7th Lecture

Chapter Two

القواعد والتبولوجيات النسبية

Bases and Relative Topologies

Definition (2.1): Let (X, τ) be a topological space and β be a family of subsets of X. We say that β is a **base** for τ iff every element of τ is equal to the union of a number of elements of β.

In other word: β is a **base** for τ iff $\forall\ G \in \tau, \exists\ \beta_i \in \beta, G = \bigcup_i \beta_i$.

Example (2.1):

Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$.

Let $\beta_1 = \{\{a\}, \{b\}\}$ and $\beta_2 = \{\{a\}, \{a, b\}, \{c\}\}$ be two families of subsets of X which of them (β_1, β_2) form a base for ?

Solution:

If $\beta = \beta_1$

Since $X = \bigcup_{\beta_i \in \beta_1} \beta_i$

$\Rightarrow\ \beta_1$ is not a base for τ

If $\beta = \beta_2$

Since $G = \bigcup_{\beta_i \in \beta_2} \beta_i, \forall\ G \in \tau$

$\Rightarrow\ \beta_2$ is a base for τ

Example (2.2):

Let $X = \{a, b, c, d, e\}$ and τ is the discrete topology on X. Then

$\beta = \{\{a\}, \{b\}, \{c\}, \{d\}, \{e\}\}$ is a base for τ.

Theorem (2.1): Let β be a family subset of X. Then β is a base for a topology on X iff the following conditions satisfied

(i) $X = \bigcup\{B : B \in \beta\}$

(ii) $\forall\ \beta_1, \beta_2 \in \beta, \forall\ x \in \beta_1 \cap \beta_2, \exists\ B \in \beta; x \in B \subset \beta_1 \cap \beta_2$

Subbases القواعد الجزئية

Definition (2.2): Let (X, τ) be a topological space. We say the family Ψ of subsets of X is a **subbase** iff the finite intersection of elements of Ψ form a base for τ.

That is: Ψ is a **subbase** for τ iff $\bigcap_{finite} \Psi_i$ is a base for τ.

Example (2.3):

Let $X = \{a, b, c, d\}$ and $\Psi = \{\{a, b\}, \{b, c\}, \{d\}\}$.

Determine wither Ψ is a subbase or no.

Solution:

$\Omega = \bigcap_{finite} \Psi_i = \bigcap_{finite} \{\{a, b\}, \emptyset, \{b, c\}, \{d\}\}$

$\Omega = \{\{a, b\}, \{b\}, \{b, c\}, \{d\}, \emptyset\}$

Now

$\beta = \bigcup \Omega = \{\{a, b\}, \{b\}, \{b, c\}, \{d\}, \emptyset, \{a, b, c\}, \{a, b, d\}, \{b, d\}, \{b, c, d\}, \{a, b, c, d\}\}$

$= \{\emptyset, \{b\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, X\}$

Since $\beta = \tau$ is a topology on X

$= \Omega$ is a base for τ

$\Rightarrow\ \Psi$ is a subbase for τ

Remark (2.1):

The family Ψ is a subbase $\Leftrightarrow$ $\cup[\bigcap_{finite}\{k_i : k_i \in \Psi\}]$ for a topology.

The Relative Topologies التبولوجيا النسبية

Definition (2.3): Let (X, τ) be a topological space and $X^* \subset X$. The topology on X^* is called the **relative topology** (denoted by τ^*) and defined as follow:

$$\tau^* = \{G^* : G^* = X^* \cap G , G \in \tau\}$$

Example (2.4):

Let $X = \{a, b, c, d, e\}$ and

$\tau = \{\emptyset, \{a\}, \{a, b\}, \{c, d\}, \{b, c, d\}, \{a, c, d\}, \{a, b, c, d\}, \{b, c, d, e\}, X\}$.

Let $X^* = \{a, d, e\}$, the topology on X^* is $\tau^* = \{\emptyset, \{a\}, \{d\}, \{a, d\}, \{d, e\}, X^*\}$

(X^*, τ^*) is the relative topology.
