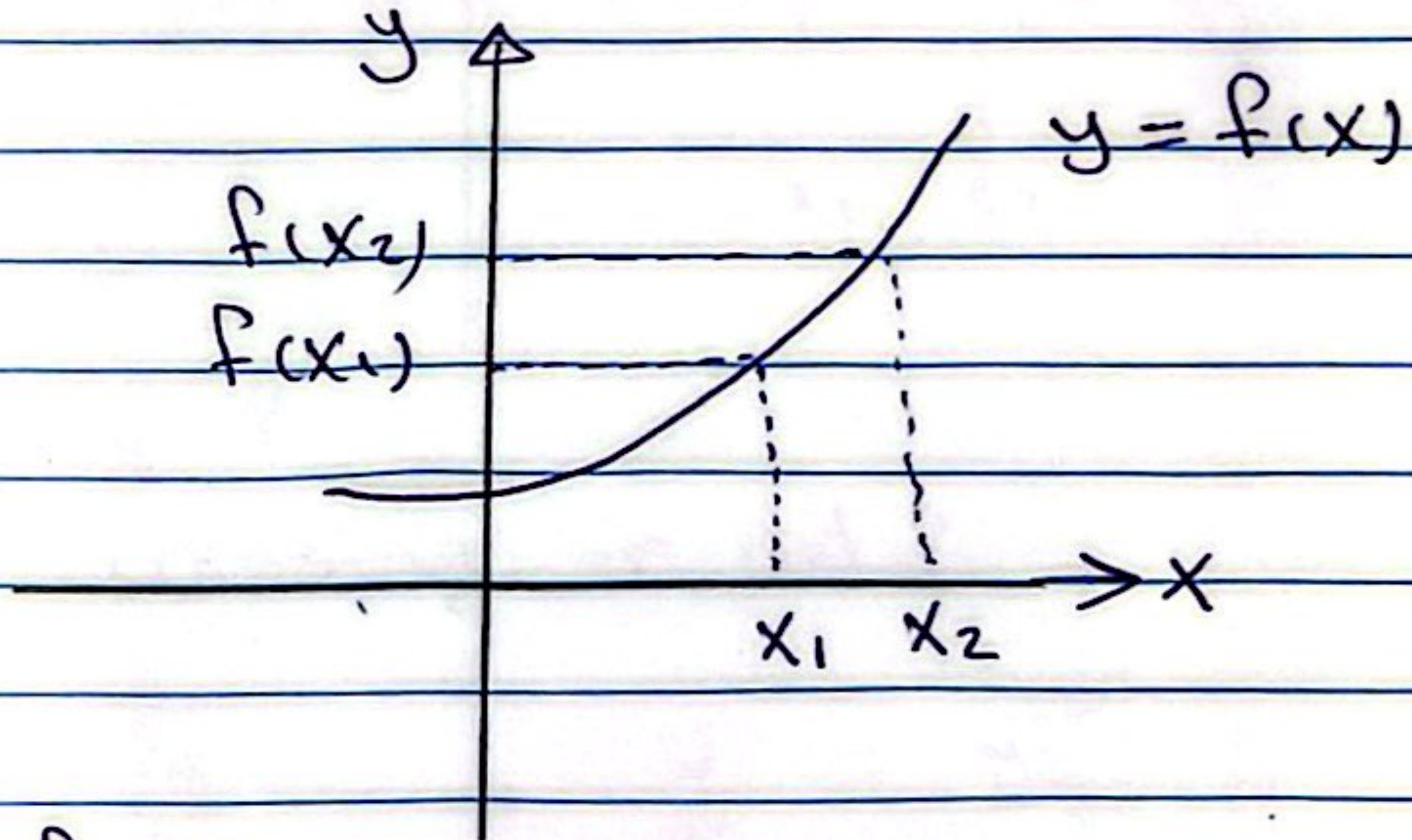
Applications on the derivative

- Increasing and Decreasing Functions:-

Def.: we say that $f(x)$ is increasing function iff satisfy :-

$$f(x_1) \le f(x_2)$$ to all values of x_1 and x_2

such that $x_1 \le x_2$



Ex let the function $y = 2 + x$

is y Increasing or not ?

Sol:

x	y	x	y
1	3	0	2
2	4	-1	1
3	5	-2	0
4	6	-3	-1
تزايد	تزايد	تناقص	تناقص

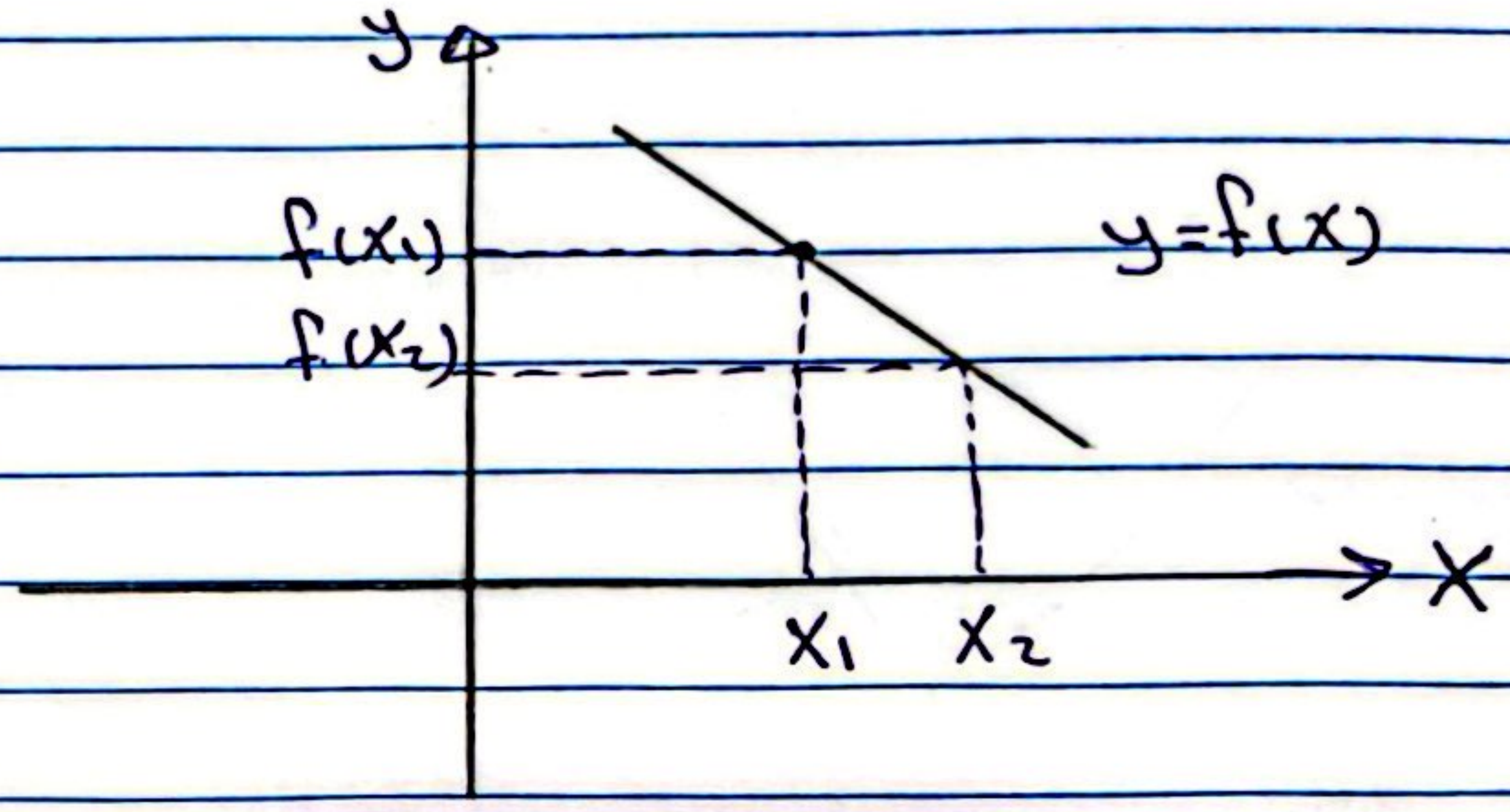
كلما تزداد x تزداد y — كلما تقل x تقل y

$\therefore$ y is Increasing

Def.: we say that $f(x)$ is Decreasing function iff

$$f(x_1) \geq f(x_2) \quad \text{for all values of } x_1 \text{ and } x_2$$

$$\text{Such that} \quad x_1 \leq x_2$$



Ex If $y = 4 - x$

Is y decreasing or not?

Solu

x	y	x	y
1	3	0	4
2	2	-1	5
3	1	-2	6
4	0	-3	7
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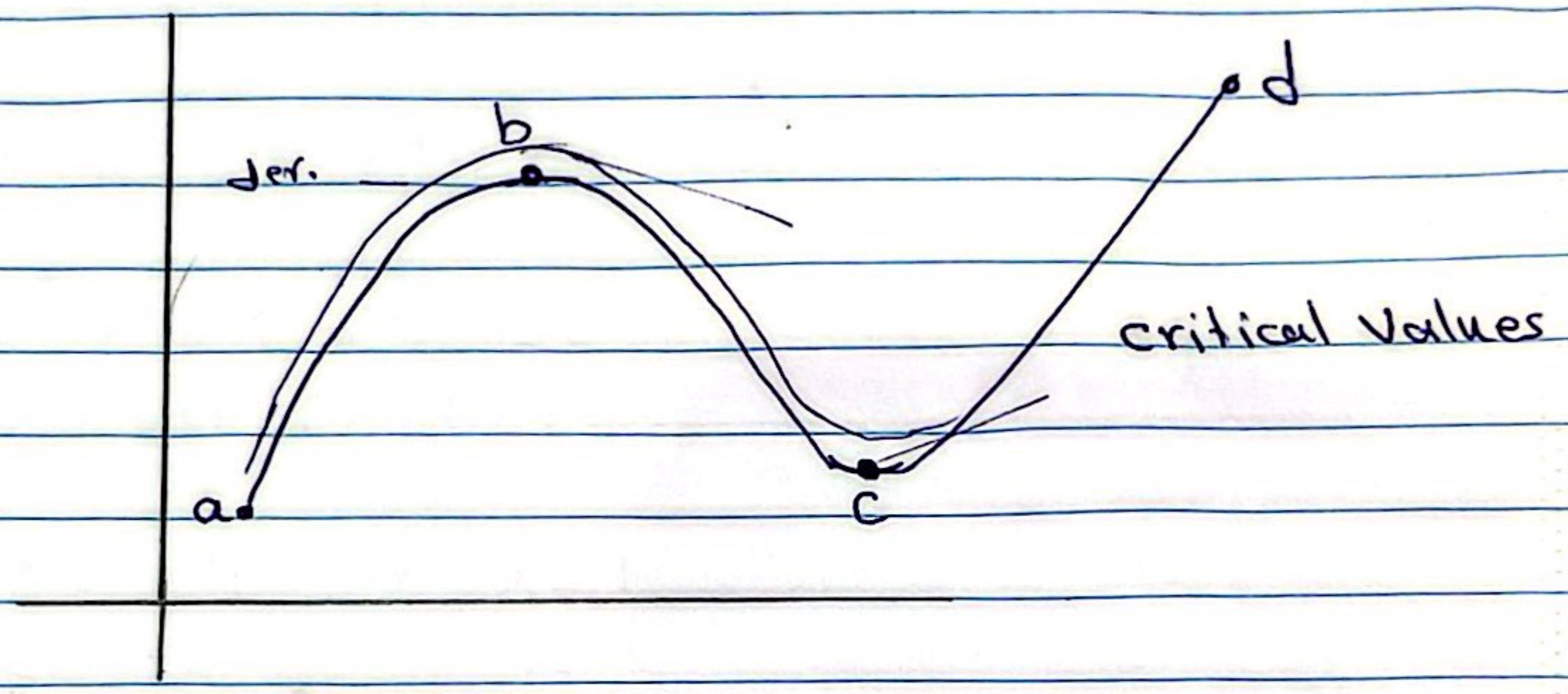
$\therefore$ y is decreasing

<u>Def.:</u> we say that $f(x)$ is increasing iff $f'(x)$ is positive for all values of x

and $f(x)$ is decreasing iff $f'(x)$ is negative for all values of x



<u>Ex1.</u> If $f(x) = 2x^3 - 6x$ is increasing or not ?
and identify the poinst of decreasing and increasing in the functi-

<u>Solu</u> $f'(x) = 6x^2 - 6$

$= 6(x^2 - 1) > 0$ positive

$f'(x) = 0 \Rightarrow 6(x^2 - 1) = 0$

$6 = 0$

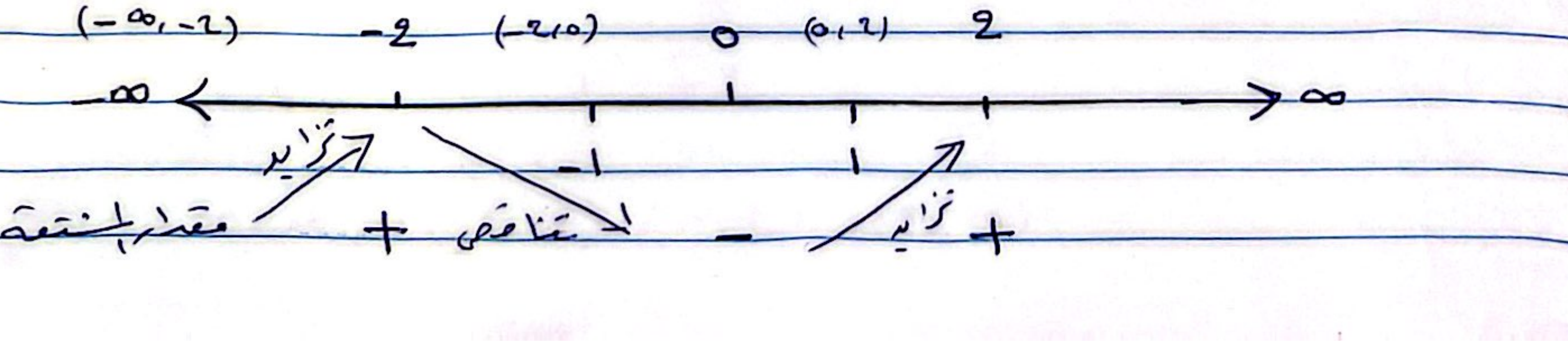
$x^2 - 1 = 0 \Rightarrow x = \mp 1$

$\therefore$ the critical values are $(-1, +1)$

$(-\infty, -1)$, $(-1, 1)$, $(1, \infty)$

لتحديد التزايد والتناقص في الدالة: نأخذ قيم بين النقاط الحرجة وهي (-1,1) وهو 0 ونعوض في المشتقة لنحصل على قيمة المشتقة بالسالب ثم قيم أقل من -1 وأكبر من 1 لنحصل على موجب المشتقة .

$\therefore y$ is increasing



2. $f(x) = 2x^3 + 9x^2 - 24x - 10$

حدد نقاط التزايد والتناقص في الدالة

<u>Sol</u>

$f'(x) = 6x^2 + 18x - 24 \quad > 0$

$f'(x) = 0$

$6x^2 + 18x - 24 = 0 \qquad \div 6$

$x^2 + 3x - 4 = 0$

$(x+4)(x-1) = 0$

$x = -4 \quad \text{or} \quad x = 1$

$\in R \qquad \in R$

$\because$ The critical values are $(-4, 1)$

$(-\infty, -4)$ increasing $(1, \infty)$ increasing , $(-4, 1)$ decreasing

$-5 \qquad -2 \qquad 2$

$-\infty \longleftarrow \text{———————————————} \longrightarrow \infty$

$-4 \qquad 1$

الاشارة $+ \; + \; + \quad 0 \quad - \; - \; - \quad 0 \quad + \; + \; +$

تزايد $\longrightarrow$ تناقص $\longrightarrow$ تزايد

$\because f(x)$ is increasing.

H.W. Find increasing and decreasing value in the function:

1. $f(x) = x^4 - 8x^2 + 1$

$f'(x) = 4x^3 - 16x \qquad ; f'(x) = 0$

$4x^3 - 16x = 0 \qquad \div 4$

$x^3 - 4x = 0$

$x(x^2 - 4) = 0$

$x = 0 \quad \text{or} \quad x^2 = \mp 2$

تناقص تزايد تناقص تزايد

$(-\infty, -2), (-2, 0)\ (0, 2), (2, \infty)$

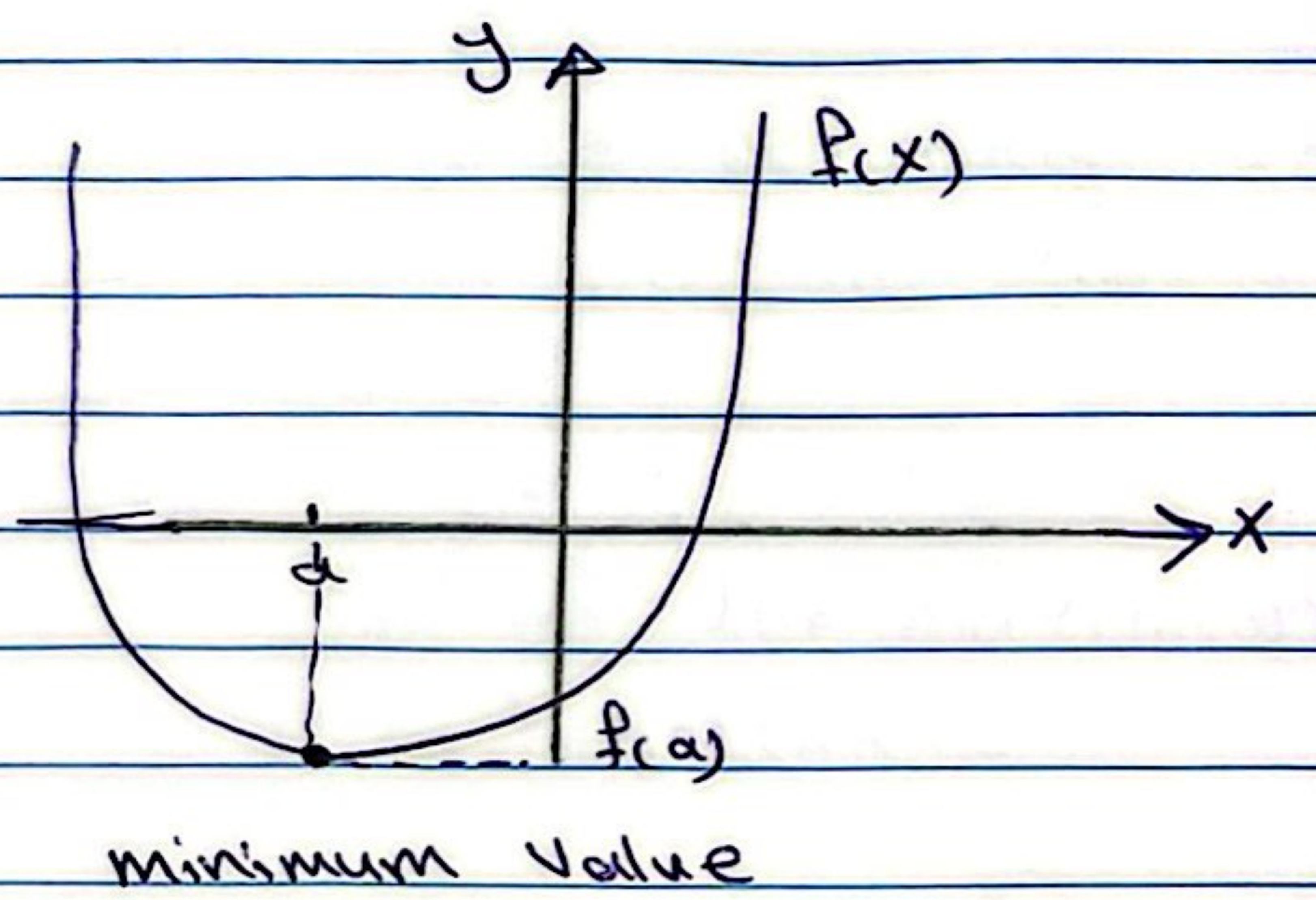
Maximum and Minimum Values:-

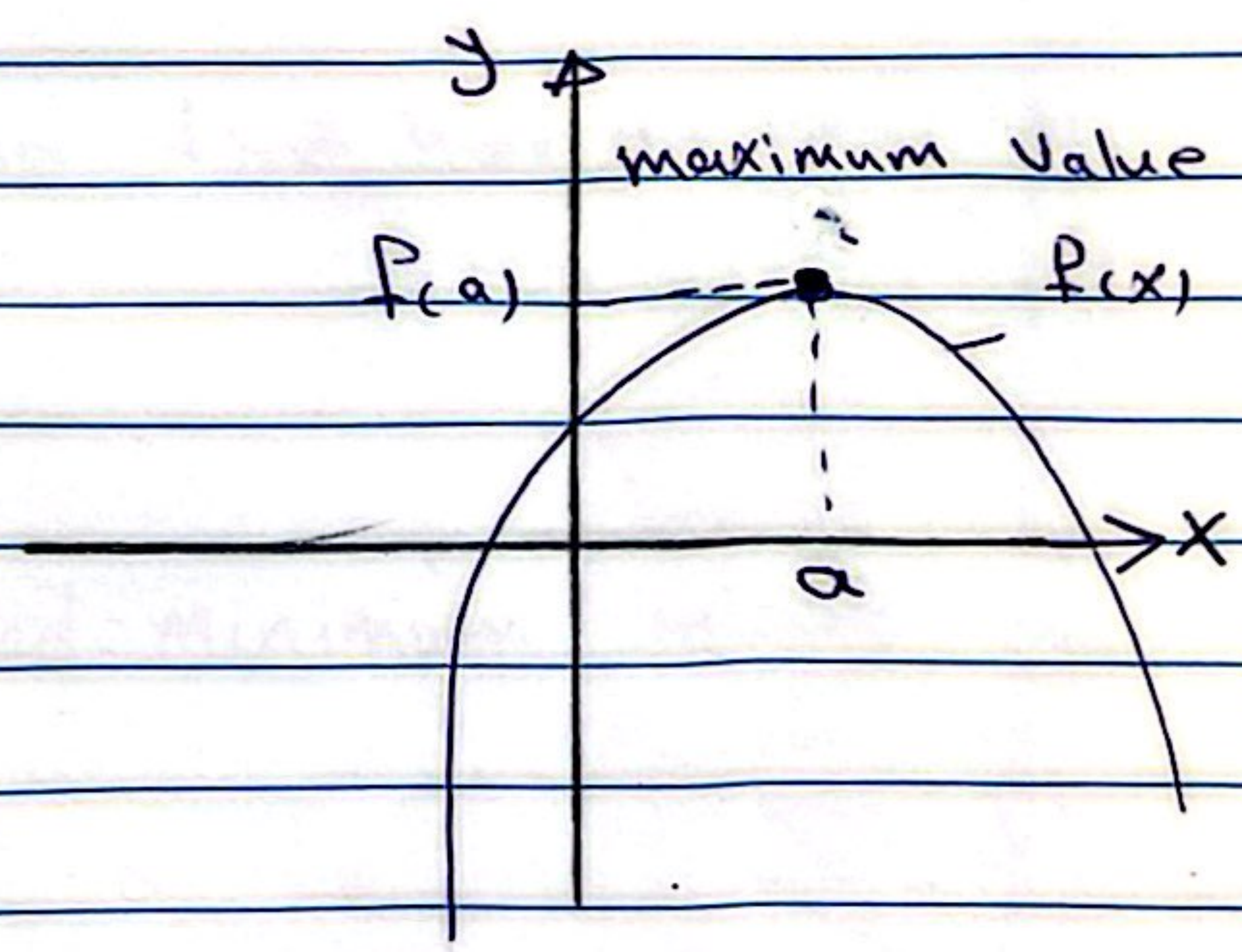
Def.: we say that $f(x)$ has relative maximum value at $x=a$ iff:

$$f(a) \geq f(x) \quad \forall x \in \overset{\text{Domain}}{D} f$$

Def. we say that $f(x)$ has relative minimum value at $x=a$ iff:

$$f(a) < f(x) \quad \forall x \in D f$$





Ex

1. $f(x) = x^2 - 9$; $a \in (-\infty, \infty)$; $[-3, 3]$

Find the maximum value of $f(x)$ in interval $[-3, 3]$
or minimum

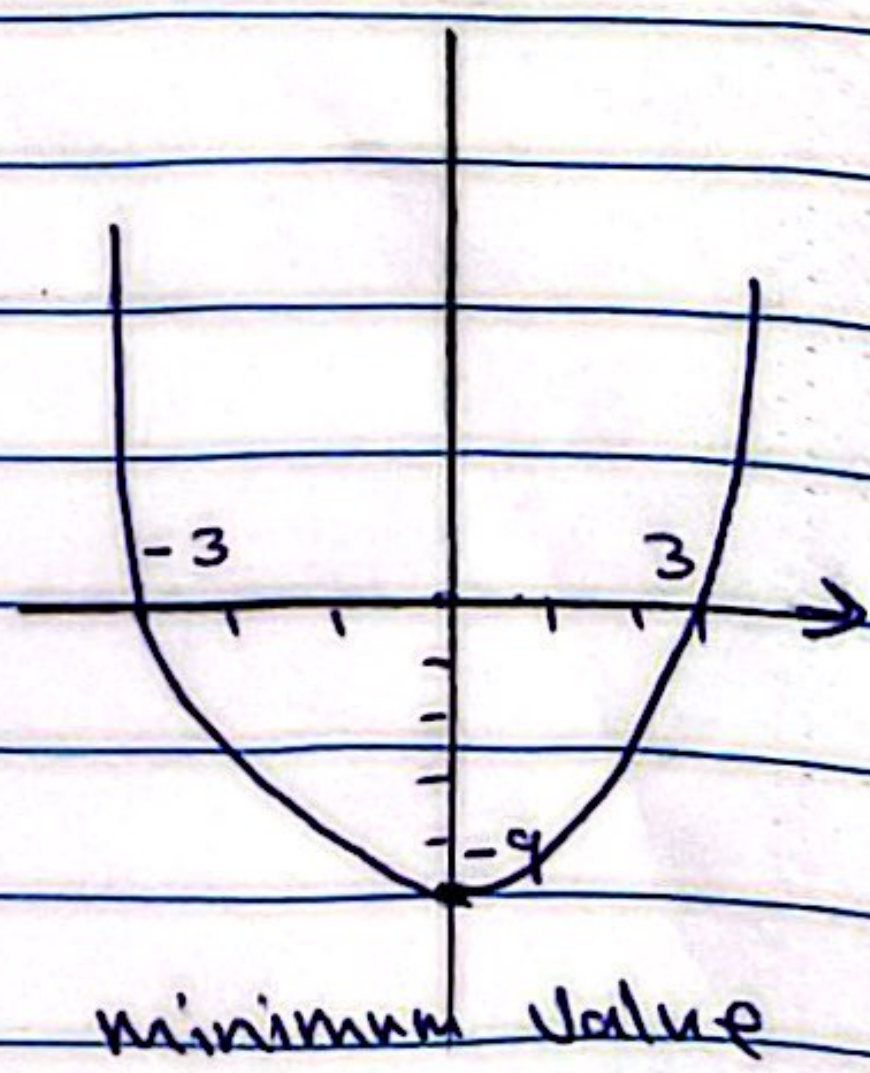
Solu

$\cdot f(x) = x^2 - 9$

$f'(x) = 2x \quad > 0$

$f'(x) = 0 \Rightarrow 2x = 0 \Rightarrow x = 0$

from $f(x) \Rightarrow x = 0 \Rightarrow y = -9 \quad (0, -9)$



2. $f(x) = \frac{1}{x} \quad ; x \neq 0$

Sol/

$f(x) = x^{-1}$

$f'(x) = -x^{-2} = \frac{-1}{x^2}$

$f'(x) = 0 \Rightarrow \frac{-1}{x^2} = 0$

$f'(x) \neq 0$

$\therefore$ There are no maxinum and minimum points for the function

$\frac{1}{x}$ It is the only function that has no maximum or minimum points

H.w.

Find the maximum and minimum values of the function:-

1. $f(x) = (x+2)^3 (x-2)^2$

2. $f(x) = \frac{x^2+16}{x^2}$

3. $f(x) = x^4 - 8x^2$

Roll's Theorem

Theorem:- let f be differentiable on (a,b) and continuous on $[a,b]$. If:

$$f(a) = f(b) = 0$$

Then there is at least one number c in (a,b) such that $f'(c) = 0$

Ex Does the following function satisfies Roll's theorem

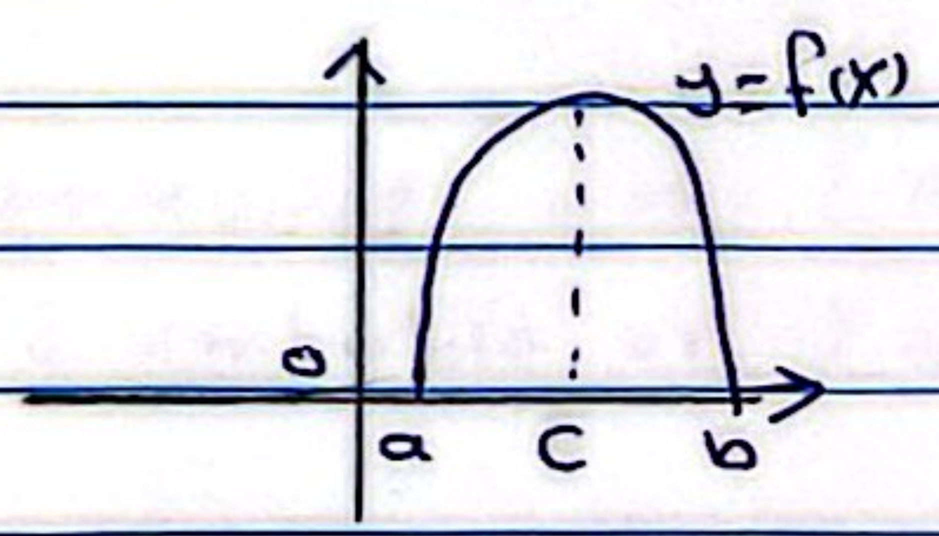
1.

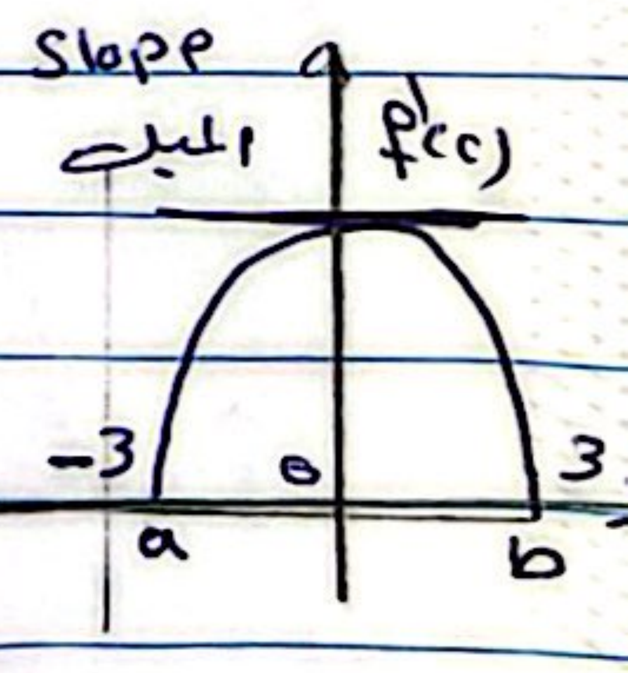
$$f(x) = 9 - x^2 \qquad [-3,3]$$

Sol ① The function is continous on $[-3,3]$
② The function is differentable on $[-3,3]$

$f'(x) = -2x$ exists on $(-3,3)$

$f(-3) = 9-(-3)^2 = 0$
$f(3) = 9-(3)^2 = 0$
$\therefore f(a) = f(b) = 0$





$\therefore$ The function is satisfies the Roll's Theorem

and to find c?

Then there exist c ; $-3 < c < 3$

Such that:

$$f'(x) = 0 \Rightarrow f'(c) = 0$$

$$\therefore -2c = 0 \Rightarrow c = 0 \quad ; -3 < 0 < 3$$

2. $f(x) = x^2 - 5x + 7 \qquad [0,5]$

Solu ① The function is continuous on $[0,5]$
② The function is differentable on $[0,5]$

$f'(x) = 2x - 5$ exist on $(0,5)$

$f(a) = f(0) = 7 \;;\; f(b) = f(5) = 7$

$\because f(a) = f(b) = 7$

$\therefore f(x)$ is satisfy the Roll's Theorem

To find c ; $f'(x) = 0$

$2x - 5 = 0 \qquad 0 < c < 5$

$2x = 5 \Rightarrow x = c = \frac{5}{2} = 2.5$

$\therefore c \in [0,5]$

3. $f(x) = \sin x \quad ; \quad [0, 2\pi]$

Solu ① The function is continuous on $[0,2\pi]$
② The function is differentable on $[0,2\pi]$

$f'(x) = \cos x$ exist on $(0, 2\pi)$

$f(a) = \sin 0 = 0 \;;\; f(b) = \sin(2\pi) = \sin(360) = 0$

$\because f(a) = f(b) = 0$

$\therefore f(x)$ is satisfy the Roll's Theorem

To find c

Then there exists a number c.

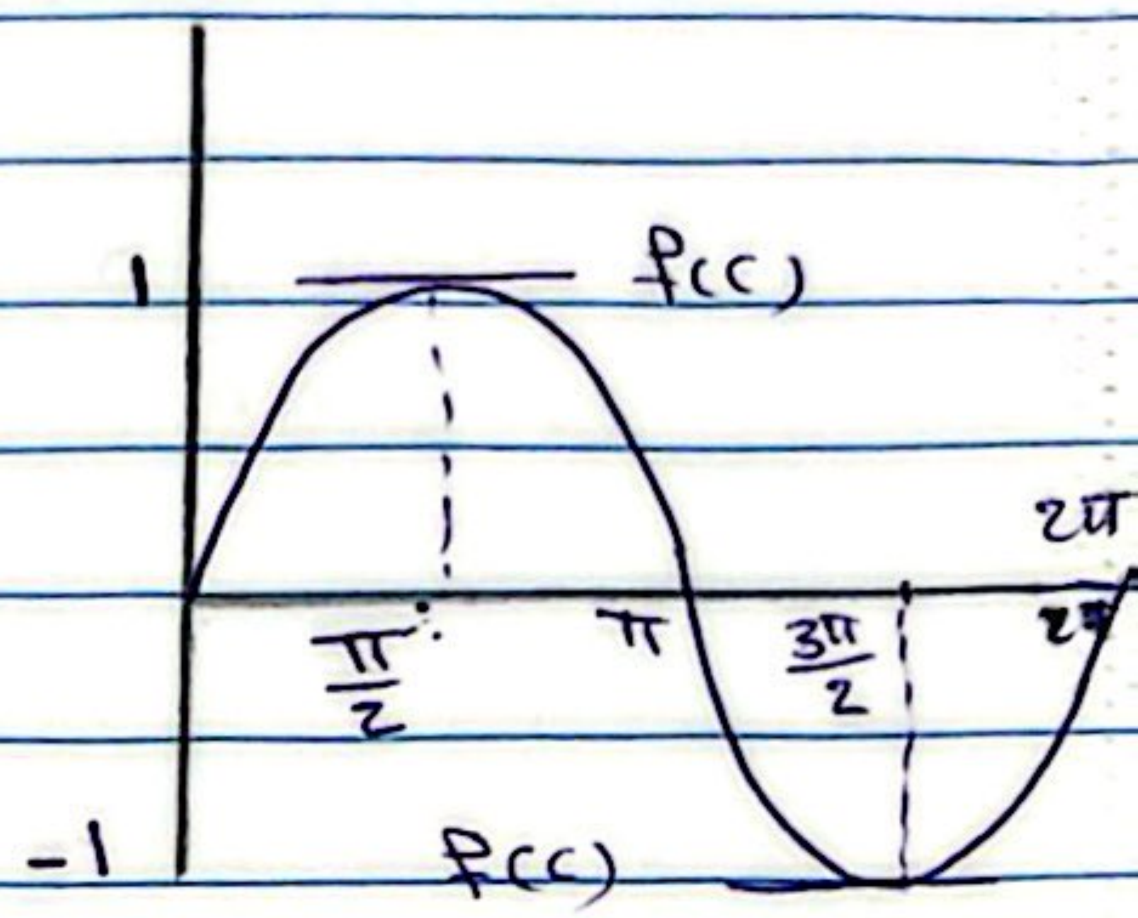
S.t. $0 < c < 2\pi$

$f'(x) = 0$

$\cos x = 0 \Rightarrow \cos(c) = 0$

$c_1 = \frac{\pi}{2}$; $c_2 = \frac{3\pi}{2}$

$\because c_1$ and $c_2 \in (0, 2\pi)$



H.W.

1) Does the function $f(x) = \frac{x^3}{3} - 3x$; Satisfies Roll's Theorem on the interval $[-3, 3]$? if yes find c ?

2) Does the function $f(x) = x^{\frac{2}{3}}$; Satisfy Roll's Theorem on the interval $[-1, 1]$? If yes find c ?

3. $f(x) = x^2 + 3x - 1$; $[-3, -1]$

Solu

1. $f(x)$ is continuous on $[-3, -1]$
2. $f(x)$ is differentable on $[-3, -1]$

$f'(x) = 2x + 3$; exist on $(-3, -1)$

$f(a) = f(-3) = -1$; $f(b) = f(-1) = -3$

$\because f(a) \neq f(b)$

$\because$ The function is not satisfies Roll's Theorem

4. $f(x) = x^2 - 2$; $[-2, 2]$

Solu 1) The function is continuous on $[-2, 2]$
2) The function is differentable on $[-2, 2]$

$f'(x) = 2x$

$f(a) = f(-2) = 2$; $f(b) = f(2) = 2$

$f(a) = f(b) = 2$

To find c

$f'(x) = 0 \Rightarrow 2x = 0 \Rightarrow c = 0$

$0 \in [-2, 2]$

$\therefore$ $f(x)$ is satisfies the Roll's Theorem

5. $f(x) = \dfrac{x}{x-1}$ on $[0, 1]$

Solu

1) $f(1) = \dfrac{1}{0}$ is not defined

$\therefore$ $f(x)$ is not continuous on $[0, 1]$

$\therefore$ The function is not satisfies the Roll's Theorem

6. $f(x) = x^3 - 2x + 1$ on $[0, 2]$

1) $f(x)$ is continuous on $[0, 2]$ because $f(x)$ is polynomial
2) $f(x)$ is differentable on $[0, 2]$

$f'(x) = 3x^2 - 2$

$f(a) = f(0) = 1$; $f(b) = f(2) = 5$; $f(a) \neq f(b)$

$\therefore$ $f(x)$ is not satisfies the Roll's Theorem