Using this information, answer these questions.

1. What hypotheses would you use?
2. Is the sample considered small or large?
3. What assumption must be met before the hypothesis test can be conducted?
4. Which probability distribution would you use?
5. Would you select a one- or two-tailed test? Why?
6. What critical value(s) would you use?
7. Conduct a hypothesis test. Use $\sigma = 30.3$.
8. What is your decision?
9. What is your conclusion?
10. Write a brief statement summarizing your conclusion.
11. If you lived in a city whose population was about 50,000, how many automobile thefts per year would you expect to occur?

See page 469 for the answers.

Exercises 8–2

For Exercises 1 through 13, perform each of the following steps.

a. State the hypotheses and identify the claim.
b. Find the critical value(s).
c. Compute the test value.
d. Make the decision.
e. Summarize the results.

Use diagrams to show the critical region (or regions), and use the traditional method of hypothesis testing unless otherwise specified.

1. **Warming and Ice Melt** The average depth of the Hudson Bay is 305 feet. Climatologists were interested in seeing if the effects of warming and ice melt were affecting the water level. Fifty-five measurements over a period of weeks yielded a sample mean of 306.2 feet. The population variance is known to be 3.57. Can it be concluded at the 0.05 level of significance that the average depth has increased? Is there evidence of what caused this to happen?

   Source: *World Almanac and Book of Facts 2010.*

2. **Credit Card Debt** It has been reported that the average credit card debt for college seniors at the college book store for a specific college is \$3262. The student senate at a large university feels that their seniors have a debt much less than this, so it conducts a study of 50 randomly selected seniors and finds that the average debt is \$2995, and the population standard deviation is \$1100. With $\alpha = 0.05$, is the student senate correct?

3. **Revenue of Large Businesses** A researcher estimates that the average revenue of the largest businesses in the United States is greater than \$24 billion. A sample of 50 companies is selected, and the revenues (in billions of dollars) are shown. At $\alpha = 0.05$, is there enough evidence to support the researcher's claim? Assume $\sigma = 28.7$.

178	122	91	44	35
61	56	46	20	32
30	28	28	20	27
29	16	16	19	15
41	38	36	15	25
31	30	19	19	19
24	16	15	15	19
25	25	18	14	15
24	23	17	17	22
22	21	20	17	20

   Source: *New York Times Almanac.*

4. **Moviegoers** The average "moviegoer" sees 8.5 movies a year. A *moviegoer* is defined as a person who sees at least one movie in a theater in a 12-month period. A random sample of 40 moviegoers from a large university revealed that the average number of movies seen per person was 9.6. The population standard deviation is 3.2 movies. At the 0.05 level of significance, can it be concluded that this represents a difference from the national average?

   Source: *MPAA Study.*

5. **Nonparental Care** According to the *Digest of Educational Statistics,* a certain group of preschool children under the age of one year each spends an average of 30.9 hours per week in nonparental care. A study of state university center-based programs indicated that a random sample of 32 infants spent an average of 32.1 hours per week in their care. The standard deviation of the population is 3.6 hours. At $\alpha = 0.01$ is there sufficient evidence to conclude that the sample mean differs from the national mean?

   Source: *www.nces.ed.gov*

6. Peanut Production in Virginia The average production of peanuts in Virginia is 3000 pounds per acre. A new plant food has been developed and is tested on 60 individual plots of land. The mean yield with the new plant food is 3120 pounds of peanuts per acre, and the population standard deviation is 578 pounds. At $\alpha = 0.05$, can you conclude that the average production has increased?

Source: *The Old Farmer's Almanac.*

7. Heights of 1-Year-Olds The average 1-year-old (both genders) is 29 inches tall. A random sample of 30 1-year-olds in a large day care franchise resulted in the following heights. At $\alpha = 0.05$, can it be concluded that the average height differs from 29 inches? Assume $\sigma = 2.61$.

25	32	35	25	30	26.5	26	25.5	29.5	32
30	28.5	30	32	28	31.5	29	29.5	30	34
29	32	27	28	33	28	27	32	29	29.5

Source: www.healthepic.com

8. Salaries of Government Employees The mean salary of federal government employees on the General Schedule is \$59,593. The average salary of 30 state employees who do similar work is \$58,800 with $\sigma = \$1500$. At the 0.01 level of significance, can it be concluded that state employees earn on average less than federal employees?

Source: *New York Times Almanac.*

9. Operating Costs of an Automobile The average cost of owning and operating an automobile is \$8121 per 15,000 miles including fixed and variable costs. A random survey of 40 automobile owners revealed an average cost of \$8350 with a population standard deviation of \$750. Is there sufficient evidence to conclude that the average is greater than \$8121? Use $\alpha = 0.01$.

Source: *New York Times Almanac 2010.*

10. Home Prices in Pennsylvania A real estate agent claims that the average price of a home sold in Beaver County, Pennsylvania, is \$60,000. A random sample of 36 homes sold in the county is selected, and the prices in dollars are shown. Is there enough evidence to reject the agent's claim at $\alpha = 0.05$? Assume $\sigma = \$76,025$.

9,500	54,000	99,000	94,000	80,000
29,000	121,500	184,750	15,000	164,450
6,000	13,000	188,400	121,000	308,000
42,000	7,500	32,900	126,900	25,225
95,000	92,000	38,000	60,000	211,000
15,000	28,000	53,500	27,000	21,000
76,000	85,000	25,225	40,000	97,000
284,000				

Source: *Pittsburgh Tribune-Review.*

11. Use of Disposable Cups The average college student goes through 500 disposable cups in a year. To raise environmental awareness, a student group at a large university volunteered to help count how many cups were used by students on their campus. A random sample of 50 students' results found that they used a mean of 476 cups with $\sigma = 42$ cups. At $\alpha = 0.01$, is there sufficient evidence to conclude that the mean differs from 500?

Source: www.esc.mtu.edu/SFES.php

12. Student Expenditures The average expenditure per student (based on average daily attendance) for a certain school year was \$10,337 with a population standard deviation of \$1560. A survey for the next school year of 150 randomly selected students resulted in a sample mean of \$10,798. Do these results indicate that the average expenditure has changed? Choose your own level of significance.

Source: *World Almanac.*

13. Ages of U.S. Senators The mean age of Senators in the 109th Congress was 60.35 years. A random sample of 40 senators from various state senates had an average age of 55.4 years, and the population standard deviation is 6.5 years. At $\alpha = 0.05$, is there sufficient evidence that state senators are on average younger than the Senators in Washington?

Source: *CG Today.*

14. What is meant by a P-value? The P-value is the actual probability of getting the sample mean if the null hypothesis is true.

15. State whether the null hypothesis should be rejected on the basis of the given P-value.

a. P-value $= 0.258$, $\alpha = 0.05$, one-tailed test Do not reject.

b. P-value $= 0.0684$, $\alpha = 0.10$, two-tailed test Reject.

c. P-value $= 0.0153$, $\alpha = 0.01$, one-tailed test Do not reject.

d. P-value $= 0.0232$, $\alpha = 0.05$, two-tailed test Reject.

e. P-value $= 0.002$, $\alpha = 0.01$, one-tailed test Reject.

16. Soft Drink Consumption A researcher claims that the yearly consumption of soft drinks per person is 52 gallons. In a sample of 50 randomly selected people, the mean of the yearly consumption was 56.3 gallons. The standard deviation of the population is 3.5 gallons. Find the P-value for the test. On the basis of the P-value, is the researcher's claim valid?

Source: U.S. Department of Agriculture.

17. Stopping Distances A study found that the average stopping distance of a school bus traveling 50 miles per hour was 264 feet. A group of automotive engineers decided to conduct a study of its school buses and found that for 20 buses, the average stopping distance of buses traveling 50 miles per hour was 262.3 feet. The standard deviation of the population was 3 feet. Test the claim that the average stopping distance of the company's buses is actually less than 264 feet. Find the P-value. On the basis of the P-value, should the null hypothesis be rejected at $\alpha = 0.01$? Assume that the variable is normally distributed.

Source: Snapshot, *USA TODAY.*

18. Copy Machine Use A store manager hypothesizes that the average number of pages a person copies on the store's copy machine is less than 40. A sample of 50 customers' orders is selected. At $\alpha = 0.01$, is there enough evidence to support the claim? Use the P-value hypothesis-testing method. Assume $\sigma = 30.9$.

2	2	2	5	32
5	29	8	2	49
21	1	24	72	70
21	85	61	8	42
3	15	27	113	36
37	5	3	58	82
9	2	1	6	9
80	9	51	2	122
21	49	36	43	61
3	17	17	4	1

19. Burning Calories by Playing Tennis A health researcher read that a 200-pound male can burn an average of 546 calories per hour playing tennis. Thirty-six males were randomly selected and tested. The mean of the number of calories burned per hour was 544.8. Test the claim that the average number of calories burned is actually less than 546, and find the *P*-value. On the basis of the *P*-value, should the null hypothesis be rejected at $\alpha = 0.01$? The standard deviation of the population is 3. Can it be concluded that the average number of calories burned is less than originally thought?

20. Breaking Strength of Cable A special cable has a breaking strength of 800 pounds. The standard deviation of the population is 12 pounds. A researcher selects a sample of 20 cables and finds that the average breaking strength is 793 pounds. Can he reject the claim that the breaking strength is 800 pounds? Find the *P*-value. Should the null hypothesis be rejected at $\alpha = 0.01$? Assume that the variable is normally distributed.

21. Farm Sizes The average farm size in the United States is 444 acres. A random sample of 40 farms in Oregon indicated a mean size of 430 acres, and the population standard deviation is 52 acres. At $\alpha = 0.05$, can it be concluded that the average farm in Oregon differs from the national mean? Use the *P*-value method.

Source: *New York Times Almanac.*

22. Farm Sizes Ten years ago, the average acreage of farms in a certain geographic region was 65 acres. The standard deviation of the population was 7 acres. A recent study consisting of 22 farms showed that the average was 63.2 acres per farm. Test the claim, at $\alpha = 0.10$, that the average has not changed by finding the *P*-value for the test. Assume that σ has not changed and the variable is normally distributed.

23. Transmission Service A car dealer recommends that transmissions be serviced at 30,000 miles. To see whether her customers are adhering to this recommendation, the dealer selects a sample of 40 customers and finds that the average mileage of the automobiles serviced is 30,456. The standard deviation of the population is 1684 miles. By finding the *P*-value, determine whether the owners are having their transmissions serviced at 30,000 miles. Use $\alpha = 0.10$. Do you think the α value of 0.10 is an appropriate significance level?

24. Speeding Tickets A motorist claims that the South Boro Police issue an average of 60 speeding tickets per day. These data show the number of speeding tickets issued each day for a period of one month. Assume σ is 13.42. Is there enough evidence to reject the motorist's claim at $\alpha = 0.05$? Use the *P*-value method.

72	45	36	68	69	71	57	60
83	26	60	72	58	87	48	59
60	56	64	68	42	57	57	
58	63	49	73	75	42	63	

25. Sick Days A manager states that in his factory, the average number of days per year missed by the employees due to illness is less than the national average of 10. The following data show the number of days missed by 40 employees last year. Is there sufficient evidence to believe the manager's statement at $\alpha = 0.05$? $\sigma = 3.63$. Use the *P*-value method.

0	6	12	3	3	5	4	1
3	9	6	0	7	6	3	4
7	4	7	1	0	8	12	3
2	5	10	5	15	3	2	5
3	11	8	2	2	4	1	9

Extending the Concepts

26. Suppose a statistician chose to test a hypothesis at $\alpha = 0.01$. The critical value for a right-tailed test is +2.33. If the test value were 1.97, what would the decision be? What would happen if, after seeing the test value, she decided to choose $\alpha = 0.05$? What would the decision be? Explain the contradiction, if there is one.

27. Hourly Wage The president of a company states that the average hourly wage of her employees is \$8.65. A sample of 50 employees has the distribution shown. At $\alpha = 0.05$, is the president's statement believable? Assume $\sigma = 0.105$.

Class	Frequency
8.35–8.43	2
8.44–8.52	6
8.53–8.61	12
8.62–8.70	18
8.71–8.79	10
8.80–8.88	2

Technology *Step by Step*

MINITAB
Step by Step

Hypothesis Test for the Mean and the *z* Distribution

MINITAB can be used to calculate the test statistic and its *P*-value. The *P*-value approach does not require a critical value from the table. If the *P*-value is smaller than α, the null hypothesis is rejected. For Example 8–4, test the claim that the mean shoe cost is less than $80.

1. Enter the data into a column of MINITAB. Do not try to type in the dollar signs! Name the column **ShoeCost.**
2. If sigma is known, skip to step 3; otherwise estimate sigma from the sample standard deviation *s*.

Calculate the Standard Deviation in the Sample

a) Select **Calc>Column Statistics.**

b) Check the button for Standard deviation.

c) Select ShoeCost for the Input variable.

d) Type **s** in the text box for Store the result in:.

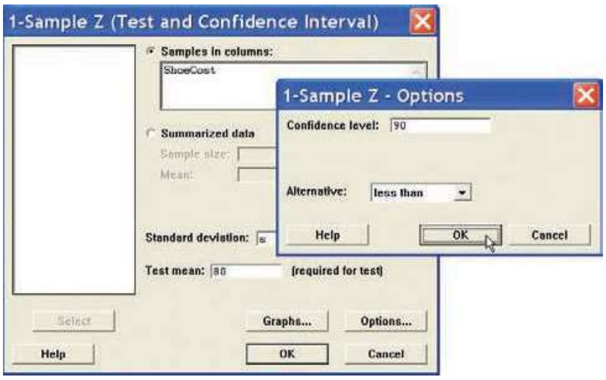
e) Click [OK].

Calculate the Test Statistic and *P*-Value

3. Select **Stat>Basic Statistics>1 Sample Z,** then select ShoeCost in the Variable text box.
4. Click in the text box and enter the value of sigma or type *s*, the sample standard deviation.
5. Click in the text box for Test mean, and enter the hypothesized value of **80.**
6. Click on [Options].

   a) Change the Confidence level to **90.**

   b) Change the Alternative to less than. This setting is crucial for calculating the *P*-value.
7. Click [OK] twice.



One-Sample Z: ShoeCost

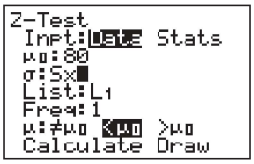
```
Test of mu = 80 vs < 80
The assumed sigma 19.161
                                                 90%
                                               Upper
Variable   N     Mean    StDev  SE Mean        Bound      Z      P
ShoeCost  36  75.0000  19.1610   3.1935      79.0926  -1.57  0.059
```

Since the *P*-value of 0.059 is less than α, reject the null hypothesis. There is enough evidence in the sample to conclude the mean cost is less than $80.

TI-83 Plus or TI-84 Plus Step by Step

Hypothesis Test for the Mean and the *z* Distribution (Data)

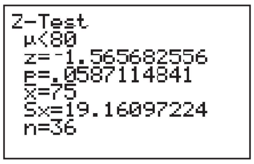
1. Enter the data values into L_1.
2. Press **STAT** and move the cursor to TESTS.
3. Press **1** for ZTest.
4. Move the cursor to Data and press **ENTER.**
5. Type in the appropriate values.
6. Move the cursor to the appropriate alternative hypothesis and press **ENTER.**
7. Move the cursor to Calculate and press **ENTER.**



Example TI8–1

This relates to Example 8–4 from the text. At the 10% significance level, test the claim that $\mu < 80$ given the data values.

60	70	75	55	80	55	50	40	80	70	50	95
120	90	75	85	80	60	110	65	80	85	85	45
75	60	90	90	60	95	110	85	45	90	70	70

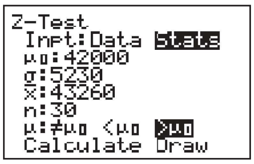


The population standard deviation σ is unknown. Since the sample size $n = 36 \geq 30$, you can use the sample standard deviation s as an approximation for σ. After the data values are entered in L_1 (step 1), press **STAT,** move the cursor to CALC, press **1** for 1-Var Stats, then press **ENTER.** The sample standard deviation of 19.16097224 will be one of the statistics listed. Then continue with step 2. At step 5 on the line for σ press **VARS** for variables, press **5** for Statistics, press **3** for S_x.

The test statistic is $z = -1.565682556$, and the P-value is 0.0587114841.

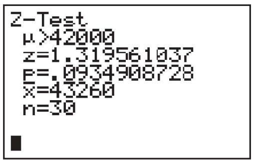
Hypothesis Test for the Mean and the *z* Distribution (Statistics)

1. Press **STAT** and move the cursor to TESTS.
2. Press **1** for ZTest.
3. Move the cursor to Stats and press **ENTER.**
4. Type in the appropriate values.
5. Move the cursor to the appropriate alternative hypothesis and press **ENTER.**
6. Move the cursor to Calculate and press **ENTER.**



Example TI8–2

At the 5% significance level, test the claim that $\mu > 42{,}000$ given $\sigma = 5230$, $\bar{X} = 43{,}260$, and $n = 30$.



The test statistic is $z = 1.319561037$, and the P-value is 0.0934908728.

Excel Step by Step

Hypothesis Test for the Mean: *z* Test

Excel does not have a procedure to conduct a hypothesis test for the mean. However, you may conduct the test of the mean by using the MegaStat Add-in available on your CD. If you have not installed this add-in, do so, following the instructions from the Chapter 1 Excel Step by Step.

Example XL8–1

This example relates to Example 8–4 from the text. At the 10% significance level, test the claim that $\mu < 80$. The MegaStat z test uses the P-value method. Therefore, it is not necessary to enter a significance level.

1. Enter the data into column A of a new worksheet.
2. From the toolbar, select Add-Ins, **MegaStat>Hypothesis Tests>Mean vs. Hypothesized Value.** *Note:* You may need to open MegaStat from the MegaStat.xls file on your computer's hard drive.

3. Select data input and type **A1:A36** as the Input Range.
4. Type **80** for the Hypothesized mean and select the "less than" Alternative.
5. Select z test and click [OK].

The result of the procedure is shown next.

Hypothesis Test: Mean vs. Hypothesized Value

```
80.000  Hypothesized value
75.000  Mean data
19.161  Standard deviation
 3.193  Standard error
    36  n

 −1.57  z
0.0587  P-value (one-tailed, lower)
```

8–3 *t* Test for a Mean

Objective 6

Test means when σ is unknown, using the t test.

When the population standard deviation is unknown, the z test is not normally used for testing hypotheses involving means. A different test, called the *t test,* is used. The distribution of the variable should be approximately normal.

As stated in Chapter 7, the t distribution is similar to the standard normal distribution in the following ways.

1. It is bell-shaped.
2. It is symmetric about the mean.
3. The mean, median, and mode are equal to 0 and are located at the center of the distribution.
4. The curve never touches the x axis.

The t distribution differs from the standard normal distribution in the following ways.

1. The variance is greater than 1.
2. The t distribution is a family of curves based on the *degrees of freedom,* which is a number related to sample size. (Recall that the symbol for degrees of freedom is d.f. See Section 7–2 for an explanation of degrees of freedom.)
3. As the sample size increases, the t distribution approaches the normal distribution.

The t test is defined next.

The ***t* test** is a statistical test for the mean of a population and is used when the population is normally or approximately normally distributed, and σ is unknown.

The formula for the t test is

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}}$$

The degrees of freedom are d.f. $= n - 1$.

The formula for the t test is similar to the formula for the z test. But since the population standard deviation σ is unknown, the sample standard deviation s is used instead.

The critical values for the t test are given in Table F in Appendix C. For a one-tailed test, find the α level by looking at the top row of the table and finding the appropriate column. Find the degrees of freedom by looking down the left-hand column.

Notice that the degrees of freedom are given for values from 1 through 30, then at intervals above 30. When the degrees of freedom are above 30, some textbooks will tell you to use the nearest table value; however, in this textbook, you should always round down to the nearest table value. For example, if d.f. = 59, use d.f. = 55 to find the critical value or values. This is a conservative approach.

As the degrees of freedom get larger, the critical values approach the z values. Hence the bottom values (large sample size) are the same as the z values that were used in the last section.

Example 8–8

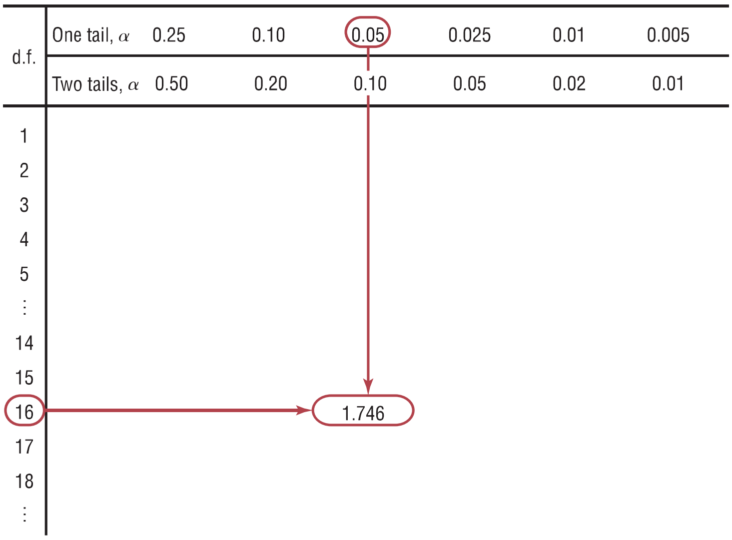
Find the critical t value for $\alpha = 0.05$ with d.f. = 16 for a right-tailed t test.

Solution

Find the 0.05 column in the top row and 16 in the left-hand column. Where the row and column meet, the appropriate critical value is found; it is +1.746. See Figure 8–21.

Figure 8–21

Finding the Critical Value for the *t* Test in Table F (Example 8–8)



Example 8–9

Find the critical t value for $\alpha = 0.01$ with d.f. = 22 for a left-tailed test.

Solution

Find the 0.01 column in the row labeled One tail, and find 22 in the left column. The critical value is −2.508 since the test is a one-tailed left test.

Example 8–10

Find the critical values for $\alpha = 0.10$ with d.f. = 18 for a two-tailed t test.

Solution

Find the 0.10 column in the row labeled Two tails, and find 18 in the column labeled d.f. The critical values are +1.734 and −1.734.

Example 8–11

Find the critical value for $\alpha = 0.05$ with d.f. = 28 for a right-tailed t test.

Solution

Find the 0.05 column in the One-tail row and 28 in the left column. The critical value is +1.701.

Assumptions for the t Test for a Mean When σ Is Unknown

1. The sample is a random sample.
2. Either $n \geq 30$ or the population is normally distributed if $n < 30$.

When you test hypotheses by using the t test (traditional method), follow the same procedure as for the z test, except use Table F.

Step 1 State the hypotheses and identify the claim.

Step 2 Find the critical value(s) from Table F.

Step 3 Compute the test value.

Step 4 Make the decision to reject or not reject the null hypothesis.

Step 5 Summarize the results.

Remember that the t test should be used when the population is approximately normally distributed and the population standard deviation is unknown.

Examples 8–12 through 8–14 illustrate the application of the t test.

Example 8–12 Hospital Infections

A medical investigation claims that the average number of infections per week at a hospital in southwestern Pennsylvania is 16.3. A random sample of 10 weeks had a mean number of 17.7 infections. The sample standard deviation is 1.8. Is there enough evidence to reject the investigator's claim at $\alpha = 0.05$?

Source: Based on information obtained from Pennsylvania Health Care Cost Containment Council.

Solution

Step 1 H_0: $\mu = 16.3$ (claim) and H_1: $\mu \neq 16.3$.

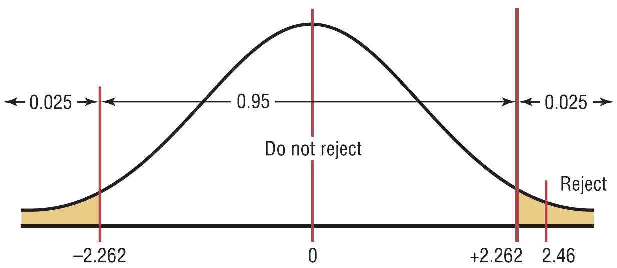
Step 2 The critical values are $+2.262$ and -2.262 for $\alpha = 0.05$ and d.f. $= 9$.

Step 3 The test value is

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}} = \frac{17.7 - 16.3}{1.8/\sqrt{10}} = 2.46$$

Step 4 Reject the null hypothesis since $2.46 > 2.262$. See Figure 8–22.

Figure 8–22
Summary of the t Test of Example 8–12



Step 5 There is enough evidence to reject the claim that the average number of infections is 16.3.

Example 8–13 Substitute Teachers' Salaries

An educator claims that the average salary of substitute teachers in school districts in Allegheny County, Pennsylvania, is less than \$60 per day. A random sample of eight school districts is selected, and the daily salaries (in dollars) are shown. Is there enough evidence to support the educator's claim at $\alpha = 0.10$?

60 56 60 55 70 55 60 55

Source: *Pittsburgh Tribune-Review.*

Solution

Step 1 H_0: μ = \$60 and H_1: μ < \$60 (claim).

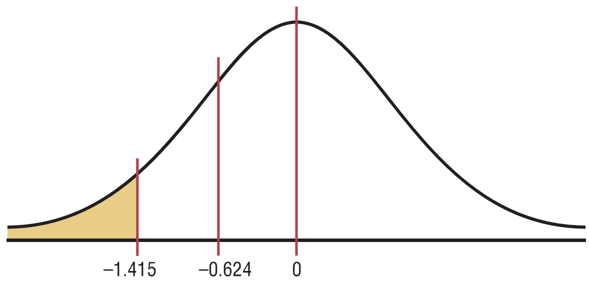
Step 2 At $\alpha = 0.10$ and d.f. = 7, the critical value is −1.415.

Step 3 To compute the test value, the mean and standard deviation must be found. Using either the formulas in Chapter 3 or your calculator, $\overline{X}$ = \$58.88, and $s = 5.08$, you find

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}} = \frac{58.88 - 60}{5.08/\sqrt{8}} = -0.624$$

Step 4 Do not reject the null hypothesis since −0.624 falls in the noncritical region. See Figure 8–23.

Figure 8–23
Critical Value and Test Value for Example 8–13



Step 5 There is not enough evidence to support the educator's claim that the average salary of substitute teachers in Allegheny County is less than \$60 per day.

The *P*-values for the *t* test can be found by using Table F; however, specific *P*-values for *t* tests cannot be obtained from the table since only selected values of α (for example, 0.01, 0.05) are given. To find specific *P*-values for *t* tests, you would need a table similar to Table E for each degree of freedom. Since this is not practical, only *intervals* can be found for *P*-values. Examples 8–14 to 8–16 show how to use Table F to determine intervals for *P*-values for the *t* test.

Example 8–14

Find the *P*-value when the *t* test value is 2.056, the sample size is 11, and the test is right-tailed.

Solution

To get the *P*-value, look across the row with 10 degrees of freedom (d.f. = $n - 1$) in Table F and find the two values that 2.056 falls between. They are 1.812 and 2.228. Since this is a right-tailed test, look up to the row labeled One tail, α and find the two α values corresponding to 1.812 and 2.228. They are 0.05 and 0.025, respectively. See Figure 8–24.

Figure 8–24

Finding the *P*-Value for Example 8–14

	Confidence intervals	50%	80%	90%	95%	98%	99%
	One tail, α	0.25	0.10	0.05	0.025	0.01	0.005
d.f.	Two tails, α	0.50	0.20	0.10	0.05	0.02	0.01
1		1.000	3.078	6.314	12.706	31.821	63.657
2		0.816	1.886	2.920	4.303	6.965	9.925
3		0.765	1.638	2.353	3.182	4.541	5.841
4		0.741	1.533	2.132	2.776	3.747	4.604
5		0.727	1.476	2.015	2.571	3.365	4.032
6		0.718	1.440	1.943	2.447	3.143	3.707
7		0.711	1.415	1.895	2.365	2.998	3.499
8		0.706	1.397	1.860	2.306	2.896	3.355
9		0.703	1.383	1.833	2.262	2.821	3.250
10		0.700	1.372	1.812 *	2.228	2.764	3.169
11		0.697	1.363	1.796	2.201	2.718	3.106
12		0.695	1.356	1.782	2.179	2.681	3.055
13		0.694	1.350	1.771	2.160	2.650	3.012
14		0.692	1.345	1.761	2.145	2.624	2.977
15		0.691	1.341	1.753	2.131	2.602	2.947
⋮		⋮	⋮	⋮	⋮	⋮	⋮
(z) ∞		0.674	1.282	1.645	1.960	2.326	2.576

*2.056 falls between 1.812 and 2.228.

Hence, the *P*-value would be contained in the interval 0.025 < *P*-value < 0.05. This means that the *P*-value is between 0.025 and 0.05. If α were 0.05, you would reject the null hypothesis since the *P*-value is less than 0.05. But if α were 0.01, you would not reject the null hypothesis since the *P*-value is greater than 0.01. (Actually, it is greater than 0.025.)

Example 8–15

Find the *P*-value when the *t* test value is 2.983, the sample size is 6, and the test is two-tailed.

Solution

To get the *P*-value, look across the row with d.f. = 5 and find the two values that 2.983 falls between. They are 2.571 and 3.365. Then look up to the row labeled Two tails, α to find the corresponding α values.

In this case, they are 0.05 and 0.02. Hence the *P*-value is contained in the interval 0.02 < *P*-value < 0.05. This means that the *P*-value is between 0.02 and 0.05. In this case, if $\alpha = 0.05$, the null hypothesis can be rejected since *P*-value < 0.05; but if $\alpha = 0.01$, the null hypothesis cannot be rejected since *P*-value > 0.01 (actually *P*-value > 0.02).

Note: **Since many of you will be using calculators or computer programs that give the specific *P*-value for the *t* test and other tests presented later in this textbook, these specific values, in addition to the intervals, will be given for the answers to the examples and exercises.**

The *P*-value obtained from a calculator for Example 8–14 is 0.033. The *P*-value obtained from a calculator for Example 8–15 is 0.031.

To test hypotheses using the *P*-value method, follow the same steps as explained in Section 8–2. These steps are repeated here.

Step 1 State the hypotheses and identify the claim.

Step 2 Compute the test value.

Step 3 Find the *P*-value.

Step 4 Make the decision.

Step 5 Summarize the results.

This method is shown in Example 8–16.

Example 8–16 Jogger's Oxygen Uptake

A physician claims that joggers' maximal volume oxygen uptake is greater than the average of all adults. A sample of 15 joggers has a mean of 40.6 milliliters per kilogram (ml/kg) and a standard deviation of 6 ml/kg. If the average of all adults is 36.7 ml/kg, is there enough evidence to support the physician's claim at $\alpha = 0.05$?

Solution

Step 1 State the hypotheses and identify the claim.

H_0: $\mu = 36.7$ and H_1: $\mu > 36.7$ (claim)

Step 2 Compute the test value. The test value is

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}} = \frac{40.6 - 36.7}{6/\sqrt{15}} = 2.517$$

Step 3 Find the *P*-value. Looking across the row with d.f. = 14 in Table F, you see that 2.517 falls between 2.145 and 2.624, corresponding to $\alpha = 0.025$ and $\alpha = 0.01$ since this is a right-tailed test. Hence, *P*-value > 0.01 and *P*-value < 0.025, or 0.01 < *P*-value < 0.025. That is, the *P*-value is somewhere between 0.01 and 0.025. (The *P*-value obtained from a calculator is 0.012.)

Step 4 Reject the null hypothesis since *P*-value < 0.05 (that is, *P*-value < α).

Step 5 There is enough evidence to support the claim that the joggers' maximal volume oxygen uptake is greater than 36.7 ml/kg.

Interesting Fact

The area of Alaska contains $\frac{1}{6}$ of the total area of the United States.

Students sometimes have difficulty deciding whether to use the *z* test or *t* test. The rules are the same as those pertaining to confidence intervals.

1. If σ is known, use the *z* test. The variable must be normally distributed if $n < 30$.
2. If σ is unknown but $n \geq 30$, use the *t* test.
3. If σ is unknown and $n < 30$, use the *t* test. (The population must be approximately normally distributed.)

These rules are summarized in Figure 8–25.

Speaking of Statistics

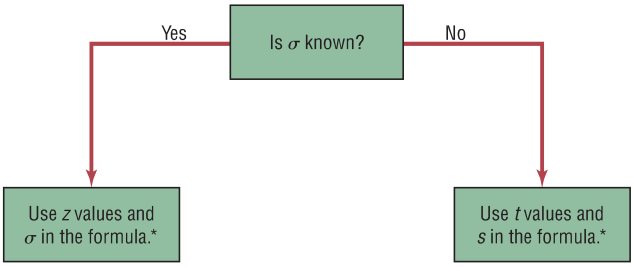
Can Sunshine Relieve Pain?

A study conducted at the University of Pittsburgh showed that hospital patients in rooms with lots of sunlight required less pain medication the day after surgery and during their total stay in the hospital than patients who were in darker rooms.

Patients in the sunny rooms averaged 3.2 milligrams of pain reliever per hour for their total stay as opposed to 4.1 milligrams per hour for those in darker rooms. This study compared two groups of patients. Although no statistical tests were mentioned in the article, what statistical test do you think the researchers used to compare the groups?

Figure 8–25

Using the *z* or *t* Test



*If $n < 30$, the variable must be normally distributed.

Applying the Concepts 8–3

How Much Nicotine Is in Those Cigarettes?

A tobacco company claims that its best-selling cigarettes contain at most 40 mg of nicotine. This claim is tested at the 1% significance level by using the results of 15 randomly selected cigarettes. The mean is 42.6 mg and the standard deviation is 3.7 mg. Evidence suggests that nicotine is normally distributed. Information from a computer output of the hypothesis test is listed.

Sample mean = 42.6	*P*-value = 0.008
Sample standard deviation = 3.7	Significance level = 0.01
Sample size = 15	Test statistic t = 2.72155
Degrees of freedom = 14	Critical value t = 2.62610

1. What are the degrees of freedom?
2. Is this a z or t test?
3. Is this a comparison of one or two samples?
4. Is this a right-tailed, left-tailed, or two-tailed test?