2DTransformations

1- Rotation

In rotation, we rotate the object at particular angle θthetathetafrom its origin. From the following figure, we can see that the point P(x,y)is located at angle φ from the horizontal X coordinate with distance r from the origin.

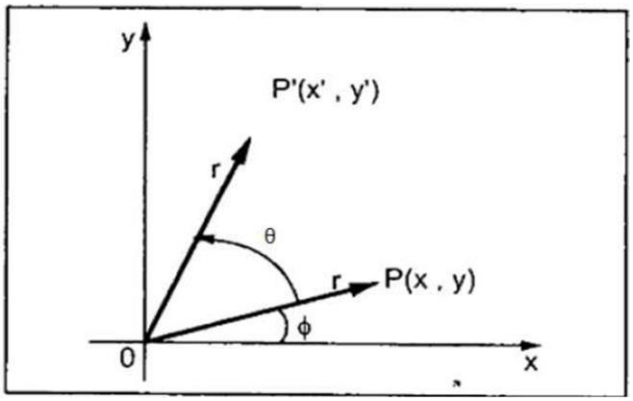
Let us suppose you want to rotate it at the angle θ. After rotating it to a new location, you will get a new point P′(x′,y′).

Therotatedirectionabouttheoriginiseitherpositiveoriented(anticlockwise)where angle is positive or negative oriented (clockwise) where angle is negative .

Thepolarrepresentationofcircles:

A circle can be described by tracing and the path of the point (x, y) keeping R fixed and making one complete revolution of the angle θ. One revolution is measured from0to360orradians.Whichisthemeasureusuallyemployedbycomputersfrom0 to 2π Radians. Such that:

$$x=R*\cos(\theta)$$
$$y=R*\sin(\theta)$$



Using standard trigonometric the original coordinate of point P(x,y) can be represented as:

$$x = r\cos\phi \quad .....(1)$$
$$y = r\sin\phi \quad .....(2)$$

Samewaywecanrepresentthepoint P'(x′,y′)as:

$$x' = r\cos(\phi+\theta) = r\cos\phi\cos\theta - r\sin\phi\sin\theta \quad .....(3)$$
$$y' = r\sin(\phi+\theta) = r\cos\phi\sin\theta + r\sin\phi\cos\theta \quad .....(4)$$

Substitutingequation1&2in3&4respectively,wewillget:

$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$

Matrix representation:

$$(\acute{x}\acute{y}1) = (xy1) * \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

ex.3: Rotatethefollowingpoints $p_1(5,5), p_2(10,5), p_3(5,10)$, for θ=90degreecounter clockwise.

Sol.:

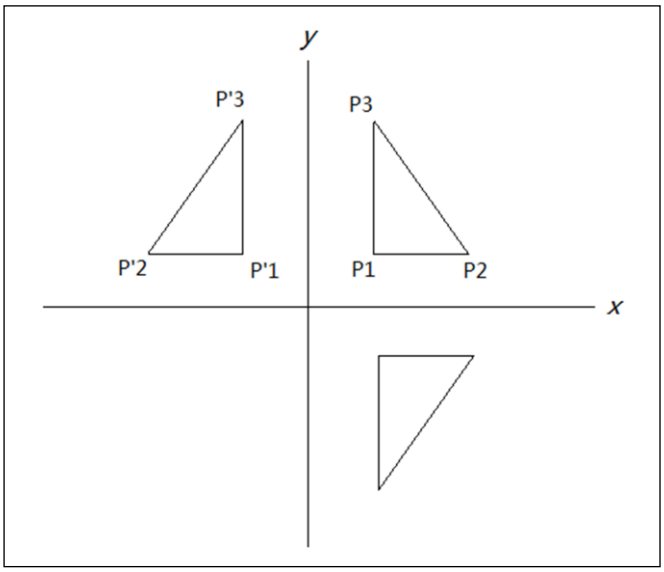
$x' = x\cos\theta - y\sin\theta$

$x' = x(0) - y(1)$

$y' = x\sin\theta + y\cos\theta$

$y' = x(1) + y(0)$

$p'_1(-5,5), p'_2(-10,5), p'_3(-5,10)$,



degrees	sin	cos
30	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
45	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
60	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
90	1	0
0	0	1