The Continuity

we Say that $f(x)$ is Continuous if and only if satisfy the following Conditions:-

1. $f(a) \in R$ is defined

2. $\lim_{x \to a} f(x)$ exist

3. 1. = 2. or $f(a) = \lim_{x \to a} f(x)$

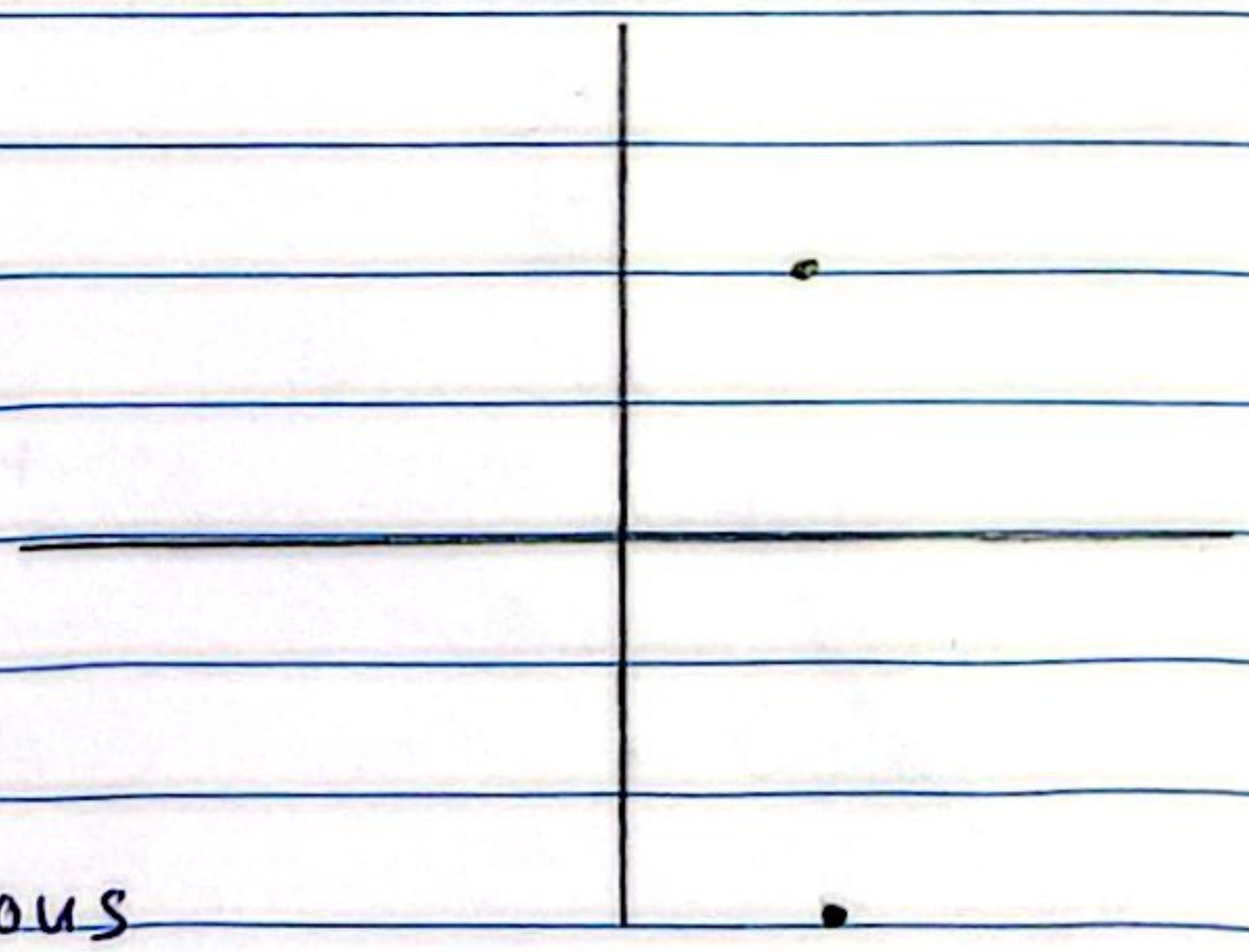
Ex 1. The $f(x)$ is Continuous or not ?

$$f(x) = \begin{cases} 2x - 5 & ; \quad x \neq 1 \\ 2 & ; \quad x = 1 \end{cases}$$

Solu $\lim_{x \to 1} f(x) = \lim_{x \to 1} (2x - 5)$

$= 2(1) - 5$

$= -3$



$f(x = 1) = 2$

$\because f(a) \neq \lim_{x \to a} f(x)$

$\therefore f(x)$ is not Continuous

2. Is the function $f(x) = x^2 + 3$ is Continuous if $x = 1$?

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Solu

① $f(x) = x^2 + 3$

$f(1) = 1 + 3 = 4 \in R$ is defined

② $\lim\limits_{x \to 1} f(x) = \lim\limits_{x \to 1} (x^2 + 3)$

$= 1 + 3 = 4$ exist

③ $f(1) = 4 = \lim\limits_{x \to 1} f(x) = x^2 + 3 = 4$

$\therefore$ $f(x)$ is Continuous at $x = 1$

3. $f(x) = \dfrac{x}{x+1}$ at $x = 3$

مجال الدالة $= R / (-1)$

① $f(3) = \dfrac{3}{3+1} = \dfrac{3}{4}$

② $\lim\limits_{x \to 3} \dfrac{x}{x+1} = \dfrac{3}{3+1} = \dfrac{3}{4}$

$\because$ ① = ②

$\therefore$ The function is continuous.

4. If $f(x)$ is continuous or not?

$f(x) = \sqrt{x} \quad ; \quad x \geq 0$

$\lim_{x \to 0} \sqrt{x} = \sqrt{0}$ is not found

$\therefore$ The function is not continuous.

5. $f(x) = \begin{cases} \dfrac{x^2-9}{x+3} & ; \quad x \neq -3 \\ -6 & ; \quad x = -3 \end{cases}$

Solu

$$\lim_{x \to -3} \frac{x^2-9}{x+3} = \lim_{x \to -3} \frac{(x-3)(x+3)}{x+3} = -6$$

$f(-3) = -6$

$\therefore f(x)$ is continuous

Techniques of differentiation

① Derivative of a constant function

$f(x) = c$; c : any constant

$f'(x) = 0$

Ex
$f(x) = \sqrt{2} \Rightarrow f'(x) = 0$

$f(x) = 2\sqrt{5} + 6 \Rightarrow f'(x) = 0$

② Derivative of alinear function

$f(x) = mx + c$; m, c real no.

$f'(x) = m$

Ex
$f(x) = 7x - 2 \Rightarrow f'(x) = 7$

③ Derivative of Power functions

$f(x) = x^n$ $\quad$ n: integer no.

$f'(x) = n x^{n-1}$ $\quad$ $n = \mp 1, \mp 2, \ldots$

Proof

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

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$$f'(x) = \lim_{h \to 0} \frac{[\cancel{x^n} + n x^{n-1} h + \frac{n(n-1)}{2!} x^{n-2} h^2 + \cdots + n x h^{n-1} + h^n] - \cancel{x^n}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}[n x^{n-1} + \frac{n(n-1)}{2!} x^{n-2} h + \cdots + n x h^{n-2} + h^{n-1}]}{\cancel{h}}$$

$$\therefore f'(x) = n x^{n-1}$$

Ex

1. $f(x) = x^3 \Rightarrow f'(x) = 3x^2$

2. $f(x) = x^{\frac{1}{2}} = \sqrt{x} \Rightarrow f'(x) = \frac{1}{2} x^{\frac{1}{2}-1}$

$$= \frac{1}{2\sqrt{2}}$$

3. $f(x) = \sqrt[3]{x^2} = x^{\frac{2}{3}}$

$$f'(x) = \frac{2}{3} x^{\frac{2}{3}-1}$$

$$= \frac{2}{3} x^{-\frac{1}{3}} \Rightarrow f'(x) = \frac{2}{3\sqrt[3]{x}}$$

4. $f(x) = x^8 + 12x^5 - 4x^4 + 10x^3 - 6x + 5$

$$f'(x) = 8x^7 + 60x^4 - 16x^3 + 30x^2 - 6$$

1. **Ex** $f(X) = (X^3+1)(2X^2+8X-5)$

Find $f'(X)$?

Solu $f'(X) = (X^3+1)(4X+8) + (2X^2+8X-5)(3X^2)$

2. $f(X) = (3X^4-5X)(X^2+\sqrt{X})$

$f'(X) = (12X^3-5)(X^2+\sqrt{X}) + (3X^4-5X)\left(2X+\frac{1}{2\sqrt{X}}\right)$

3. $f(X) = \frac{1+3X}{3X}(3-X)$

$f'(X) = \left(\frac{1+3X}{3X}\right)(-1) + (3-X)\left(\frac{3X\cdot 3-(1+3X)\cdot 3}{(3X)^2}\right)$

$= \frac{-1-3X}{3X} + (3-X)\cdot\frac{\cancel{9X}-3-\cancel{9X}}{9X^2}$

$= \frac{-1-3X}{3X} + (3-X)\cdot\frac{-\cancel{3}}{{}^{3}\cancel{9}X^2}$

$= \frac{-1-3X}{3X} + (3-X)\cdot\frac{-1}{3X^2}$

$= \frac{-1-3X}{3X} + \frac{(-3+X)}{3X^2}$

$= \frac{-1}{3X} - \frac{3X}{3X} - \frac{3}{3X^2} + \frac{X}{3X^2}$

$= \cancel{\frac{-1}{3X}} - 1 - \frac{1}{X^2} + \cancel{\frac{1}{3X}} = -1 - \frac{1}{X^2}$

(4) The derivative of the product of two functions

$$D_x(f(x)\cdot g(x)) = f(x)\cdot g'(x) + g(x)\cdot f'(x)$$

Proof

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$g'(x) = \lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$$

Then:-

$$(f(x)\cdot g'(x))' = \lim_{h\to 0} \frac{f(x+h)\cdot g(x+h) - f(x)\cdot g(x)}{h}$$

we subtract and add the term $f(x+h)\cdot g(x)$ in the numerator :-

$$= \lim_{h\to 0} \frac{f(x+h)\cdot g(x+h) + f(x+h)\cdot g(x) - f(x+h)\cdot g(x) - f(x)\cdot g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)\cdot g(x+h) - f(x+h)\, g(x) - f(x+h)\, g(x) - f(x)\, g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)[g(x+h) - f(x+h)] + g(x)[f(x+h) - f(x)]}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)[g(x+h) - f(x+h)]}{h} + \lim_{h\to 0} \frac{g(x)[f(x+h) - f(x)]}{h}$$

$$= \lim_{h\to 0} f(x+h) \cdot \lim_{h\to 0} \frac{g(x+h) - f(x+h)}{h} + \lim_{h\to 0} g(x) \lim \frac{f(x+h) - f(x)}{h}$$

$$= f(x)\, g'(x) + g(x)\cdot f'(x)$$