$$w_k = \sqrt[n]{r}\left(\cos\frac{\theta+2\pi k}{n} + i\sin\frac{\theta+2\pi k}{n}\right)$$

$$w_0 = \sqrt[3]{3^3}\left(\cos\frac{90}{3} + i\sin\frac{90}{3}\right) \quad , k=0$$

$$= 3\left(\cos 30 + i\sin 30\right)$$

$$= 3\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)$$

$$w_1 = 3\left(\cos\frac{90+2\pi}{3} + i\sin\frac{90+2\pi}{3}\right)$$

$$= 3\left(\cos\frac{\cancel{3}(30+120)}{3} + i\sin\frac{\cancel{3}(30+120)}{\cancel{3}}\right)$$

$$= 3\left(\cos 150^\circ + i\sin 150^\circ\right)$$

$$= 3\left(\frac{-\sqrt{3}}{2} + \frac{1}{2}i\right)$$

$$= \frac{-3\sqrt{3}}{2} + \frac{3}{2}i.$$

$$w_2 = 3\left(\cos\frac{90+4\pi}{3} + i\sin\frac{90+4\pi}{3}\right)$$

$$= 3\left(\cos\frac{\cancel{3}(30+240)}{\cancel{3}} + i\sin\frac{3(30+240)}{\cancel{3}}\right)$$

$$= 3\left(\cos 270 + i\sin 270\right)$$

$$= 3\left(0 + (-1)i\right)$$

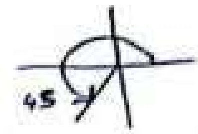
$$= 0 - 3i.$$

Ex :- Find $z^3 = \frac{1}{-2+2i}$

$$z^3 = \frac{1}{-2+2i} \times \frac{-2-2i}{-2-2i} = \frac{-2-2i}{8} = \frac{-2}{8} - \frac{2}{8}i = \frac{-1}{4} - \frac{1}{4}i .$$

$$r = \sqrt{\left(\frac{-1}{4}\right)^2 + \left(\frac{-1}{4}\right)^2} = \frac{1}{2\sqrt{2}} = \frac{1}{\sqrt{2}\sqrt{2}\sqrt{2}} = \left(\frac{1}{\sqrt{2}}\right)^3$$

$$\theta = \tan^{-1} \frac{-\frac{1}{4}}{-\frac{1}{4}} = 1$$



$$\theta = \pi + \frac{\pi}{4} = 225^\circ$$

$$W_k = \sqrt[n]{r} \left(\cos \frac{\theta + 2\pi k}{n} + i \sin \frac{\theta + 2\pi k}{n} \right) , \quad k = 0,1,2$$

$$W_0 = \sqrt[3]{\left(\frac{1}{\sqrt{3}}\right)^3} \left(\cos \frac{225}{3} + i \sin \frac{225}{3} \right)$$

$$= \frac{1}{\sqrt{3}} (\cos 75 + i \sin 75).$$

$$W_1 = \frac{1}{\sqrt{3}} \left(\cos \frac{225 + 2\pi}{3} + i \sin \frac{225 + 2\pi}{3} \right)$$

$$= \frac{1}{\sqrt{3}} \left(\cos \frac{\cancel{3}(75+120)}{\cancel{3}} + i \sin \frac{\cancel{3}(75+120)}{\cancel{3}} \right)$$

$$= \frac{1}{\sqrt{3}} (\cos 195 + i \sin 195).$$

$$W_2 = \frac{1}{\sqrt{3}} \left(\cos \frac{225 + 4\pi}{3} + i \sin \frac{225 + 4\pi}{3} \right)$$

$$= \frac{1}{\sqrt{3}} \left(\cos \frac{\cancel{3}(75+240)}{\cancel{3}} + i \sin \frac{\cancel{3}(75+240)}{\cancel{3}} \right)$$

$$= \frac{1}{\sqrt{3}} (\cos 315 + i \sin 315) .$$

المبرهنة الاساسية في الجبر

The Fundemental theorem of Algebra :-

let $p(z) = a_0 z^n + a_1 z^{n-1} + \cdots\cdots + a_n$ be a polynomial of degree n where $a_j \in \mathbb{C}$, $0 \le j \le n$ and $a_0 \ne 0$.

Then $p(z) = 0$ has n.th roots and these all roots is a complex numbers.

Ex.:- Solve the equation $\overset{a}{i} z^2 - \overset{b}{(2+2i)} z + \overset{c}{(2-i)} = 0$

$\Rightarrow ax^2 + bx + c = 0 \quad , \; z = a + ib$

هذه المعادلة من الدرجة الثانية نحلها بالدستور

$$z = \frac{(2+2i) \mp \sqrt{(2+2i)^2 - 4i(2-i)}}{2i} \qquad x = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{(2+2i) \mp \sqrt{8i - 4 - 8i}}{2i} = \frac{(2+2i) \mp 2i}{2i}$$

$$z_1 = \frac{2+2i+2i}{2i} = \frac{2(1+2i)}{2i} = \frac{1+2i}{i}$$

$$z_1 = \frac{1+2i}{i} \times \frac{-i}{-i} = \frac{i-2}{-i} = 2 - i$$

$$z_2 = \frac{2+2i-2i}{2i} = \frac{2}{2i} = \frac{1}{i}$$

$$z_2 = \frac{1}{i} \times \frac{-i}{-i}$$

$$= \frac{-i}{-i^2} = \frac{-i}{1} = -i$$

لاحظ $x^2 + 1 = 0$

$\Rightarrow a = 1 , b = 0 , c = 1$

$$x = \frac{-0 \mp \sqrt{0-4}}{2} = \frac{\mp\sqrt{-4}}{2} = \mp i$$

EX.:- Find the roots of $Z^2=-1$

$Z^2=-1+0i$, $a=-1$, $b=0$, $n=2$

$r=\sqrt{(-1)^2+0^2}=1$

$\theta=\tan^{-1}\frac{0}{1}=0 \Rightarrow \theta=\cancel{0}\ ,180$

$w_k=\sqrt[n]{r}\left(\cos\frac{\theta+2\pi k}{n}+i\sin\frac{\theta+2\pi k}{n}\right)$, $k=0,1$

$w_0=\sqrt[2]{1}\left(\cos\frac{180+2\pi(0)}{2}+i\sin\frac{180+2\pi(0)}{2}\right)$

$=\left(\cos\frac{180}{2}+i\sin\frac{180}{2}\right)$

$=\cos 90^\circ+i\sin 90^\circ=0+i=\underline{\underline{i}}$

$w_1=\left(\cos\frac{180+2(180)}{2}+i\sin\frac{180+2(180)}{2}\right)$

$=\left(\cos\frac{\overset{90}{\cancel{180}}(1+2)}{\cancel{2}_1}+i\sin\frac{\overset{90}{\cancel{180}}(1+2)}{\cancel{2}_1}\right)$

$=(\cos 270+i\sin 270)$

$=0+i(-1)=-i_{//}$

Ex.: Find the polar representation $Z=5i$

$a=0 \quad b=5$

$r=\sqrt{0^2+5^2}=5$

$\theta=\tan^{-1}\frac{b}{a}=\tan^{-1}\frac{5}{0} \Rightarrow \theta=90^\circ$

$\therefore Z=5(\cos 90+i\sin 90)$.

Ex.:- Find the solution of the equation $Z^3+1=0$.

Sol.:- $Z^3=-1 \Rightarrow a=-1,\ b=0,\ n=3$

$r=\sqrt{(-1)^2+0^2}=\sqrt{1}=1$

$\theta=\tan^{-1}\frac{b}{a}=\tan^{-1}(0)=180^\circ$

$W_k=\sqrt[n]{r}\left(\cos\frac{\theta+2\pi k}{n}+i\sin\frac{\theta+2\pi k}{n}\right) \quad k=0,1,2$

$W_0=1\left(\cos\frac{180+2\pi(0)}{3}+i\sin\frac{180+2\pi(0)}{3}\right)$

$=(\cos 60+i\sin 60)=\frac{1}{2}+\frac{\sqrt{3}}{2}i.$

$W_1=\left(\cos\frac{180+2\pi}{3}+i\sin\frac{180+2\pi}{3}\right)$

$=\left(\cos\frac{\cancel{3}(60+120)}{\cancel{3}}+i\sin\frac{\cancel{3}(60+120)}{\cancel{3}}\right)$

$=\cos 180+i\sin 180=-1+0=-1.$

$W_2=\left(\cos\frac{180+4\pi}{3}+i\sin\frac{180+4\pi}{3}\right)$

$=\left(\cos\frac{\cancel{3}(60+240)}{\cancel{3}}+i\sin\frac{\cancel{3}(60+240)}{\cancel{3}}\right)$

$=\cos 300+i\sin 300$

$=\frac{1}{2}-\frac{\sqrt{3}}{2}i$