Minterm and Maxterm

There are two ways in which we can put the Boolean function. These ways are the minterm canonical form and maxterm canonical form.

Literal

A Literal signifies the Boolean variables including their complements. Such as B is a boolean variable and its complements are ~B or B', which are the literals.

Minterm

The product of all literals, either with complement or without complement, is known as **minterm**.

Example

The minterm for the Boolean variables A and B is:

1. A.B
2. A.~B
3. ~A.B

The complement variables ~A and ~B can also be written as A' and B' respectively. Thus, we can write the minterm as:

1. A.B'
2. A'.B

Minterm from values

Using variable values, we can write the minterms as:

1. If the variable value is 1, we will take the variable without its complement.
2. If the variable value is 0, take its complement.

Example

Let's assume that we have three Boolean variables A, B, and C having values

A=1
B=0
C=0

Now, we will take the complement of the variables B and C because these values are 0 and will take A without complement. So, the minterm will be:

Minterm=A.B'C'

Let's take another example in which we have two variables B and C having the value

B = 0
C = 1
Minterm=B'C

Shorthand notation for minterm

We know that, when Boolean variables are in the form of minterm, the variables will appear in the product. There are the following steps for getting the shorthand notation for minterm.

- In the first step, we will write the term consisting of all the variables
- Next, we will write 0 in place of all the complement variables such as ~A or A'.
- We will write 1 in place of all the non-complement variables such as A or b.
- Now, we will find the decimal number of the binary formed from the above steps.
- In the end, we will write the decimal number as a subscript of letter **m**(minterm). Let's take some example to understand the theory of shorthand notation

Example 1: Minterm = AB'

- First, we will write the minterm: Minterm = AB'
- Now, we will write 0 in place of complement variable B'. Minterm = A0
- We will write 1 in place of non-complement variable A. Minterm = 10
- The binary number of the minterm AB' is 10. The decimal point number of $(10)_2$ is 2. So, the shorthand notation of AB' is Minterm = m_2

Example 2: Minterm = AB'C'

- First, we will write the minterm: Minterm = AB'C'
- Now, we will write 0 in place of complement variables B' and C'. Minterm = A00

- We will write 1 in place of non-complement variable A. Minterm = 100
- The binary number of the minterm AB'C' is 100. The decimal point number of $(100)_2$ is 4. So, the shorthand notation of AB'C' is
  Minterm = m_4

Maxterm

The sum of all literals, either with complement or without complement, is known as **maxterm**.

Example:

The maxterm for the Boolean variables A and B will be:

1. A+B
2. A+~B
3. ~A+B

We know that the complement variables ~A and ~B can be written as A' and B' respectively. So, the above maxterm can be written as

1. A+B'
2. A'+B

Maxterm from values

Using the given variable values, we can write the maxterm as:

1. If the variable value is 1, then we will take the variable without a complement.
2. If the variable value is 0, take the complement of the variable.

Example

Let's assume that we have three Boolean variables A, B., and C having values

A=1
B=0
C=0
Now, we will take the complement of the variables B and C because these values are 0 and will take A without complement. So, the maxterm will be:

Maxterm=A+B'+C'

Let's take another example in which we have two variables B and C having the value

B = 0
C = 1
Maxterm=B'+C

Shorthand notation for maxterm

We know that, when Boolean variables are in the form of maxterm, the variables will appear in sum. The steps for the maxterm are same as minterm:

- In the first step, we will write the term consisting of all the variables
- Next, we will write 0 in place of all the complement variables such as ~A or A'.
- We will write 1 in place of all the non-complement variables such as A or b.
- Now, we will find the decimal number of the binary formed from the above steps.
- In the end, we will write the decimal number as a subscript of letter Here, M denotes maxterm.

Let's take some example to understand the theory of shorthand notation

Example 1: Maxterm = A+B'

- First, we will write the minterm: Maxterm = A+B'
- Now, we will write 0 in place of complement variable B'.
- We will write 1 in place of non-complement variable A.
- The binary number of the maxterm A+B' is 10. The decimal point number of $(10)_2$ is 2. So, the shorthand notation of A+B' is Maxterm = M_2

Example 2: Maxterm = A+B'+C'

- First, we will write the maxterm: Maxterm = A+B'+C'
- Now, we will write 0 in place of complement variables B' and C'.
- We will write 1 in place of non-complement variable A.
- The binary number of the maxterm A+B'+C' is 100. The decimal point number of $(100)_2$ is 4. So, the maxterm of A+B'+C' is m_4.

Sum of product(SOP)

A canonical sum of products is a boolean expression that entirely consists of minterms. The Boolean function F is defined on two variables X and Y. The X and Y are the inputs of the boolean function F whose output is true when any one of the inputs is set to true. The truth table for Boolean expression F is as follows:

X	Y	F
0	0	0
0	1	1
1	0	1
1	1	1

In our previous section, we learned about how we can form the minterm from the variable's value. Now, a column will be added for the minterm in the above table. The complement of the variables is taken whose value is 0, and the variables whose value is 1 will remain the same.

X	Y	F	M
0	0	0	X'Y'
0	1	1	X'Y
1	0	1	XY'
1	1	1	XY

Now, we will add all the minterms for which the output is true to find the desired canonical SOP(Sum of Product) expression.

F=X' Y+XY'+XY

Converting Sum of Products (SOP) to shorthand notation

The process of converting SOP form to shorthand notation is the same as the process of finding shorthand notation for minterms. There are the following steps to find the shorthand notation of the given SOP expression.

- Write the given SOP expression.
- Find the shorthand notation of all the minterms.
- Replace the minterms with their shorthand notations in the given expression.

Example: F = X'Y+XY'+XY

1. Firstly, we write the SOP expression:

F = X'Y+XY'+XY

2. Now, we find the shorthand notations of the minterms X'Y, XY', and XY.

$X'Y = (01)_2 = m_1$
$XY' = (10)_2 = m_2$
$XY = (11)_2 = m_3$

3. In the end, we replace all the minterms with their shorthand notations:

F=m1+m2+m3

Converting shorthand notation to SOP expression

The process of converting shorthand notation to SOP is the reverse process of converting SOP expression to shorthand notation. Let's see an example to understand this conversion.

Example:

Let us assume that we have a boolean function F, which defined on two variables X and Y. The minterms for the function F are expressed as shorthand notation is as follows:

F=∑(1,2,3)

Now, from this expression, we will find the SOP expression. The Boolean function F has two input variables X and y and the output of F=1 for m1, m2, and m3, i.e., 1^{st}, 2^{nd}, and 3^{rd} combinations. So,

$F=\sum(1,2,3)$
$F= m1 + m2 + m3$
$F= 01 + 10 + 11$

Now, we replace zeros with either X' or Y' and ones with either X or Y. Simply, the complement variable is used when the variable value is 1 otherwise the non-complement variable is used.

$F = \sum(1,2,3)$
$F=01+10+11$
$F= A'B + AB' + AB$

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X	Y	F	M
0	0	0	X'Y'
0	1	1	X'Y
1	0	1	XY'
1	1	1	XY

Now, we will add all the minterms for which the output is true to find the desired canonical SOP(Sum of Product) expression.

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F=∑(1,2,3)
F= m1 + m2 + m3
F= 01 + 10 + 11

Now, we replace zeros with either X' or Y' and ones with either X or Y. Simply, the complement variable is used when the variable value is 1 otherwise the non-complement variable is used.

F = ∑(1,2,3)
F=01+10+11
F= A'B + AB' + AB

Product of Sum (POS)

A canonical product of sum is a boolean expression that entirely consists of maxterms. The Boolean function F is defined on two variables X and Y. The X and Y are the inputs of the boolean function F whose output is true when only one of the inputs is set to true. The truth table for Boolean expression F is as follows:

X	**Y**	**F**
0	0	0

0	1	1
1	0	1
1	1	0

In our minterm and maxterm section, we learned about how we can form the maxterm from the variable's value. A column will be added for the maxterm in the above table. The complement of the variables is taken whose value is 0, and the variables whose value is 1 will remain the same.

X	Y	F	M
0	0	0	X'+Y'
0	1	1	X'+Y
1	0	1	X+Y'
1	1	1	X+Y

Now, we will multiply all the minterms for which the output is false to find the desired canonical POS(Product of sum) expression.

F=(X'+Y').(X+Y)

Converting Product of Sum (POS) to shorthand notation

The process of converting POS form to shorthand notation is the same as the process of finding shorthand notation for maxterms. There are the following steps used to find the shorthand notation of the given POS expression.

- Write the given POS expression.
- Find the shorthand notation of all the maxterms.

- Replace the minterms with their shorthand notations in the given expression.

Example: F = (X'+Y').(X+Y)

1. Firstly, we will write the POS expression:

F = (X'+Y').(X+Y)

2. Now, we will find the shorthand notations of the maxterms X'+Y' and X+Y.

X'+Y' = $(00)_2 = M_0$
X+Y = $(11)_2 = M_3$

3. In the end, we will replace all the minterms with their shorthand notations:

$F=M_0.M_3$

Converting shorthand notation to POS expression

The process of converting shorthand notation to POS is the reverse process of converting POS expression to shorthand notation. Let's see an example to understand this conversion.

Example:

Let us assume that we have a boolean function F, defined on two variables X and Y. The maxterms for the function F are expressed as shorthand notation is as follows:

$F=\prod(1,2,3)$

Now, from this expression, we find the POS expression. The Boolean function F has two input variables X and Y and the output of F=0 for M1, M2, and M3, i.e., 1st, 2nd, and 3rd combinations. So,

$F=\prod(1,2,3)$
F= M1.M2.M3
F= 01.10.11

Next, we replace zeros with either X or Y and ones with either X' or Y'. Simply, if the value of the variable is 1, then we take the complement of that variable, and if the value of the variable is 0, then we take the variable "as is".

F = ∑(1,2,3)
F=01.10.11
F=(A+B').(A'+B).(A'+B')

Similarly, we add p*p' = 1 in this term for getting the term containing all the variables.

(q' + r + s' + p*p') = (p + q' + r + s') * (p' + q' + r + s')
3. Term (q' + r + s')

Now, there is no need to add anything because all the variables are contained in this term.

So, the standard POS form equation of the function is

F = (p' + q + r + s)* (p' + q + r + s')* (p + q' + r + s')* (p' + q' + r + s') * (p + q' + r' + s)

Conversion between Canonical Forms

In our previous section, we learned about SOP(sum of product) and POS(product of sum) expressions and calculated POS and SOP forms for different Boolean functions. In this section, we will learn about how we can represent the POS form in the SOP form and SOP form in the POS form.

For converting the canonical expressions, we have to change the symbols ∏, ∑. These symbols are changed when we list out the index numbers of the equations. From the original form of the equation, these indices numbers are excluded. The SOP and POS forms of the boolean function are duals to each other.

There are the following steps using which we can easily convert the canonical forms of the equations:

1. Change the operational symbols used in the equation, such as ∑, ∏.
2. Use the Duality's De-Morgan's principal to write the indexes of the terms that are not presented in the given form of an equation or the index numbers of the Boolean function.

Conversion of POS to SOP form

For getting the SOP form from the POS form, we have to change the symbol ∏ to ∑. After that, we write the numeric indexes of missing variables of the given Boolean function.

There are the following steps to convert the POS function F = Π x, y, z (2, 3, 5) = x y' z' + x y' z + x y z' into SOP form:

1. In the first step, we change the operational sign to Σ.
2. Next, we find the missing indexes of the terms, 000, 110, 001, 100, and 111.
3. Finally, we write the product form of the noted terms.

000 = x' * y' * z'

001 = x' * y' * z

100 = x * y' * z'

110 = x * y* z'

111 = x * y * z

So the SOP form is:

F = Σ x, y, z (0, 1, 4, 6, 7) = (x' * y' * z') + (x' * y' * z) + (x * y' * z') + (x * y* z') + (x * y * z)

Conversion of SOP form to POS form

For getting the POS form of the given SOP form expression, we will change the symbol ∏ to ∑. After that, we will write the numeric indexes of the variables which are missing in the boolean function.

There are the following steps used to convert the SOP function F = ∑ x, y, z (0, 2, 3, 5, 7) = x' y' z' + z y' z' + x y' z + xyz' + xyz into POS:

- In the first step, we change the operational sign to ∏.
- We find the missing indexes of the terms, 001, 110, and 100.
- We write the sum form of the noted terms.

001 = (x + y + z)

100 = (x + y' + z')

110 = (x + y' + z')

So, the POS form is:

F = Π x, y, z (1, 4, 6) = (x + y + z) * (x + y' + z') * (x + y' + z')

Conversion of SOP form to standard SOP form or Canonical SOP form

For getting the standard SOP form of the given non-standard SOP form, we will add all the variables in each product term which do not have all the variables. By using the Boolean algebraic law, (x + x' = 0) and by following the below steps we can easily convert the normal SOP function into standard SOP form.

- Multiply each non-standard product term by the sum of its missing variable and its complement.
- Repeat step 1, until all resulting product terms contain all variables
- For each missing variable in the function, the number of product terms doubles.

Example:

Convert the non standard SOP function F = AB + A C + B C

Sol:

F = A B + A C + B C
= A B (C + C') + A (B + B') C + (A + A') B C
= A B C + A B C' + A B C + A B' C + A B C + A' B C
= A B C + A B C' + A B' C + A' B C
So, the standard SOP form of non-standard form is F = A B C + A B C' + A B' C + A' B C

Conversion of POS form to standard POS form or Canonical POS form

For getting the standard POS form of the given non-standard POS form, we will add all the variables in each product term that do not have all the variables. By using the Boolean algebraic law (x * x' = 0) and by following the below steps, we can easily convert the normal POS function into a standard POS form.

- By adding each non-standard sum term to the product of its missing variable and its complement, which results in 2 sum terms
- Applying Boolean algebraic law, x + y z = (x + y) * (x + z)
- By repeating step 1, until all resulting sum terms contain all variables

By these three steps, we can convert the POS function into a standard POS function.

Example:

F = (p' + q + r) * (q' + r + s') * (p + q' + r' + s)

1. Term (p' + q + r)

As we can see that the variable s or s' is missing in this term. So we add s*s' = 1 in this term.

(p' + q + r + s*s') = (p' + q + r + s) * (p' + q + r + s')

2. Term (q' + r + s')

Similarly, we add p*p' = 1 in this term for getting the term containing all the variables.

(q' + r + s' + p*p') = (p + q' + r + s') * (p' + q' + r + s')

3. Term (q' + r + s')

Now, there is no need to add anything because all the variables are contained in this term.

So, the standard POS form equation of the function is

F = (p' + q + r + s)* (p' + q + r + s')* (p + q' + r + s')* (p' + q' + r + s') * (p + q' + r' + s)