\Rightarrow f$ is bijective.

هل ممكن ان تكتب الدالة
بصورة مثال على ان
تكون دالة onto
او 1-1 او
bijective

ارسم
اكتب

الدالة المحايدة

Ex.:- ① Identity function $I_A : A \to A$, $I_A(x) = x$, $\forall x \in A$.

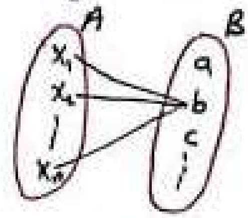


f is onto.

f is one to one.

الدالة الثابتة

② Constant function $K_b : A \to B$, $K_b(x) = b$, $\forall x \in A$.



f is not onto.

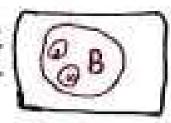
f is not one to one.

دالة الاحتواء

③ Inclusion function $B \subseteq A$, $E_B : B \to A$, $E_B(x) = x$, $\forall x \in B$.

f is not onto.

f is one to one. يروح ويرجع ياخذ صورة نفسو



Composition of functions تركيب الدوال

Def.:- Let $f : A \to B$ be a function and $g : B \to C$ be a function by the composition of A into C we mean $g \circ f : A \to C$ is a function

$$(g \circ f)(x) = g(f(x)) \ , \forall x \in A.$$