2DTransformations

Transformationmeanschangingsomegraphicsintosomethingelsebyapplyingrules.
There are various types of transformations such as translation, scaling up or down, rotation, shearing, etc. When a transformation takes place on a 2D plane, it is called 2D transformation.

Transformations play an important role in computer graphics to reposition the graphics on the screen and change their size or orientation.

FundamentalTransformation

Thereareseveralbasictransformations:

1- Translation(ShiftorMove).
2- Scaling.
3- Rotation.
4- Reflection.
5- Shear.

HomogenousCoordinates

To shorten theprocess of sequence of transformations, we have to use 3×3 transformation matrix instead of 2×2 transformation matrix. To convert a 2×2 matrix to 3×3 matrix, we have to add an extra dummy coordinate W.

In this way, we can represent the point by 3 numbers instead of 2 numbers, which is called**Homogenous Coordinate**system. In this system, we can represent all the transformation equations in matrix multiplication. Any Cartesian point P(X,Y)can be converted to homogenous coordinates by P\` (X_h, Y_h, h).

1- Translation

Considerapointp(x,y).wecantranslateditmeansshiftittonew position p\`(x\`, y\`)by adding tx and tyin y where Tx and Ty are translating factor. Mathematically this can be represented as:

$$x` = x + Tx$$
$$y` = y + Ty$$

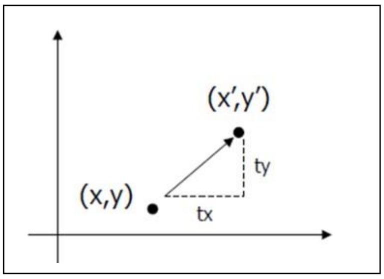
Notethat:Usingcoordinatesystemthetranslatingfactorare: If

Tx>0 then point moves to the right.

IfTx<0thenpointmovestotheleft. If

Ty>0 then point moves to the up.

IfTy<0thenpointmovestothedown.



Matrixrepresentation:Eachoftwodimensionaltransformationscanbe presented asa product of the row vector ($xy1$) and an 3*3 matrix.

$$(x'y'1)=(xy1)*\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ tx & ty & 1\end{bmatrix}$$

$$[x', \quad y', \quad 1]=[x+tx, \quad y+ty, \quad 1]$$

Ex.1: Translatethefollowingpoints:(5,10),(50,10),(30,20),as:

1- 10pointsto<u>right</u>,5points<u>up</u>,

2- 5 pointsto<u>left</u>, 7points<u>down</u>,

3- 2 pointsto<u>right</u>,0points<u>down</u>,

Sol.:

1-tx=10,ty=5,

(5+10,10+5), (50+10,10+5), (30+10,20+5)

(15,15) (60,15) (40,25)

2-tx=-5,ty=-7

(0,3) (45,3) (25,13)

3- tx=2,ty=0

(7,10) (52,10) (32,20)

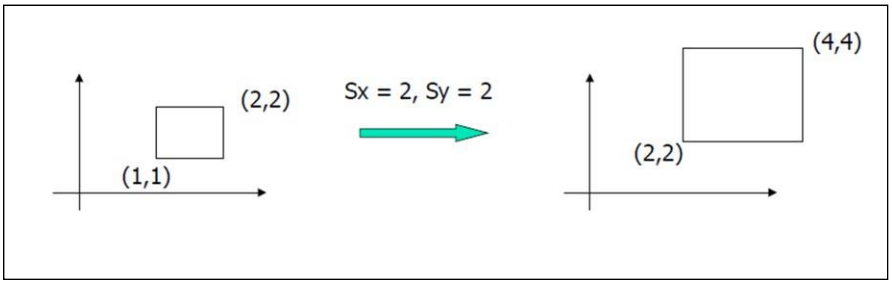
2- Scaling

To changes the size of an object such that we can magnify the size or reduce it. This process is called scaling. Suppose p(x,y) is the point which we want to scale, after scaling we get new point having coordinates as p`(x`,y`) ⇒ x` = x* Sx,y` =y* Sy where Sx and Sy are scaling factors.

Whenever scaling is preformed there is one point that remains of the same location called the fixed point of scaling. If the fixed point is at the origin(0,0) a point (x, y) can be scaled by a factor Sx in the x direction and by Sy in the y direction.

x`=x*Sx

y`=y*Sy



Matrix representation:

$$(x'\ y'\ 1) = (x\ y\ 1) * \begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[x', \quad y', \quad 1] = [x * Sx, \quad y * Sy, \quad 1]$$

Ex.2: Scale the following points (5,5),(10,5),(5,10),(10,10), for $Sx = Sy = {}^{1}$, (uniform scaling).

Sol.: $p`_1$ (2.5,2.5), $p`_2$ (2.5,2.5), $p`_3$(2.5,2.5), $p`_4$(2.5,2.5)