Ex(1):- Let $f: \mathbb{R} \to \mathbb{R}$, $f(x) = 3x^2+5$.

$g: \mathbb{R} \to \mathbb{R}$, $g(x) = x$.

$(g \circ f)(x) = g(f(x)) = g(3x^2+5) = 3x^2+5$.

$(f \circ g)(x) = f(g(x)) = f(x) = 3x^2+5$.

Ex(2):- Let $S = \{x, y, z\}$, $T = \{1, 2, 3\}$, $U = \{a, b, c\}$

$\alpha(x) = 2$, $\alpha(y) = 1$, $\alpha(z) = 3$.

$\beta(1) = b$, $\beta(2) = c$, $\beta(3) = a$.

$(\beta \circ \alpha)(x) = \beta(\alpha(x)) = \beta(2) = c$.

$(\beta \circ \alpha)(y) = \beta(\alpha(y)) = \beta(1) = b$.

$(\beta \circ \alpha)(z) = \beta(\alpha(z)) = \beta(3) = a$.

Ex(3):- give example show that $f \circ g \neq g \circ f$.

$f: \mathbb{Z} \to \mathbb{Z}$, $f(x) = x-1$.

$g: \mathbb{Z} \to \mathbb{Z}$, $g(x) = x^2$.

$(f \circ g)(x) = f(g(x)) = f(x^2) = x^2-1$.

$(g \circ f)(x) = g(f(x)) = g(x-1) = (x-1)^2$.

$\therefore f \circ g \neq g \circ f$.

Ex.(4):- $f: E \to Z$, $f(x) = \frac{x}{2}$.

$g: Z \to N$, $g(x) = x^2$.

find $(f \circ g)(x)$, $(g \circ f)(x)$.

$(f \circ g)(x) = f(g(x)) = f(x^2) = \frac{x^2}{2}$.

$(g \circ f)(x) = g(f(x)) = g\left(\frac{x}{2}\right) = \left(\frac{x}{2}\right)^2 = \frac{x^2}{4}$

Ex.(5):- Let $f: Z \to E$ given by $f(x) = 2x$.

and $g: E \to Z$ given by $g(u) = \frac{u}{2}$ $\forall u \in E$

$(g \circ f)(x) = g(f(x)) = g(2x) = \frac{2x}{2} = x$.

هذه اسميها الدالة الذاتية

لأن صورة الدالة تساوي العنصر نفسه

$\therefore (g \circ f)(x) = x = I_Z(x)$.

$(f \circ g)(u) = f(g(u)) = f\left(\frac{u}{2}\right) = 2\left(\frac{u}{2}\right) = u$.

$\therefore (f \circ g) = I_E$.

$\forall y \in$ المدى $, \exists x \in$ المجال $,$ s.t $f(x) = y$

Lemma:- Let $f: A \to B$ is a function, then $g: A \to f(A)$ is bijective function.

صور

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Proof:- Defined $g: A \to f(A)$ by $g(x) = (x, f(x))$

First we must prove that g is well defined

بعكس المتباينة

let $x_1, x_2 \in A$ s.t. $x_1 = x_2$

$\because f$ is function

$\Rightarrow f$ is well-defined $f(x_1) = f(x_2)$

$\Rightarrow (x_1, f(x_1)) = (x_2, f(x_2))$

$\Rightarrow g(x_1) = g(x_2)$ $\therefore g$ is well defined.

Ex.(4):- $f: E \to Z$, $f(x) = \frac{x}{2}$.

$g: Z \to N$, $g(x) = x^2$.

Find $(f \circ g)(x)$, $(g \circ f)(x)$.

$(f \circ g)(x) = f(g(x)) = f(x^2) = \frac{x^2}{2}$.

$(g \circ f)(x) = g(f(x)) = g\left(\frac{x}{2}\right) = \left(\frac{x}{2}\right)^2 = \frac{x^2}{4}$

Ex.(5):- Let $f: Z \to E$ given by $f(x) = 2x$.

and $g: E \to Z$ given by $g(u) = \frac{u}{2}$ $\forall u \in E$

$(g \circ f)(x) = g(f(x)) = g(2x) = \frac{2x}{2} = x$.

هذه اسميها الدالة الذاتية

$\therefore (g \circ f)(x) = x = I_Z(x)$.

لأن صورة الدالة تساوي العنصر نفسه

$(f \circ g)(u) = f(g(u)) = f\left(\frac{u}{2}\right) = 2\left(\frac{u}{2}\right) = u$.

$\therefore (f \circ g) = I_E$.

$\forall y \in$ المجال, $\exists x \in$ المنطلق, s.t $f(x) = y$

Lemma:- Let $f: A \to B$ is a function, then $g: A \longrightarrow f(A)$ is bijective function.

صور

الدالة هي عبارة عن ازواج مرتبة

Proof:- Defined $g: A \to f(A)$ by $g(x) = (x, f(x))$

First we must prove that g is well defined

بعكس المتباينة

let $x_1, x_2 \in A$ s.t. $x_1 = x_2$

$\because f$ is function

$\Rightarrow f$ is well-defined $f(x_1) = f(x_2)$

$\Rightarrow (x_1, f(x_1)) = (x_2, f(x_2))$

$\Rightarrow g(x_1) = g(x_2)$ $\therefore g$ is well defined.

To prove g is 1-1 (injective)

let $g(x_1) = g(x_2)$, $\forall x_1, x_2 \in A$

$\Rightarrow (x_1, f(x_1)) = (x_2, f(x_2))$

$\Rightarrow x_1 = x_2 \Rightarrow g$ is 1-1 (one to one).

Now

$\forall (x, f(x)) \in g$, $\exists x \in A$ s.t $g(x) = (x, f(x))$

$\therefore g$ is on to.

$\therefore g$ is bijective.

Theorem :- If $f: A \to B$ and $g: B \to C$ are bijective function then $g \circ f : A \to C$ is a bijective function.

well, onto, 1-1 معاً

Proof :- first we must prove that $g \circ f$ is well defined.

let $x_1 = x_2 \Rightarrow f(x_1) = f(x_2)$ (since f is function) f is well

$\Rightarrow g(f(x_1)) = g(f(x_2))$ (since g is function) g is well

$\Rightarrow g \circ f(x_1) = g \circ f(x_2)$

الاصل متساوية او الصور متساوية

$\because g$ is bijective $\Rightarrow g$ is 1-1 $\Rightarrow f(x_1) = f(x_2)$

$\because f$ is bijective $\Rightarrow f$ is 1-1 $\Rightarrow x_1 = x_2$

$\therefore g \circ f$ is 1-1.

صورة لـ y

$\because g$ is onto $\Rightarrow \forall z \in C$, $\exists y \in B$ s.t $(y, z) \in g$ or $z = g(y)$.

$\because f$ is onto $\Rightarrow \forall y \in B$, $\exists x \in A$ s.t $(x, y) \in f$ or $y = f(x)$.

تعرف الدالة بشكل ازواج مرتبة

$\therefore \forall z \in C$, $\exists x \in A$ s.t $(z, x) \in g \circ f$ or $z = (g \circ f)(x)$.

$\therefore g \circ f$ is onto.

$\therefore g \circ f$ is bijective.

Theorem*:- If $f: A \to B$ and $g: B \to C$ are a function and $g \circ f: A \to C$ is bijective then f is 1-1 and g is onto

Proof:-

let $x_1, x_2 \in A$ s.t $f(x_1) = f(x_2)$

$\because g$ is function $\Rightarrow g(y_1) = g(y_2)$, $y_1, y_2 \in B$

$\Rightarrow g(f(x_1)) = g(f(x_2))$

$\Rightarrow (g \circ f)(x_1) = (g \circ f)(x_2)$

$\because g \circ f$ is bijective

$\Rightarrow g \circ f$ is 1-1

$\Rightarrow x_1 = x_2$

$\therefore f$ is 1-1

$\because g \circ f$ is onto

$\Rightarrow \forall z \in C, \exists x \in A$ s.t $(x, z) \in g \circ f$ or $z = (g \circ f)(x)$

$\Rightarrow \exists y \in B$ s.t $y = f(x)$ or $(x, y) \in f$.

$\therefore (x, z) \in g \circ f$ and $(x, y) \in f$

$\Rightarrow (y, z) \in g$ $[(x, y) \in f$ and $(y, z) \in g \Rightarrow (x, z) \in g \circ f]$

$\Rightarrow z = g(y)$.

$\therefore \forall z \in C, \exists y \in B$ s.t $(g(y)) = z$.

$\therefore g$ is onto