

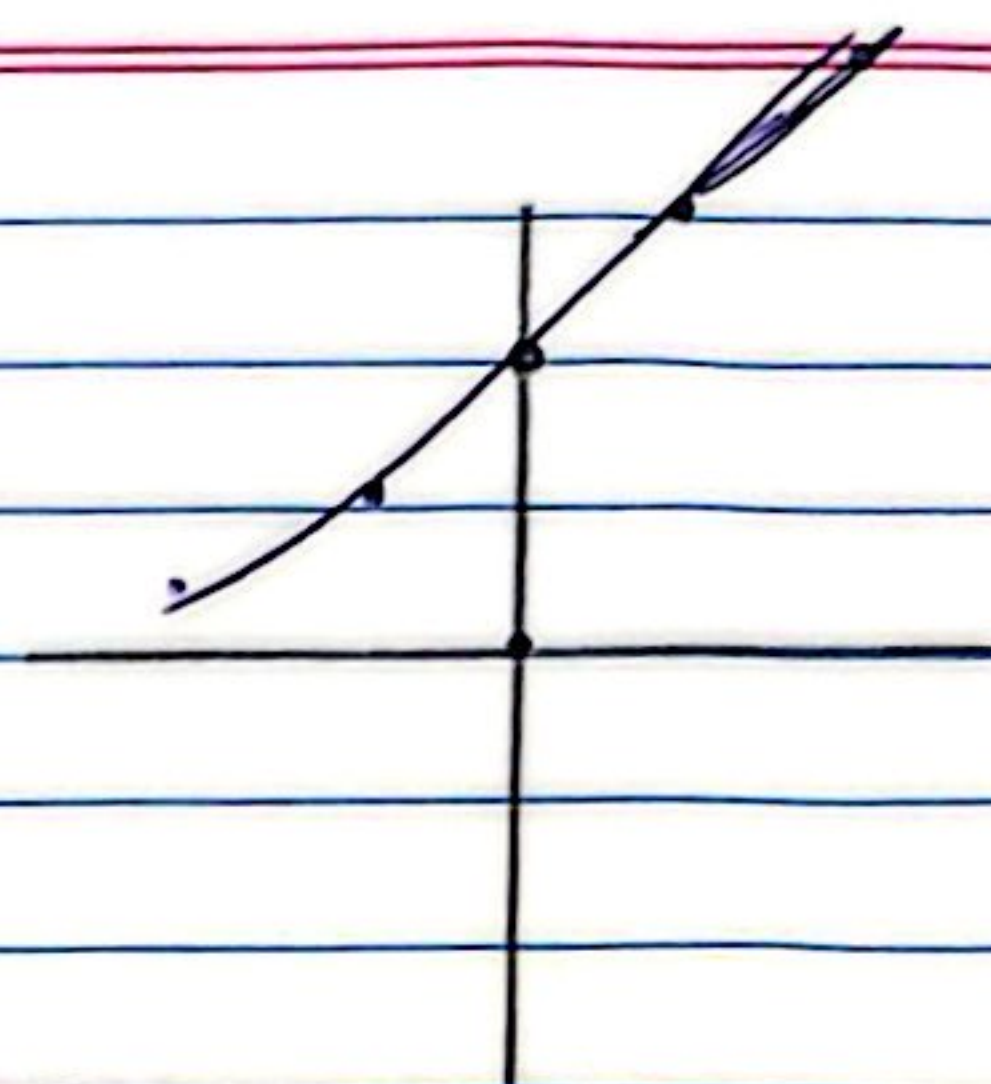
2. $y = x + 3$

$$D = \{x : x \in \mathbb{R}\}$$

حل $\Rightarrow x = y - 3$

$$R = \{y : y \in \mathbb{R}\} = (-\infty, \infty)$$

x	y
0	3
1	4
2	5
-1	2
-2	1



3. $y = x^2 + 3x + 4$

Solu $x^2 + 3x + 4 - y = 0$

$a = x^2$ معامل

$b = 3x$ معامل

$c = 4 - y$ الثابت

$$\Delta = b^2 - 4ac \geq 0$$

$$\Delta = 9 - 4(1)(4 - y) \geq 0$$

$$\therefore 9 - 4(4 - y) \geq 0$$

$$9 - 16 + 4y \geq 0$$

$$-7 + 4y \geq 0$$

$$4y \geq 7 \Rightarrow y \geq \frac{7}{4} = \text{حل}$$

4. $y = \frac{x}{x^2 + 2}$

Solu

$$x^2 y + 2y = x$$

$a = y$

$b = -1$

$c = 2y$

$$yx^2 - x + 2y = 0$$

$$\Delta = (-1)^2 - 4y(2y) \geq 0$$

$$1 - 8y^2 \geq 0$$

$$\therefore y \leq \pm \frac{1}{\sqrt{8}}$$

$$\therefore |y| \leq \frac{1}{2\sqrt{2}}$$

$$-\frac{1}{2\sqrt{2}} \leq y \leq \frac{1}{2\sqrt{2}}$$

$$\text{cs. 11} \Rightarrow R = \{y : y \in \mathbb{R}\} = \left[-\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}\right]$$

5. $y = \frac{2x-1}{x^2-9}$

Soln

$$yx^2 - 9y = 2x - 1$$

$$a = y$$

$$yx^2 - 2x + 1 - 9y = 0$$

$$b = -2$$

$$c = 1 - 9y$$

$$\Delta = (-2)^2 - 4y(1-9y) \geq 0$$

$$4 - 4y + 36y^2 \geq 0$$

$$4(1 - y + 9y^2) \geq 0$$

6. $f(x) = 4 + 5 \cos x = y$

cos دالة $(-1, 1)$

Solu

$$-1 \leq \cos x \leq 1$$

$\times 5$

$$-5 \leq 5 \cos x \leq 5$$

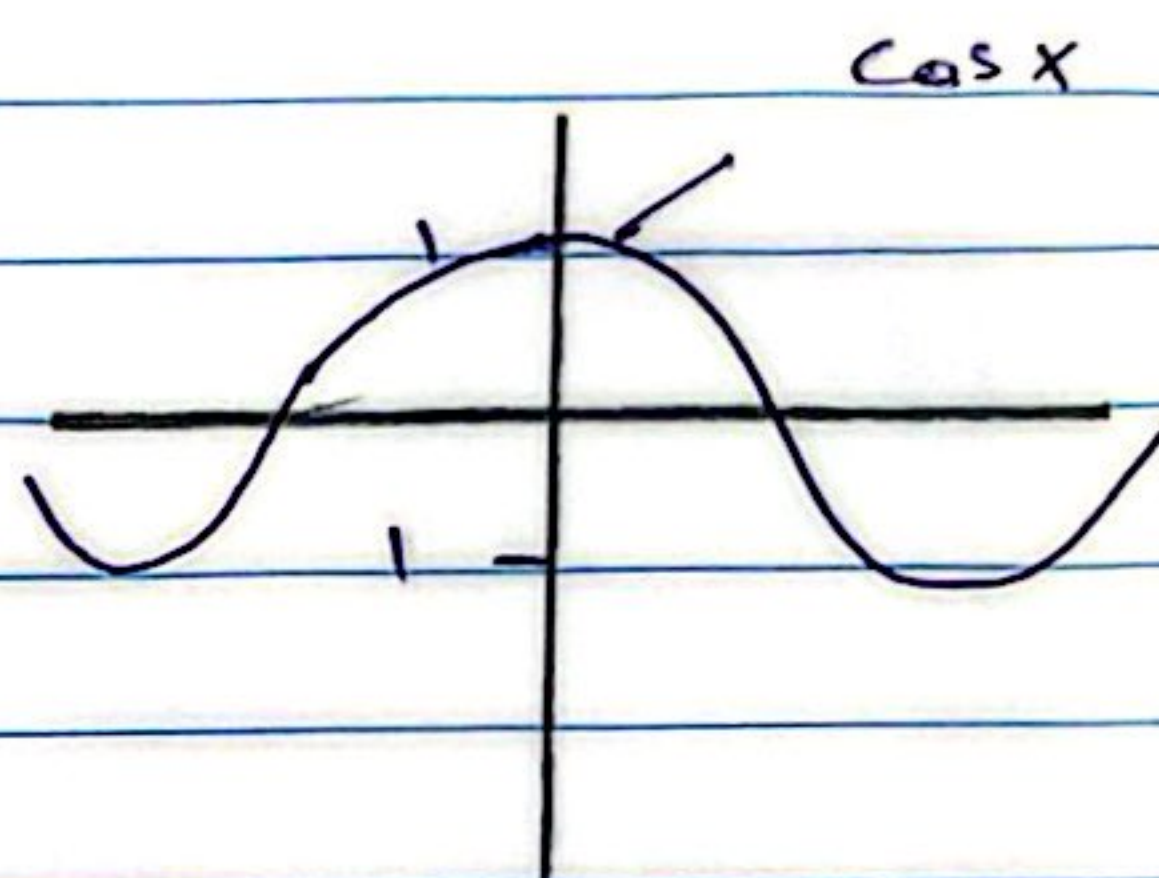
$+4$

$$-5 + 4 \leq 4 + 5 \cos x \leq 5 + 4$$

$$-1 \leq \underbrace{4 + 5 \cos x}_y \leq 9$$

$$-1 \leq y \leq 9$$

$$\text{Range} = [-1, 9]$$



7. $y = 1 + \cos x$; $x \in [0, \frac{\pi}{2}]$

Solu

$$0 \leq x \leq \frac{\pi}{2}$$

في $\cos$ المتناقص

$$\cos 0 \geq \cos x \geq \cos \frac{\pi}{2} = 0$$

لأنها متناقصة تقلب المتباينة

$$1 \geq \cos x \geq 0$$

$+1$

$$2 \geq 1 + \cos x \geq 1$$

$$2 \geq y \geq 1$$

$$\text{Range} \Rightarrow R = \{x : x \in R\} = [1, 2]$$

7. $y = -3 + \sin^2 x$

Solu

المجال لـ $\sin$ $(-1, 1)$

$$-1 \leq \sin x \leq 1$$

$$0 \leq \sin^2 x \leq 1$$

بتربيع الطرفين

$$-3 \leq -3 + \sin^2 x \leq 1 - 3 \quad \rightarrow -3$$

$$-3 \leq -3 + \sin^2 x \leq -2$$

$$-3 \leq y \leq -2$$

المجال $\Rightarrow R = \{x : x \in \mathbb{R}\} = [-3, -2]$

H.w.

$$f(x) = \frac{1}{\sqrt{\sin^2 x + 1}}$$

Solu

المجال لـ $\sin$

$$-1 \leq \sin x \leq 1$$

بتربيع الطرفين

$$0 \leq \sin^2 x \leq 1$$

إضافة 1

$$0 + 1 \leq \sin^2 x + 1 \leq 1 + 1$$

$$1 \leq \sin^2 x + 1 \leq 2$$

أخذ الجذر التربيعي

$$\sqrt{1} \leq \sqrt{\sin^2 x + 1} \leq \sqrt{2}$$

قلب المتباينة

$$1 \geq \frac{1}{\sqrt{\sin^2 x + 1}} \geq \frac{1}{\sqrt{2}}$$

$$1 \geq y \geq \frac{1}{\sqrt{2}}$$

المجال $\Rightarrow R = \{x : x \in \mathbb{R}\} = \left[\frac{1}{\sqrt{2}}, 1\right]$

[الكبير، الصغير]

H.W. Find the Range and Domain to the Figures

1.

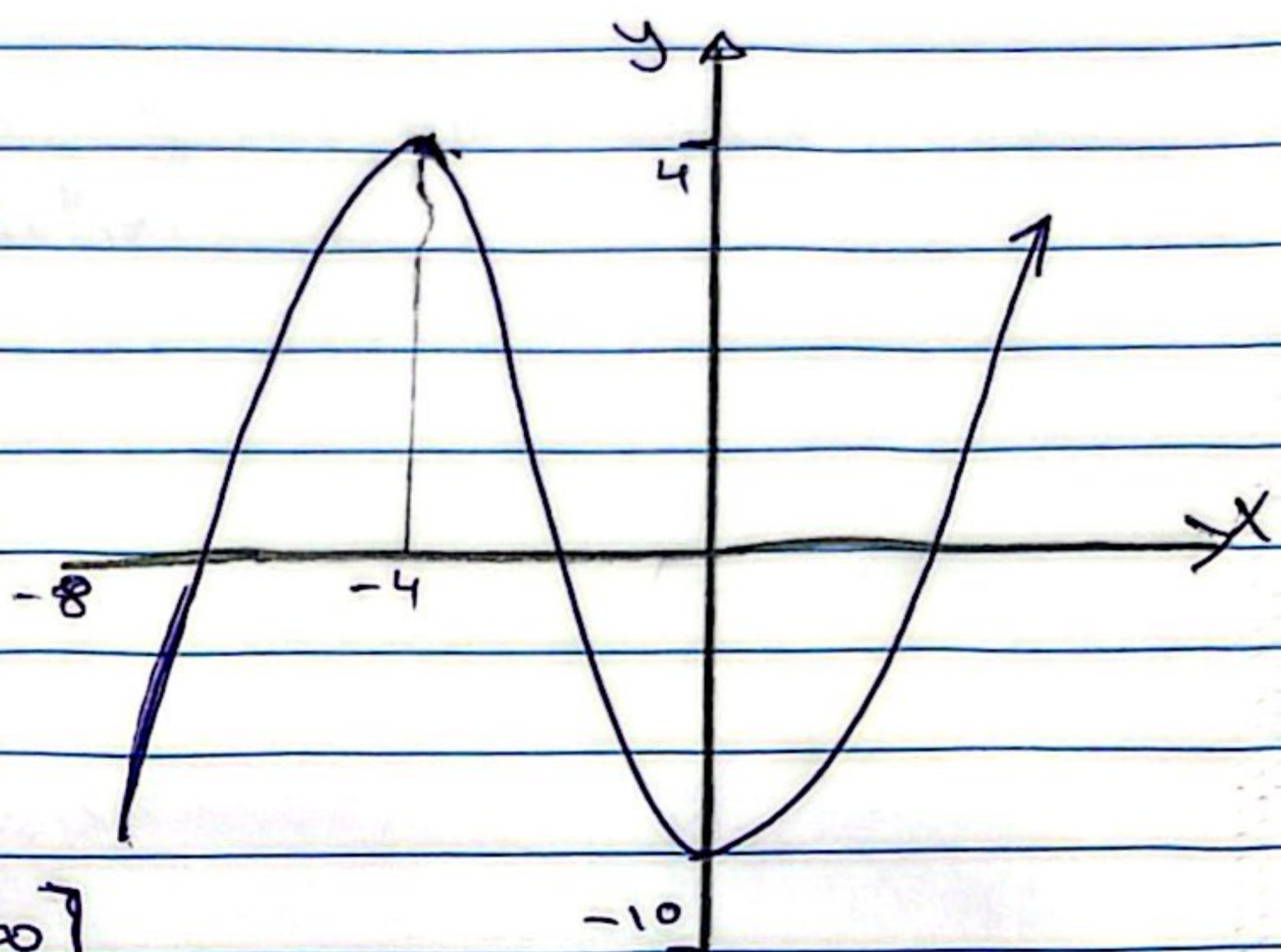
$$D = [-8, -4) \cup (-4, \infty)$$

or

$$D = \{x : x \in \mathbb{R}; x \neq -4\}$$

$$x \neq -4$$

$$R = \{y : y \in \mathbb{R}\} = [-10, \infty]$$



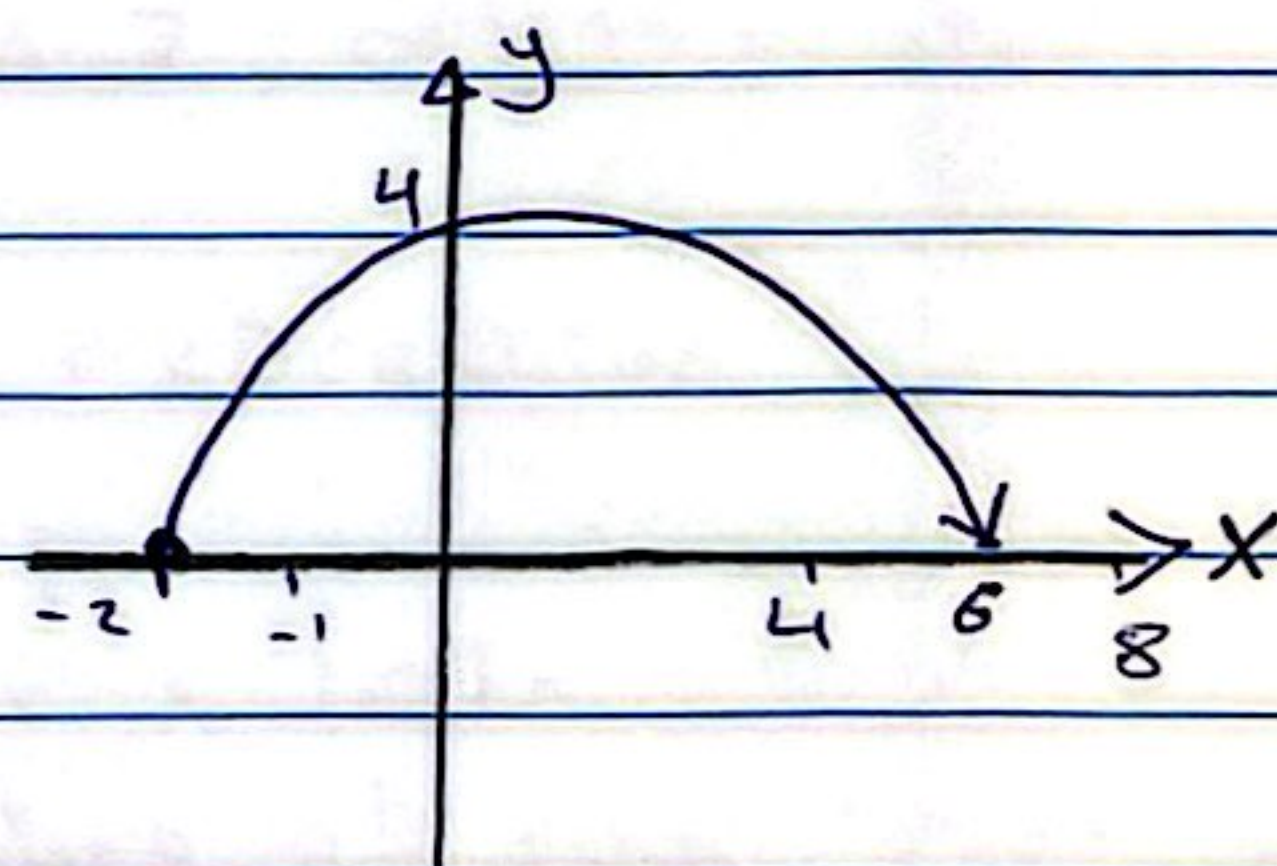
2.

$$D = \{x : x \in \mathbb{R}\} = [-2, 6]$$

or

$$D = [-2, 6)$$

$$R = \{y : y \in \mathbb{R}\} = [0, 4]$$



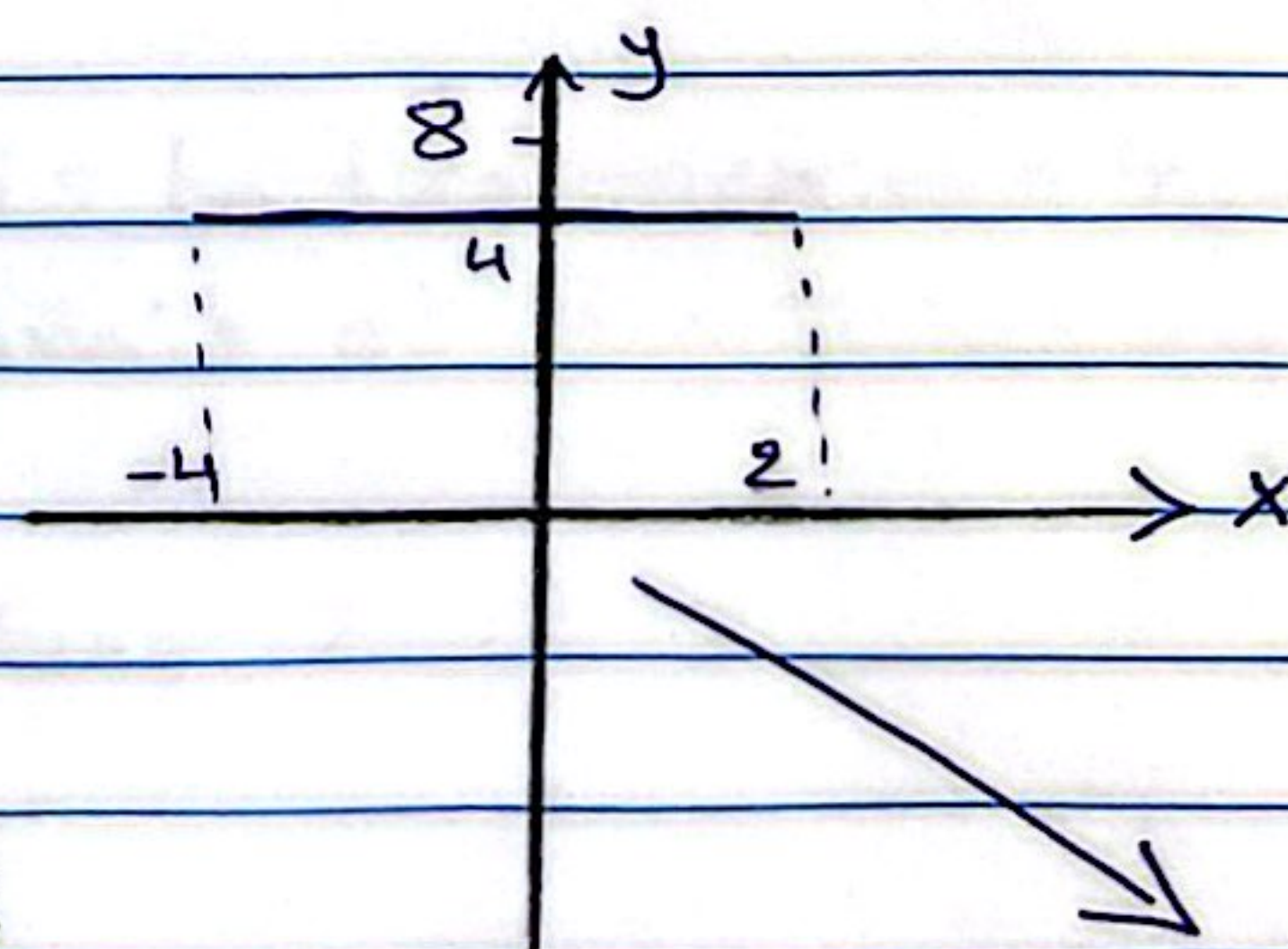
3.

$$D = \{x : x \in \mathbb{R}; x > -4; x \neq 2\}$$

$$\text{or } D = (-4, \infty) \cup (2, \infty)$$

$$R = \{y : y \in \mathbb{R}; y < -2; y = 6\}$$

$$\text{c.s.t.} = (-\infty, -2) \cup (6)$$



limits

To explain the limit of goals, we consider the following mathematical definition:-

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

let have the function

$$f(x) = x^2 - 1 \quad ; x \in \mathbb{R}$$

we want to know what happens to the value of the function $f(x)$, when we give values for x that are very close to 2 on both sides that is, we give values of x are a little less than 2, a little more than 2, and so on.

This is what is meant by the expression "very close to 2" from both sides.

When we substitute values for x into the function we get the result that the value of the function is very close to the number 3.

The closer the value of x is to the number 2, that is, we can say that the limit of the function:-

$$f(x) = x^2 - 1 \quad \text{in the number 2 is 3}$$

$$\text{That is : } f(2) = 2^2 - 1 = 3$$

The limits are divided into three sections:-

The limits are :-

1. The real number

The limit approaches L when x approaches to a :

That is:

$$\lim_{x \rightarrow a} f(x) = L$$

If the value of x approaches any real number, then the Substitution is made directly with the Same Value

This is: $f(x)_{x \rightarrow a} = L$

Ex 1. Find $\lim_{x \rightarrow 2} x^3 + 4x$

Solu $\lim_{x \rightarrow 2} x^3 + 4x$

$$= (2)^3 + 4(2) = 8 + 8 = 16$$

So that the limit of the function $\lim_{x \rightarrow 2} x^3 + 4x$ when x approaches 2 is 16

2. $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$

1. التعريف

Solu

$$= \frac{4 - 4}{2 - 2} = \frac{0}{0}$$

كيفية غير معرفة $\frac{0}{1} = 0$
 $\frac{1}{0} = \infty$

$$\therefore \lim_{x \rightarrow 2} \frac{(x-2)(x+2)}{x-2}$$

2. التبسيط

$$= \lim_{x \rightarrow 2} (x+2) = 4$$

$$3. \lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - x}$$

١. التعويض

$$\frac{1+1-2}{1-1} = \frac{0}{0}$$

٢. التبسيط

يحلل بالتجزئة

$$= \lim_{x \rightarrow 1} \frac{(x+2)(x-1)}{x(x-1)}$$

$$= \lim_{x \rightarrow 1} \frac{x+2}{x}$$

$$= \frac{3}{1} = 3$$

$$4. \lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2}$$

Solu

$$\frac{8-8}{2-2} = \frac{0}{0}$$

لا يجوز

$$\therefore = \lim_{x \rightarrow 2} \frac{(x-2)(x^2+2x+4)}{(x-2)}$$

$$= \lim_{x \rightarrow 2} (x^2 + 2x + 4)$$

$$= 2^2 + 2(2) + 4 = 12$$

$$5. \lim_{x \rightarrow -1} \frac{1}{x^2 + 2}$$

Solu

$$\lim_{x \rightarrow -1} \frac{1}{x^2 + 2} = \frac{1}{1+2} = \frac{1}{3}$$

So that the limit of the function $\lim_{x \rightarrow -1} \frac{1}{x^2 + 2}$ when x approaches -1 is $\frac{1}{3}$.