SLEEPING BRAIN, NOT AT REST

Regions of the brain that have spent the day learning sleep more heavily at night.

In a study published in the journal *Nature*, Giulio Tononi, a psychiatrist at the University of Wisconsin–Madison, had subjects perform a simple point-and-click task with a computer adjusted so that its cursor didn't track in the right direction. Afterward, the subjects' brain waves were recorded while they slept, then examined for "slow wave" activity, a kind of deep sleep.

Compared with people who'd completed the same task with normal cursors, Tononi's subjects showed elevated slow wave activity in brain areas associated with spatial orientation, indicating that their brains were adjusting to the day's learning by making cellular-level changes. In the morning, Tononi's subjects performed their tasks better than they had before going to sleep.

—*Richard A. Love*

Data Projects

Use a significance level of 0.05 for all tests below.

1. **Business and Finance** Use the data collected in data project 1 of Chapter 2 to complete this problem. Test the claim that the mean earnings per share for Dow Jones stocks are greater than for NASDAQ stocks.
2. **Sports and Leisure** Use the data collected in data project 2 of Chapter 7 regarding home runs for this problem. Test the claim that the mean number of home runs hit by the American League sluggers is the same as the mean for the National League.
3. **Technology** Use the cell phone data collected for data project 2 in Chapter 8 to complete this problem. Test the claim that the mean length for outgoing calls is the same as that for incoming calls. Test the claim that the standard deviation for outgoing calls is more than that for incoming calls.
4. **Health and Wellness** Use the data regarding BMI that were collected in data project 6 of Chapter 7 to complete this problem. Test the claim that the mean BMI for males is the same as that for females. Test the claim that the standard deviation for males is the same as that for females.
5. **Politics and Economics** Use data from the last Presidential election to categorize the 50 states as "red" or "blue" based on who was supported for President in that state, the Democratic or Republican candidate. Use the data collected in data project 5 of Chapter 2 regarding income. Test the claim that the mean incomes for red states and blue states are equal.
6. **Your Class** Use the data collected in data project 6 of Chapter 2 regarding heart rates. Test the claim that the heart rates after exercise are more variable than the heart rates before exercise.

Answers to Applying the Concepts

Section 9–1 Home Runs

1. The population is all home runs hit by major league baseball players.
2. A cluster sample was used.
3. Answers will vary. While this sample is not representative of all major league baseball players per se, it does allow us to compare the leaders in each league.
4. H_0: $\mu_1 = \mu_2$ and H_1: $\mu_1 \neq \mu_2$
5. Answers will vary. Possible answers include the 0.05 and 0.01 significance levels.
6. We will use the z test for the difference in means.
7. Our test statistic is $z = \dfrac{44.75 - 42.88}{\sqrt{\dfrac{8.8^2}{40} + \dfrac{7.8^2}{40}}} = 1.01$, and our P-value is 0.3124.
8. We fail to reject the null hypothesis.

9. There is not enough evidence to conclude that there is a difference in the number of home runs hit by National League versus American League baseball players.

10. Answers will vary. One possible answer is that since we do not have a random sample of data from each league, we cannot answer the original question asked.

11. Answers will vary. One possible answer is that we could get a random sample of data from each league from a recent season.

Section 9–2 Too Long on the Telephone

1. These samples are independent.

2. We compare the P-value of 0.06317 to the significance level to check if the null hypothesis should be rejected.

3. The P-value of 0.06317 also gives the probability of a type I error.

4. Since two critical values are shown, we know that a two-tailed test was done.

5. Since the P-value of 0.06317 is greater than the significance value of 0.05, we fail to reject the null hypothesis and find that we do not have enough evidence to conclude that there is a difference in the lengths of telephone calls made by employees in the two divisions of the company.

6. If the significance level had been 0.10, we would have rejected the null hypothesis, since the P-value would have been less than the significance level.

Section 9–3 Air Quality

1. The purpose of the study is to determine if the air quality in the United States has changed over the past 2 years.

2. These are dependent samples, since we have two readings from each of 10 metropolitan areas.

3. The hypotheses we will test are H_0: $\mu_D = 0$ and H_1: $\mu_D \neq 0$.

4. We will use the 0.05 significance level and critical values of $t = \pm 2.262$.

5. We will use the t test for dependent samples.

6. There are $10 - 1 = 9$ degrees of freedom.

7. Our test statistic is $t = \dfrac{-6.7 - 0}{11.27/\sqrt{10}} = -1.879$. We fail to reject the null hypothesis and find that there is not enough evidence to conclude that the air quality in the United States has changed over the past 2 years.

8. No, we could not use an independent means test since we have two readings from each metropolitan area.

9. Answers will vary. One possible answer is that there are other measures of air quality that we could have examined to answer the question.

Section 9–4 Smoking and Education

1. Our hypotheses are H_0: $p_1 = p_2$ and H_1: $p_1 \neq p_2$.

2. At the 0.05 significance level, our critical values are $z = \pm 1.96$.

3. We will use the z test for the difference between proportions.

4. To complete the statistical test, we would need the sample sizes.

5. Knowing the sample sizes were 1000, we can now complete the test.

6. Our test statistic is $z = \dfrac{0.323 - 0.145}{\sqrt{(0.234)(0.766)\left(\dfrac{1}{1000} + \dfrac{1}{1000}\right)}} =$ 9.40, and our P-value is very close to zero. We reject the null hypothesis and find that there is enough evidence to conclude that there is a difference in the proportions of high school graduates and college graduates who smoke.

Section 9–5 Variability and Automatic Transmissions

1. The null hypothesis is that the variances are the same: H_0: $\sigma_1^2 = \sigma_2^2$ (H_1: $\sigma_1^2 \neq \sigma_2^2$).

2. We will use an F test.

3. The value of the test statistic is $F = \dfrac{s_1^2}{s_2^2} = \dfrac{514.8^2}{77.7^2} = 43.92$, and the P-value is 0.0008. There is a significant difference in the variability of the prices between the two countries.

4. Small sample sizes are highly impacted by outliers.

5. The degrees of freedom for the numerator and denominator are both 5.

6. Yes, two sets of data can center on the same mean but have very different standard deviations.

Hypothesis-Testing Summary 1

1. Comparison of a sample mean with a specific population mean.

Example: H_0: $\mu = 100$

a. Use the z test when σ is known:

$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

b. Use the t test when σ is unknown:

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}} \quad \text{with d.f.} = n - 1$$

2. Comparison of a sample variance or standard deviation with a specific population variance or standard deviation.

Example: H_0: $\sigma^2 = 225$

Use the chi-square test:

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \quad \text{with d.f.} = n - 1$$

3. Comparison of two sample means.

Example: H_0: $\mu_1 = \mu_2$

a. Use the z test when the population variances are known:

$$z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

b. Use the t test for independent samples when the population variances are unknown and assume the sample variances are unequal:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

with d.f. = the smaller of $n_1 - 1$ or $n_2 - 1$.

Formula for the t test for comparing two means (independent samples, variances equal):

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$$

with d.f. $= n_1 + n_2 - 2$.

c. Use the t test for means for dependent samples:

Example: H_0: $\mu_D = 0$

$$t = \frac{\bar{D} - \mu_D}{s_D/\sqrt{n}} \quad \text{with d.f.} = n - 1$$

where n = number of pairs.

4. Comparison of a sample proportion with a specific population proportion.

Example: H_0: $p = 0.32$

Use the z test:

$$z = \frac{X - \mu}{\sigma} \quad \text{or} \quad z = \frac{\hat{p} - p}{\sqrt{pq/n}}$$

5. Comparison of two sample proportions.

Example: H_0: $p_1 = p_2$

Use the z test:

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\,\bar{q}\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$$

where

$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} \qquad \hat{p}_1 = \frac{X_1}{n_1}$$

$$\bar{q} = 1 - \bar{p} \qquad \hat{p}_2 = \frac{X_2}{n_2}$$

6. Comparison of two sample variances or standard deviations.

Example: H_0: $\sigma_1^2 = \sigma_2^2$

Use the F test:

$$F = \frac{s_1^2}{s_2^2}$$

where

s_1^2 = larger variance d.f.N. $= n_1 - 1$

s_2^2 = smaller variance d.f.D. $= n_2 - 1$