H.W.

Find the following infinite integrals:-

1. $\int \frac{1}{\sqrt{25-x^2}}\,dx$

2. $\int \frac{1}{e^{x}+e^{-x}}\,dx$

3. $\int \frac{1+\tan^2 x}{\sqrt{1-\tan^2 x}}\,dx$

4. $\int \frac{1}{x^2+6x+10}\,dx$

5. $\int \frac{1}{\sqrt{2x-x^2}}\,dx$

6. $\int \frac{e^{2x}}{4+e^{4x}}\,dx$

Evalute the folloi.. definite integrals

1. $\int_{-\sqrt{2}}^{\frac{-2}{\sqrt{3}}} \frac{1}{x\sqrt{x^2-1}}\,dx$

2. $\int_{0}^{\frac{1}{\sqrt{2}}} \frac{1}{\sqrt{1-x^2}}$

3. $\int_{1}^{\sqrt{2}} \frac{x}{3+x^4}\,dx$

Some rules

1. $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

2. $\sec^{-1}x = \cos^{-1}(\frac{1}{x})$

3. $\sin(\cos^{-1}x) = \sqrt{1-x^2}$ ; $x \geqslant 1$

4. $\cos(\sin^{-1}x) = \sqrt{1-x^2}$ ; $x \geqslant 1$

5. $\tan(\sin^{-1}x) = \frac{x}{\sqrt{1-x^2}}$ ; $x > 1$

6. $\sec(\tan^{-1}x) = \sqrt{1+x^2}$

7. $\sin(\sec^{-1}x) = \frac{\sqrt{x^2-1}}{x}$ ; $x \geqslant 1$

8. $\sin^{-1}(-x) = -\sin^{-1}x$

9. $\tan^{-1}(-x) = -\tan^{-1}x$

limits for inverse Trigonometric Functions

1) $\lim\limits_{x \to 0} \frac{\sin^{-1}x}{x} = \frac{0}{0}$ ind.

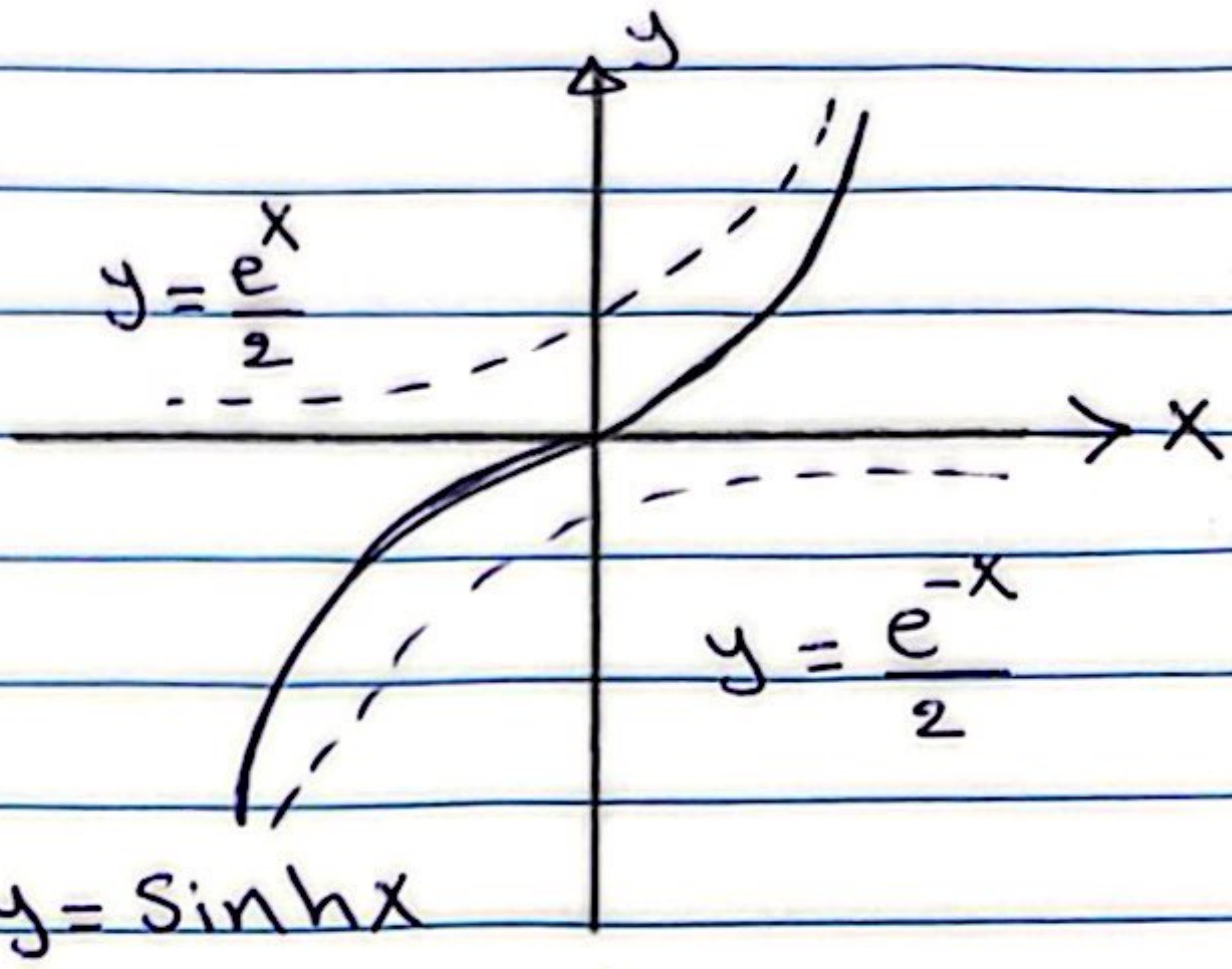
2) L.H. $\lim\limits_{x \to 0} \frac{1}{\sqrt{1-x^2}} = 1$

3. $\lim\limits_{x \to 0} \frac{1-\cos^{-1}x}{x} = \frac{0}{0}$ ind.

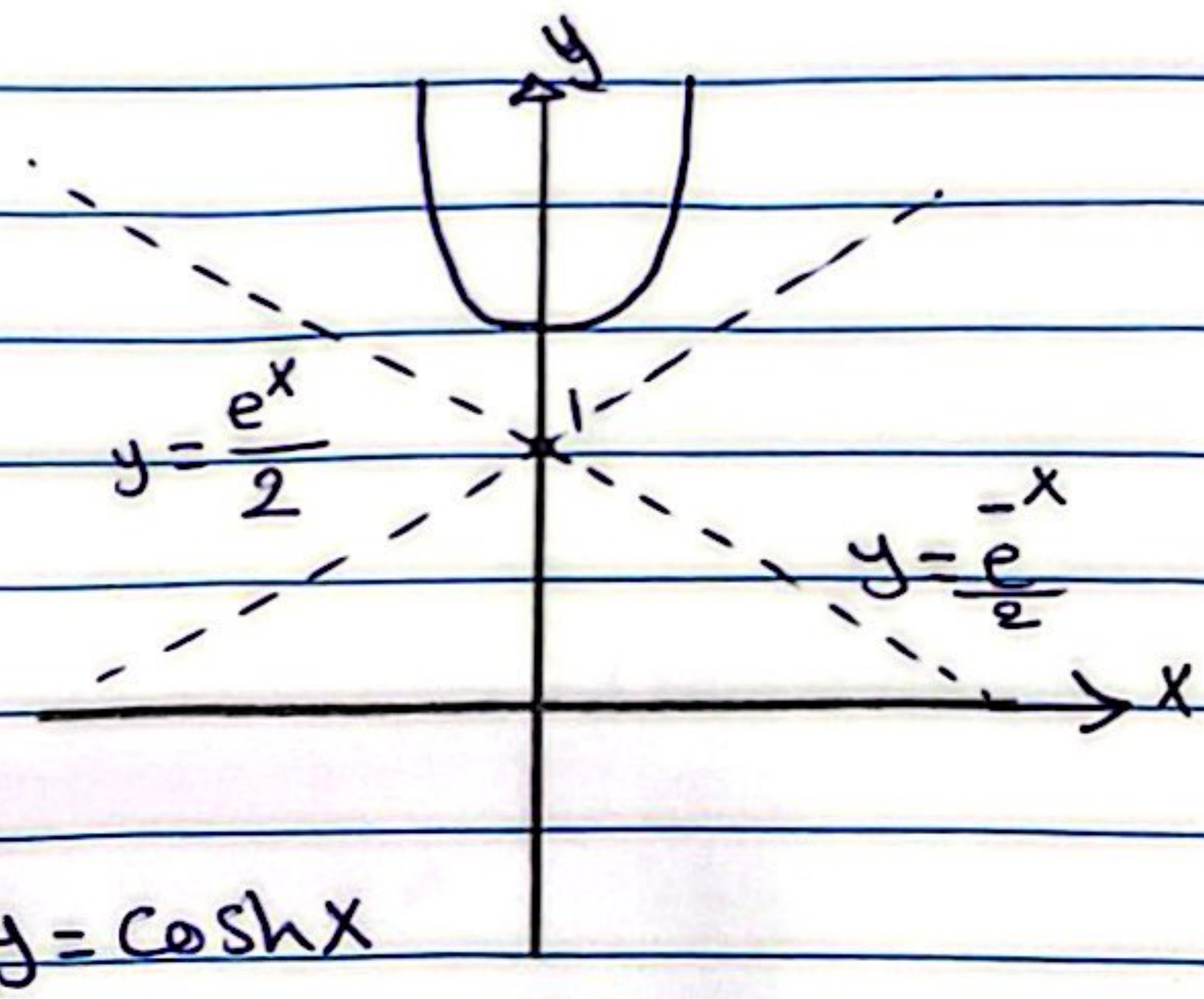
L.H. $\lim\limits_{x \to 0} -\left(\frac{-1}{\sqrt{1-x^2}}\right) = 1$

# Hyperbolic Functions

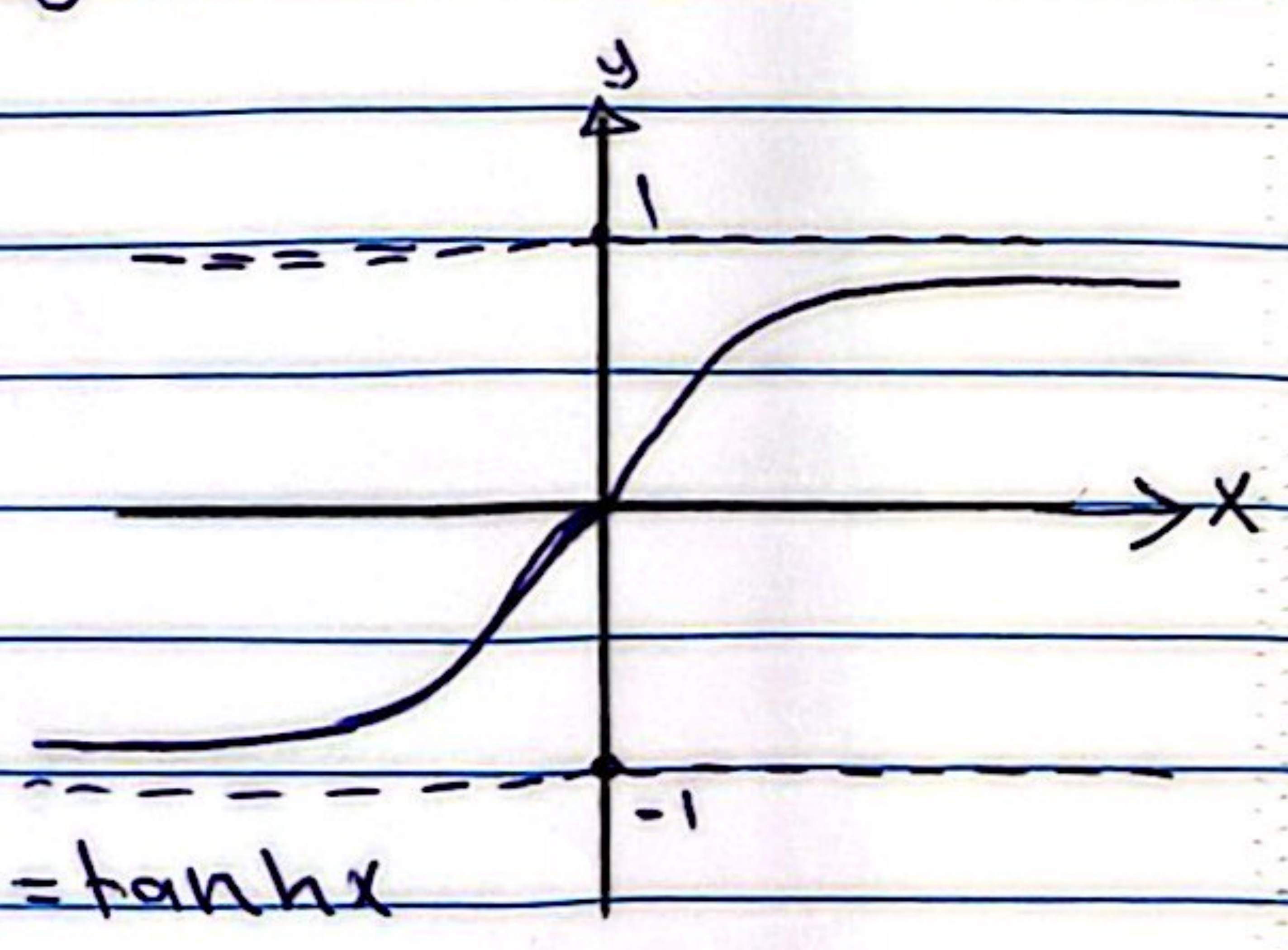
1. $\sinh x = \dfrac{e^{x} - e^{-x}}{2}$



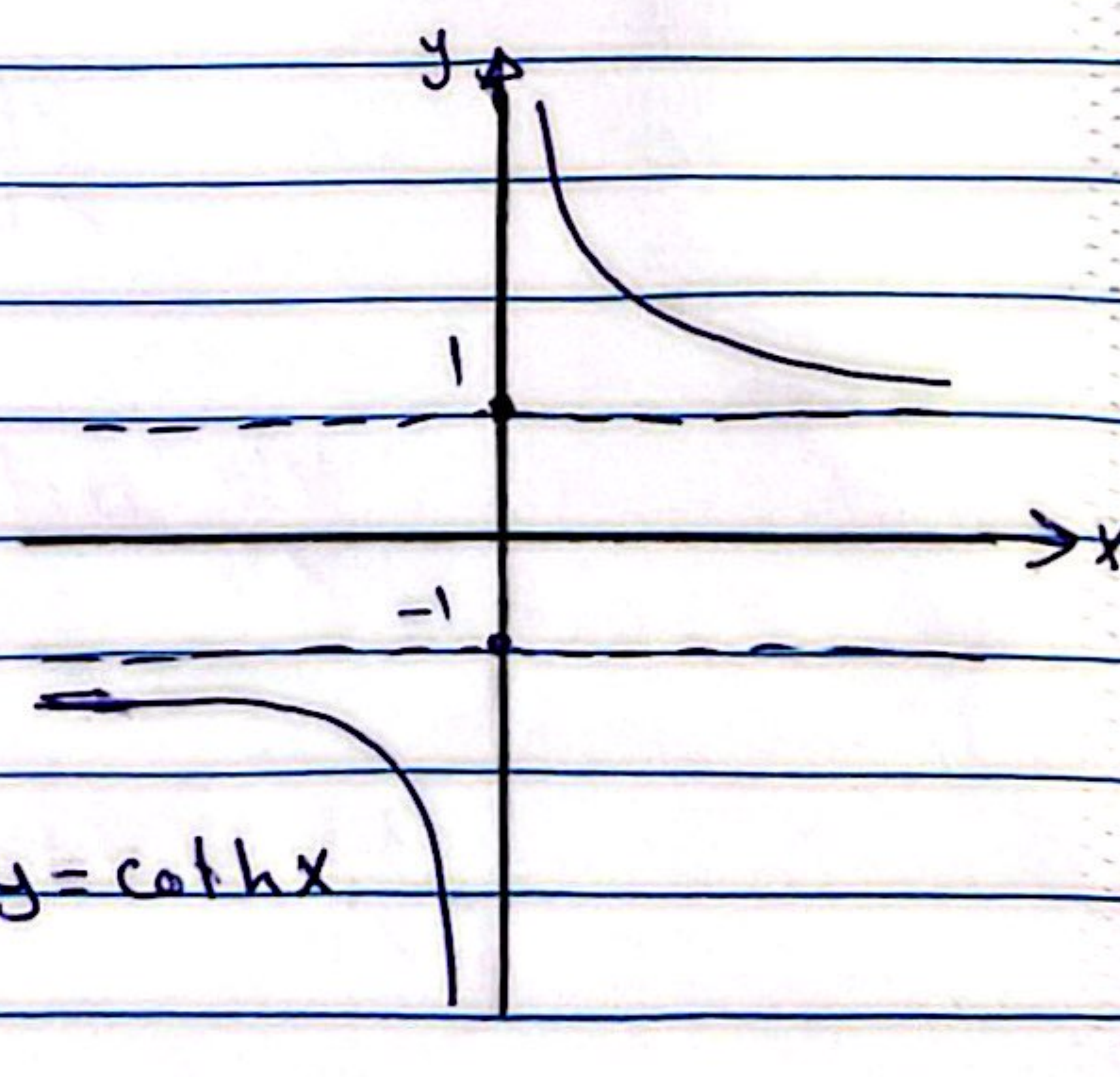

2. $\cosh x = \dfrac{e^{x} + e^{-x}}{2}$



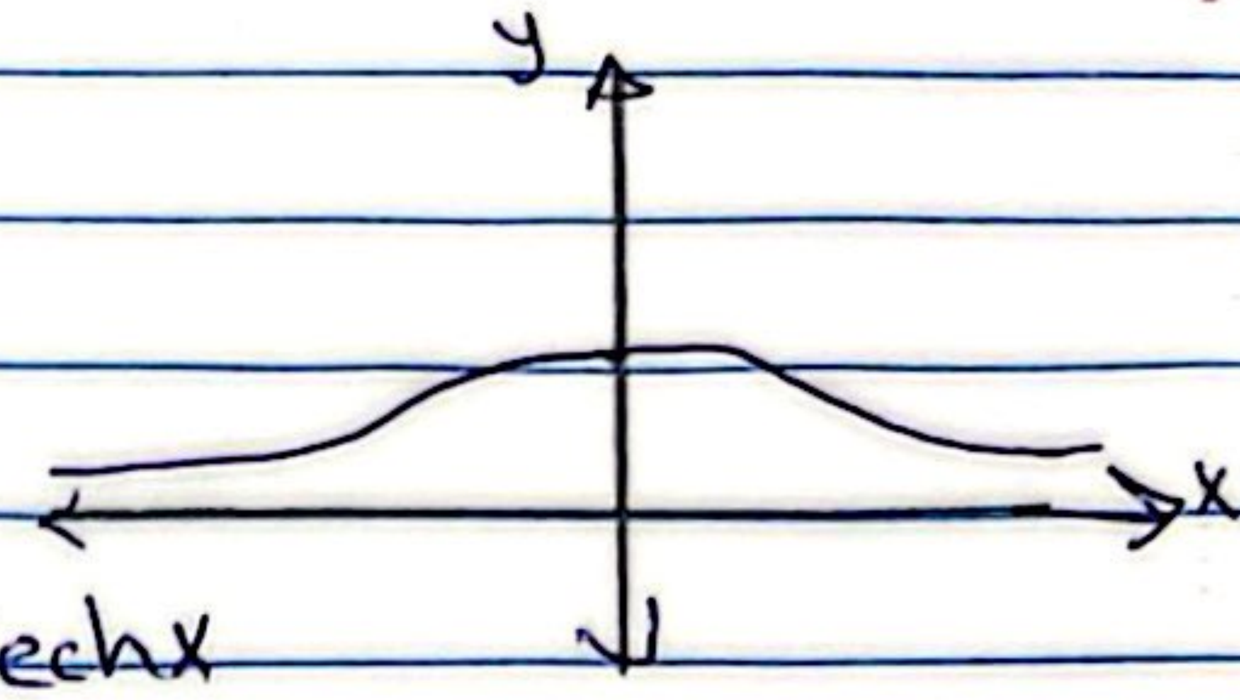

3. $\tanh x = \dfrac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$




4. $\coth x = \dfrac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$

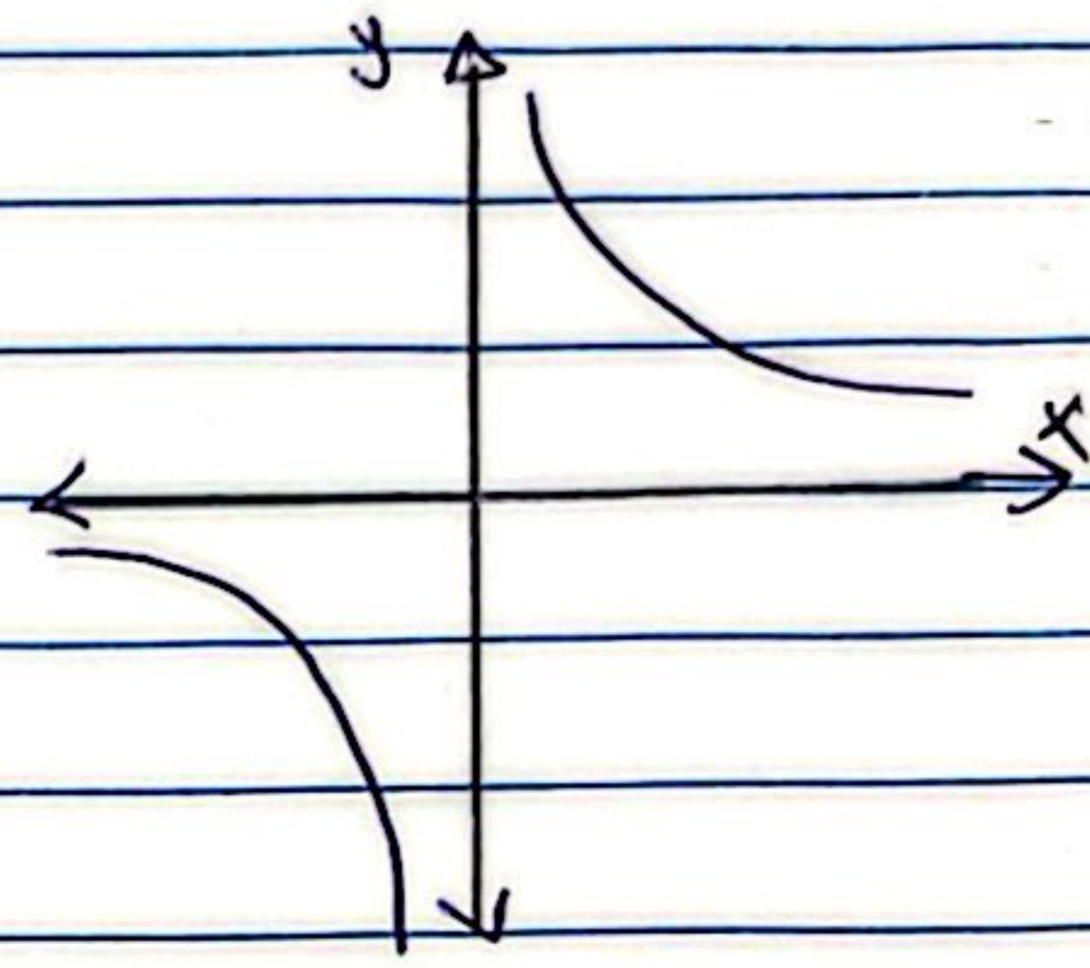


5. $\operatorname{sech} x = \dfrac{2}{e^{x} + e^{-x}}$



$y = \operatorname{sech} x$

6. $\operatorname{csch} x = \dfrac{2}{e^{x} - e^{-x}}$



$y = \operatorname{csch} x$

Theorem

1. $\dfrac{d}{dx} \sinh u = \cosh u \dfrac{du}{dx}$

2. $\dfrac{d}{dx} \cosh u = + \sinh u \dfrac{du}{dx}$

3. $\dfrac{d}{dx} \tanh u = \operatorname{sech}^2 u \dfrac{du}{dx}$

4. $\dfrac{d}{dx} \coth u = -\operatorname{csch}^2 u \dfrac{du}{dx}$

5. $\dfrac{d}{dx} \operatorname{sech} u = -\operatorname{sech} u \tanh u \dfrac{du}{dx}$

6. $\dfrac{d}{dx} \operatorname{csch} u = -\operatorname{csch} u \coth u \dfrac{du}{dx}$

1. $\int \sinh u\,du = -\cosh u + c$

2. $\int \cosh u\,du = \sinh u + c$

3. $\int \operatorname{sech}^2 u\,du = \tanh u + c$

4. $\int \operatorname{csch}^2 u\,du = -\coth u + c$

5. $\int \operatorname{sech} u \tanh u\,du = \operatorname{sech} u + c$

6. $\int \operatorname{csch} u . \coth u\,du = -\operatorname{csch} u + c$

Theorem

1. $\cosh x + \sinh x = e^{x}$

2. $\cosh x - \sinh x = e^{-x}$

3. $\cosh^2 x - \sinh^2 x = 1$

4. $1 - \tanh^2 x = \operatorname{sech}^2 x$

5. $\coth^2 x - 1 = \operatorname{csch}^2 x$

6. $\cosh(-x) = -\cosh x$

7. $\sinh(-x) = -\sinh x$

8. $\cosh 2x = 2\cosh^2(x-1)$

$= 2\sinh^2(x+1)$

$= \cosh^2(x) + \sinh^2(x)$

9. $\sinh 2x = 2\sinh x \cdot \cosh x$

10. $\cosh(x-y) = \cosh x \cosh y - \sinh x \sinh y$

11. $\sinh(x-y) = \sinh x \cosh y - \cosh x \sinh y$

12. $\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$

13. $\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y$

Prove the theorem above :-

1. $\cosh x + \sinh x \overset{?}{=} e^x$

$$\frac{e^x + e^{-x}}{2} + \frac{e^x - e^{-x}}{2} = \frac{2e^x + \cancel{e^{-x}} - \cancel{e^{-x}}}{2} = e^x$$

2. $\cosh x - \sinh x = e^{-x}$

$$\frac{e^x + e^{-x}}{2} - \frac{e^x - e^{-x}}{2} = \frac{\cancel{e^x} + e^{-x} - \cancel{e^x} + e^{-x}}{2} = e^{-x}$$

3. $\cosh^2 x - \sinh^2 x = 1$

$$\left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2 = \frac{e^{2x} + 2 + e^{-2x} - e^{2x} + 2 - e^{-2x}}{4} = \frac{4}{4} = 1$$

Prove that $\frac{d}{dx}\sinh u = \cosh u$

$$\Rightarrow \frac{d}{dx}\left[\frac{e^{u}-e^{-u}}{2}\right] = \frac{e^{u}-e^{-u}(-1)}{2} = \frac{e^{u}+e^{-u}}{2} = \cosh u$$

Prove that $\frac{d}{dx}\tanh u = \operatorname{sech}^2 u$

$$\Rightarrow \frac{d}{dx}\frac{\sinh u}{\cosh u} = \frac{\cosh u \cdot \cosh u - \sinh u \cdot \sinh u}{\cosh^2 u}$$

$$= \frac{\cosh^2 u - \sinh^2 u}{\cosh^2 u}$$

$$= \frac{1}{\cosh^2 u} = \operatorname{sech}^2 u$$

Ex

Find $y'$ for the following Functions

1. $y = \cosh(x^3)$

$y' = -\sinh(x^3) \cdot 3x^2$

2. $y = \sinh(4x-8)$

$y' = \cosh(4x-8)(4) = 4\cosh(4x-8)$

3. $y = (\coth(x^3))^2$

$y' = 2\coth(x^3)(-\operatorname{csch}^2 x^3)(3x^2)$

4. $y = \ln(\tanh x)$

$$y' = \frac{\operatorname{sech}^2 x}{\tanh x}$$

5. $\sinh xy = \tanh xy$

$$\cosh(xy)\cdot(xy' + y(1)) = \operatorname{sech}^2 h(xy)(xy' + y(1))$$

$$xy'(\cosh(xy) - \operatorname{sech}^2 h(xy)) = y(\operatorname{sech}^2 h(xy) - \cosh(xy))$$

$$\therefore y' = \frac{[\operatorname{sech}^2 h(xy) - \cosh(xy)]y}{-x[\cosh(xy) - \operatorname{sech}^2 h(xy)]}$$

$$= \frac{-y}{x}$$

6. $\cosh xy = \tanh y$

$$-\sinh(xy)(xy' + y(1)) = \operatorname{sech}^2 hy \cdot y'$$

$$-\sinh(xy)\, xy' - \sinh(xy)\, y = y' \operatorname{sech}^2 h(y)$$

$$-xy' \sinh(xy) - y' \operatorname{sech}^2 h(y) = y \sinh(xy)$$

$$y'(-x\sinh(xy) - \operatorname{sech}^2 hy) = y \sinh xy$$

$$y' = \frac{y \sinh y}{-x\sinh xy - \operatorname{sech}^2 hy}$$

$$= \frac{-y}{x}\ \frac{\sinh y}{\sinh xy - \operatorname{sech}^2 hy}$$

H.w.

7. $\operatorname{Sech} e^{xy} - \tanh e^{x} = 2$

$$\operatorname{Sech} e^{xy} \tanh e^{xy} [e^{xy} \cdot (xy' + y(1))] - \operatorname{Sec}^2 h e^{x} = 0$$

$$\operatorname{seche}^{xy} \tanh e^{xy} (xy' e^{xy} + y e^{xy}) - \operatorname{Sech}^2 e^{x} = 0$$

$$xy' e^{xy} \operatorname{seche}^{xy} \tanh e^{xy} + y e^{xy} \operatorname{seche}^{xy} \tanh e^{xy} - \operatorname{sech}^2 e^{xy} = 0$$

$$\therefore y' = \frac{-y e^{xy} \operatorname{seche}^{xy} \tanh e^{xy} + \operatorname{Sech}^2 e^{xy}}{x e^{xy} \operatorname{seche}^{xy} \tanh e^{xy}}$$

Ex: Integrate the following functions

1. $\int \cosh^5 x \sinh x \, dx$

$$= \frac{\cosh^6 x}{6} + c$$

2. $\int \coth x \, dx$

$$= \int \frac{\cosh x}{\sinh x} dx$$

$$= \ln|\cosh x| + c$$

3. $\int \tanh x \operatorname{Sech}^3 x \, dx$

$$= \int (\tanh x \operatorname{sech} x) \cdot \operatorname{sech}^2 x \, dx$$

$$= \frac{-\operatorname{sech}^3 x}{3} + C$$

4. $\displaystyle\int_0^{10} \sqrt{1+\sinh^2\left(\frac{x}{10}\right)}\,dx$

$$= \int_0^{10} \cosh\left(\frac{x}{10}\right) dx$$

$$= \sinh\left(\frac{x}{10}\right) \Big|_0^{10}$$

$$= 10\left[\sinh 1 - \sinh 0\right]$$

$$= 10\left[\frac{e^{1}-e^{-1}}{2} - \frac{e^{0}-e^{-0}}{2}\right]$$

$$= 10\left[\frac{e-e^{-1}}{2}\right]$$

H.w. Find $y'$ for the following functions:-

1- $y = \operatorname{sech}\sqrt{3x+9}$

2- $y = \operatorname{sech}^3 x \cosh^2 x$

3- $y = e^{x} \cdot \tanh\sqrt{x}$

4- $y = \ln(\cosh^2 3x)$

5- $y = \operatorname{csch}(x^2-1)$

6- $e^{y} = \ln(\tanh(xy))$

Evalute the integral for the following function

1. $\int \sqrt{\tanh x}\ \operatorname{sech}^2 x\, dx$

2. $\int \sinh x \cosh x\, dx$

3. $\int \operatorname{csch}^3(3x)\, dx$

Theorem:-

1. $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

2. $\sec^{-1}x = \cos^{-1}\left(\frac{1}{x}\right)$

3. $\sin(\cos^{-1}x) = \sqrt{1-x^2}$ ; $x \geq 1$

4. $\cos(\sin^{-1}x) = \sqrt{1-x^2}$ ; $x \geq 1$

5. $\tan(\sin^{-1}x) = \frac{x}{\sqrt{1-x^2}}$ ; $x > 1$

6. $\sec(\tan^{-1}x) = \sqrt{1+x^2}$

7. $\sin^{-1}(-x) = -\sin^{-1}x$

8. $\sin(\sec^{-1}x) = \frac{\sqrt{x^2-1}}{x}$ ; $x \geq 1$

9. $\tan^{-1}(-x) = -\tan^{-1}x$

limits for Inverese trigonometric functions:-

1. $\lim_{x \to 0} \frac{\sin^{-1}x}{x} = \frac{0}{0}$ indeterminate

2. $\lim_{x \to 0} \frac{1}{\sqrt{1-x^2}} = 1$