⑤ The derivative of the Quotient of two functions is :-

H.w.

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{(g(x))^2}$$

Ex Find the derivative of the following:-

1. $f(x) = \dfrac{3x^2 - x + 2}{4x^2 + 5}$

$$f'(x) = \frac{(4x^2+5)(6x-1) - (3x^2 - x + 2)(8x)}{(4x^2+5)^2}$$

$$= \frac{24x^3 - 4x^2 + 30x - 5 - 24x^3 + 8x^2 - 16}{16x^4 + 40x^2 + 25}$$

2. $y = x^2 + \tan x^3$

$$y' = 2x + \sec^2 x^3 (3x^2)$$

$$= 2x + 3x^2 \sec^2 x^3$$

3. $y = e^x (x^2 + 1)$

$$y' = e^x (2x) + (x^2+1) e^x \cdot 1$$

$$= 2xe^x + e^x (x^2+1)$$

4. $y = \dfrac{\sec x^2}{x}$

$$y' = \frac{x \sec x^2 \tan x^2 (2x) - \sec x^2 (1)}{x^2}$$

5. $y = e^{x+y}$

$y' = e^{x+y}(1+y')$

$y' = e^{x+y} + y' e^{x+y}$

$y' - y' e^{x+y} = e^{x+y}$

$y'(1-e^{x+y}) = e^{x+y}$

$$y' = \frac{e^{x+y}}{1-e^{x+y}}$$

H.w. Find $y'$ for the following:

1. $4x^2 + 3xy - xy^2 = 0$

2. $3x^2y^3 + 6xy - 5x^2 = 0$

3. $y^2 = \tan \frac{x}{y}$

4. $y = \frac{x^2}{xy}$

5. $y\sqrt{2+3x} + x\sqrt{1+y} = x$

Concept of derivatives and the fundemntal differetiation:-

Derivatives:-
The derivative of afunction f is denoted by $f'$ and defined by:-

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Ex Find the derivative of the function:-

1. $f(x) = x^2 + 2x$ by using the difinition:-

Solu

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$f(x) = x^2 + 2x$ ; $f(x+h) = (x+h)^2 + 2(x+h)$

$$\therefore f'(x) = \lim_{h \to 0} \frac{(x+h)^2 + 2(x+h) - (x^2 + 2x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{2x} + 2h - \cancel{x^2} - \cancel{2x}}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 + 2h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x + h + 2)}{\cancel{h}}$$

$$= 2x + 2$$

2. $f(x) = \frac{1}{x}$

Solu

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = \frac{1}{x} \quad ; \quad f(x+h) = \frac{1}{x+h}$$

$$\therefore f'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x - x - h}{x(x+h)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{-h}{x(x+h)}}{\frac{h}{1}}$$

$$= \lim_{h \to 0} \frac{-\cancel{h}}{x(x+h)} \cdot \frac{1}{\cancel{h}}$$

$$= \frac{-1}{x^2}$$

H.w.

1. $f(x) = 2x^2 + 4x$
2. $f(x) = x^2 + 1$
3. $f(x) = x^2 + 3$

H.W.

1. $y = f(x) = 2x^2 + 4x$

Solu

$$f(x+h) = 2(x+h)^2 + 4(x+h)$$

$$\therefore f'(x) = \lim_{h \to 0} \frac{2(x+h)^2 + 4(x+h) - (2x^2 + 4x)}{h}$$

$$= \lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) + 4x + 4h - 2x^2 - 4x}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 + \cancel{4x} + 4h - \cancel{2x^2} - \cancel{4x}}{h}$$

$$= \lim_{h \to 0} \frac{4xh + 2h^2 + 4h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(4x + 2h + 4)}{\cancel{h}}$$

$$= 4x + 4$$

2. $f(x) = x^2 + 1$ ; $f(x+h) = (x+h)^2 + 1$

$$f'(x) = \lim_{x \to h} \frac{(x+h)^2 + 1 - (x^2 + 1)}{h}$$

$$= \lim_{x \to h} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{1} - \cancel{x^2} - \cancel{1}}{h}$$

$$= \lim_{x \to h} \frac{\cancel{h}(2x + h)}{\cancel{h}}$$

$$= \lim_{x \to h} 2x + h = 2x$$

3. $f(x) = x^2+3$ ; $f(x+h) = (x+h)^2+3$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2+3-(x^2+3)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2}+2xh+h^2\cancel{+3}-\cancel{x^2}\cancel{-3}}{h}$$

$$= \lim_{h \to 0} \frac{2xh+h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2x+h)}{h}$$

$$= \lim_{h \to 0} 2x+h$$

$$= 2x$$

another H.w.

1. $f(x) = x^3$

Ex Find the derivative of the function :-

$f(x) = \frac{7}{\sqrt{x}}$ using the diffinition

Solu

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = \frac{7}{\sqrt{x}} \quad ; \quad f(x+h) = \frac{7}{\sqrt{x+h}}$$

$$\therefore f'(x) = \lim_{h \to 0} \frac{\frac{7}{\sqrt{x+h}} - \frac{7}{\sqrt{x}}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{7\sqrt{x} - 7\sqrt{x+h}}{\sqrt{x+h}\,\sqrt{x}}}{h}$$ multiple Conjugate

$$= \lim_{h \to 0} \frac{\frac{7\sqrt{x} - 7\sqrt{x+h}}{\sqrt{x+h}\,\sqrt{x}} \cdot \frac{7\sqrt{x} + 7\sqrt{x+h}}{7\sqrt{x} + 7\sqrt{x+h}}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{49x + \cancel{49\sqrt{x}\sqrt{x+h}} - \cancel{49\sqrt{x}\sqrt{x+h}} - 49(x+h)}{\sqrt{x+h}\,\sqrt{x}\,(7\sqrt{x} + 7\sqrt{x+h})}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{49x - 49(x+h)}{7x\sqrt{x+h} + 7(x+h)\sqrt{x}}}{h}$$

$$= \lim_{h \to 0} \frac{7(\cancel{7x} - \cancel{7x} - 7h)}{7x\sqrt{x+h} + 7\sqrt{x}(x+h)} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{\overset{7}{\cancel{-49}}\,\cancel{h}}{\cancel{7}[x\sqrt{x+h} + \sqrt{x}(x+h)} \cdot \frac{1}{\cancel{h}}$$

$$= \lim_{h \to 0} \frac{-7}{x\sqrt{x+h} + (x+h)\sqrt{x}}$$

$$\therefore f'(x) = \frac{-7}{x\sqrt{x} + \sqrt{x}\ x}$$

$$= \frac{-7}{2x\sqrt{x}}$$

or

$$f(x) = \frac{7}{\sqrt{x}}$$

$$f'(x) = \frac{\sqrt{x}(0) - 7\ \frac{1}{2}\ x^{-\frac{1}{2}}}{(\sqrt{x})^2}$$

$$= \frac{-7}{2x\sqrt{x}}$$