Power Series

Sequence : It is a function whose starting point is the set of positive integers, for example :-

$a(n)$; $a(1)=1$; $a(2)=2$; $a(3)=3$ and So on

n	$a(n)$
0	0
1	1
2	2
⋮	⋮

Ex

1. $a(n) = \frac{1}{n}$; $a(0) =$ not defined

$$a(1) = 1$$
$$a(2) = \frac{1}{2}$$
$$a(3) = \frac{1}{3}$$
$$\vdots$$

Ex

2. $a(n) = \frac{\ln(n)}{n^2}$

$$a(1) = \frac{\ln(1)}{1} = 0$$

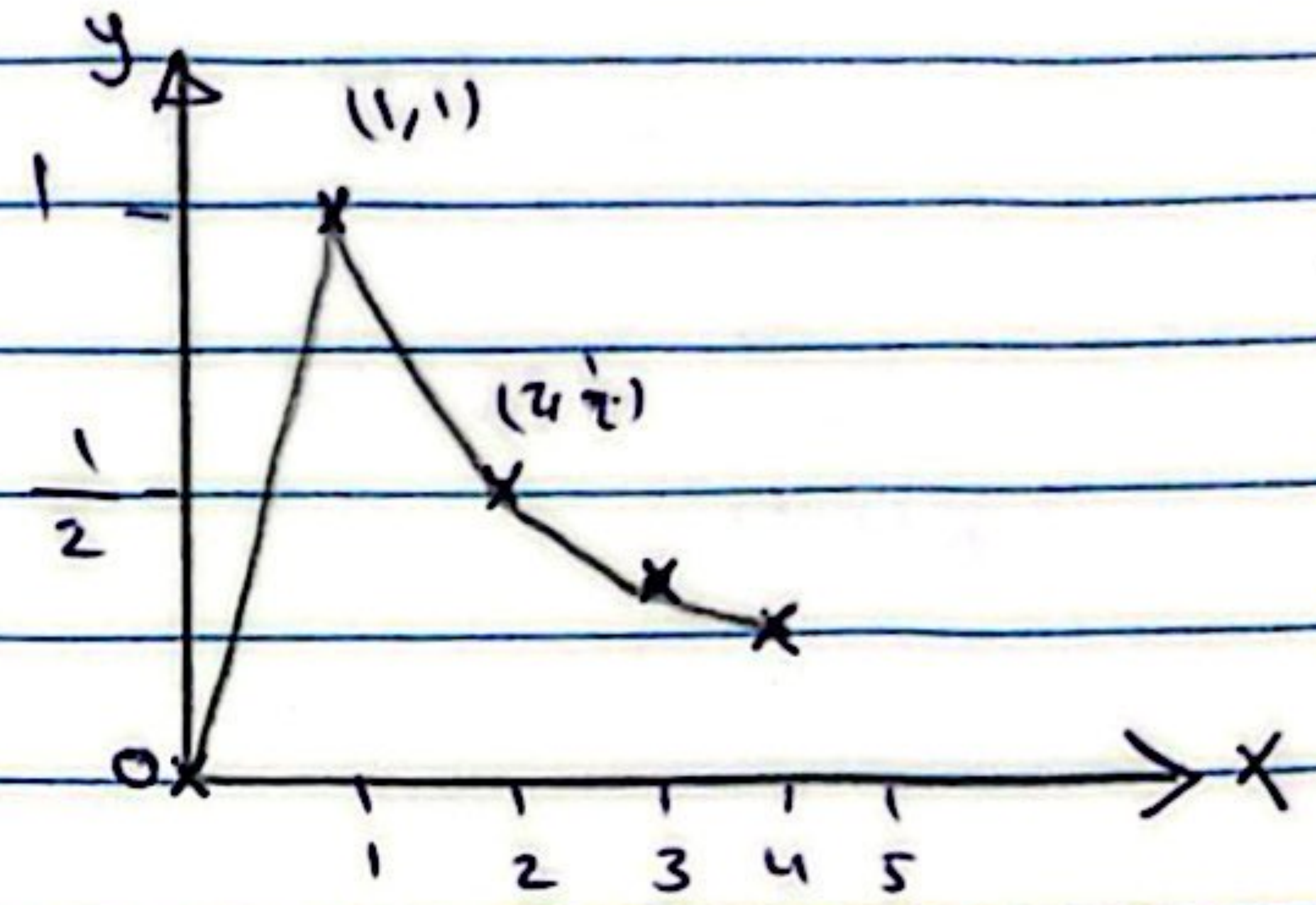
$$a(2) = \frac{\ln(2)}{4} = 0.1732$$

$$a(3) = \frac{\ln(3)}{9} = 0.1220$$

For each squence the term $a(n)$ is called the n-th term الحد النوني

Ho to draw a sequence:..

n	$a(n)=\frac{1}{n}$
0	0
1	1
2	$\frac{1}{2}$
3	$\frac{1}{3}$
4	$\frac{1}{4}$



Sequence drawing $a(n)=\frac{1}{n}$

Infinite sequences and Series

1. Sequences

المتتاليات

It is aset of Valued and numbers separated by commas and follows a speciffic rule called the general rule a_n

هي مجموعة من القيم والاعداد تفصل بينها فواصل
وتتبع قاعدة معينة تسمى بالقاعدة العامة

for example

① $\{1,4,9,16,\dots\}$ it is asequence whose general rule is:

$$a_n=\{n^2\}_{n=1}^{\infty}$$

This is ageneral term of Sequence.

② $[1+4+9+16+\dots\}$ it is asequence whose general rule is:

$$\sum_{n=1}^{\infty} n^2$$

Ex Write the first three term of the following Sequence :-

1. $a_n = \left\{ \dfrac{2n}{n^2+1} \right\}_{n=1}^{\infty}$

Solu:

$$a_1 = \frac{2(1)}{1^2+1} = 1$$

$$a_2 = \frac{2(2)}{2^2+1} = \frac{4}{5}$$

$$a_3 = \frac{2(3)}{3^2+1} = \frac{3}{5}$$

2. $a_1 = 2$

$$a_{n+1} = \frac{a_n}{1+a_n}$$

Solu

$$a_1 = 2$$

$$a_2 = \frac{a_1}{1+a_1} = \frac{2}{1+2} = \frac{2}{3}$$

$$a_3 = \frac{a_2}{1+a_2} = \frac{\frac{2}{3}}{1+\frac{2}{3}} = \frac{2}{5}$$

Ex Find a formula for the general term a_n of the Sequence

1. $\left\{ \underset{n=1}{5} \overset{3}{,} \underset{n=2}{8} \overset{3}{,} \underset{n=3}{11} \overset{3}{,} \underset{n=4}{14} \overset{3}{,} 17, \dots \right\}$

$$\therefore a_n = \{3n+2\}_{n=1}^{\infty}$$

2. $\left\{ \underset{n=1}{0}, \underset{n=2}{\frac{3}{5}}, \underset{n=3}{\frac{8}{10}}, \frac{15}{17}, \dots \right\}$

$$a_n = \left\{ \frac{n^2-1}{n^2+1} \right\}_{n=1}^{\infty}$$

3. $\left\{ 4, -1, \frac{1}{4}, \frac{-1}{16}, \frac{1}{64}, \ldots \right\}$

$n=1 \quad n=2 \quad n=3 \quad n=4 \quad n=5$

negative limit $(-1)^n$ الحد السالب

positive limit $(+1)^{n+1}$ الحد الموجب

This note is used when there are limits whose sign changes تستخدم هذه الملاحظة عند وجود حدود تتغير اشارتها

$$a_n = \left\{ \frac{(-1)^{n+1}}{(4)^{n-2}} \right\}_{n=1}^{\infty}$$

Monotoicity الترتيب

It is aproperty of sequence, i.e: whether they are ordered or not? and follow the same behavior and properties of sequences in terms of increasing or decreasing.

هي خاصية في المتتابعات اي هل هي مرتبة ام لا وتتبع نفس السلوك وخواص المتتابعات من حيث تزايدية ام متناقصة

	Increasing	Decreasing
1.	$a_{n+1} > a_n$	$a_{n+1} < a_n$

Ex If $a_n = \{ n - 2^n \}_{n=1}^{\infty}$

$a_1 = 1 - 2^1 = -1$

$a_2 = 2 - 2^2 = -2$

$a_3 = 3 - 2^2 = -1$

$a_4 = 4 - 2^2 = 0$

Sequence is decreasing (monotonic)

2. $\frac{a_{n+1}}{a_n} > 1$ $\qquad$ $\frac{a_{n+1}}{a_n} < 1$

Ex

$$b_n = \left\{\frac{e^n}{n}\right\}_{n=1}^{\infty}$$

$$b_1 = \left\{\frac{e^1}{1}\right\} = e^1$$

$$b_2 = \left\{\frac{e^2}{2}\right\} =$$

3. $a_{n+1} - a_n > 0$ $\qquad$ $a_{n+1} - a_n < 0$

4. $(a_n)' > 0$ $\qquad$ $(a_n)' < 0$

The limit of Infinite Sequences:-

$$\lim_{n\to\infty} a_n = \begin{cases} L \text{ : any number} \to \text{Converge} \\ \mp\infty \to \text{Diverge} \end{cases}$$

متقاربة من الناتج

تتباعد وغير معلومة

$$\lim_{n\to\infty} \frac{f(x)}{g(x)} = \frac{\mp\infty}{\mp\infty} =$$

- CR ln (عدم وجود) ln = $\frac{\text{اكبر اس بالبسط}}{\text{أكبر اس في المقام}}$
  log
- LR ln (عند وجود) $\frac{\text{مشتقة البسط}}{\text{مشتقة المقام}}$
  log

$\infty - \infty$ //

- توحيد مقامات $\quad \frac{\square}{\square} - \frac{\square}{\square}$
- عامل مشترك $\quad$ Polynomial
- ضرب بالمرافق $\quad \sqrt{} \mp \square$