Context-Free Grammar (CFG)

CFG stands for context-free grammar. It is a formal grammar which is used to generate all possible patterns of strings in a given formal language. A grammar G is of **type-2 (context-free grammar)** if all of its productions are of the form $A \rightarrow \beta$, where $A \in N$, $\beta \in (N \cup T)^{+}$. A production of the form $S \rightarrow \varepsilon$ can also be accepted if the start symbol S does not occur in the right hand side (R.H.S.) of any production.

Example 1:

Construct the CFG for the language having any number of a's over the set $\sum = \{a\}$.

Solution:

As we know the Regular Expression (RE) for the above language is a^* so the Production rule for the RE is as follows:

$S \rightarrow aS \mid \varepsilon$

Now if we want to derive a string "aaaaaa", we can start with start symbols:

$S \Rightarrow aS \Rightarrow aaS \Rightarrow aaaS \Rightarrow aaaaS \Rightarrow aaaaaS \Rightarrow aaaaaaS \Rightarrow aaaaaa\varepsilon \Rightarrow aaaaaa$

The RE = a^* can generate a set of string {ε, a, aa, aaa, ...}.

We can have a null string because S is a start symbol and rule 2 gives $S \rightarrow \varepsilon$.

Example 2:

Construct a CFG for the regular expression $(0+1)^*$

Solution:

The CFG can be given by Production rule (P):

$S \rightarrow 0S \mid 1S \mid \varepsilon$

The rules are in the combination of 0's and 1's with the start symbol. Since $(0+1)^*$ indicates {ε, 0, 1, 01, 10, 00, 11, ...}. In this set, ε is a string, so in the rule, we can set the rule $S \rightarrow \varepsilon$.

Example 3:

Construct a CFG for a language $L = \{w c w^R$ where $w \in \{a, b\}^*, |w| = 0\}$.

Solution:

The string that can be generated for a given language is {aacaa, bcb, abcba, bacab, abbcbba, ...}

The grammar could be:

$S \rightarrow aSa \mid bSb \mid c$

Now if we want to derive a string "abbcbba", we can start with start symbols:

$S \Rightarrow aSa \Rightarrow abSba \Rightarrow abbSbba \Rightarrow abbcbba$

Thus, any of this kind of string can be derived from the given production rules.

Example 4:

Construct a CFG for the language $L = a^n b^{2n}$ where n>=1.

Solution:

The string that can be generated for a given language is {abb, aabbbb, aaabbbbbb....}.

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The grammar could be:
$S \rightarrow aSbb \mid abb$

Now if we want to derive a string "aabbbb", we can start with start symbols:
$S \Rightarrow aSbb \Rightarrow aabbbb$

Example 5:
Construct a CFG for the language $L = b^m c^n d^p$ where m, n>=1, p=|m-n|.

Solution:
The absolute values could be:

- p=m-n   m>n $\Rightarrow$ m=p+n
- p=n-m   n>m $\Rightarrow$ n=p+m
- p=0   m=n $\Rightarrow$ p=m-m or p=n-n

The state of languages depends on the values of m and n:

$S \rightarrow A \mid B \mid C$

1. $b^{p+n} c^n d^p \Rightarrow b^p b^n c^n d^p$
   $A \rightarrow bAd \mid bXd$
   $X \rightarrow bXc \mid bc$

2. $b^m c^{p+m} d^p \Rightarrow b^m c^m c^p d^p$
   $B \rightarrow YZ$
   $Y \rightarrow bYc \mid bc$
   $Z \rightarrow cZd \mid cd$

3. $b^m c^m \Rightarrow C \rightarrow bCc \mid bc$

HW. Derive some words

Example 6:
Construct a CFG for the language $L = (d^* c)^m (c^* d)^m$ where m>=1.

Solution:
$S \rightarrow ASB \mid AB$
$A \rightarrow dA \mid c$
$B \rightarrow cB \mid d$

HW. Derive some words

Example 7:
Construct a CFG for the language $L = c^* b^{2m} c (c^* d)^m$ where m>=0.

Solution:
$S \rightarrow cS \mid B$
$B \rightarrow bbBA \mid c$
$A \rightarrow cA \mid d$

HW. Derive some words

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Example 8:
Construct a CFG for the language $L = b^{m+1} (c^* d)^{2n} b c^n e^m$ where m>=1, n>=0.

Solution:
$b^{m+1} (c^* d)^{2n} b c^n e^m \Rightarrow b b^m (c^* d)^{2n} b c^n e^m$

S → bA	S → bA
A → bAe | bBe	A → bAe | bBe
B → DDBc | b	B → CBc | b
D → cD | d	C → cC | dd

HW. Derive some words

Example 9:
Construct a CFG for the language $L = b (c^* d)^n e (d^* c)^m b$ where n, m>0.

Solution:

S → bAeBb	S → bAb
A → CA | C	A → BeC
C → cC | d	B → cB | d
B → DB | D	C → dC | c
D → dD | c	

HW. Derive some words

HW 1. *Construct a CFG for the language* $L = 0^n 2 3 1^n$ *where n>=0.*

HW 2. *Construct a CFG for the language* $L = a^{n+3} b^{n+2}$ *where n>=1.*

HW 3. *Construct a CFG for the language* $L = a^{2n} b^n$ *where n>=0.*

HW 4. *Construct a CFG for the language* $L = a^n c b^n$ *where n >0*

HW 5. *Construct a CFG for the language* $L = a^{2n+4} b^{n+1}$ *where n>=0.*

HW 6. *Construct a CFG for the language* $L = \{w c d w^R$ *where* $w \in \{a, b\}^*, |w| \neq 0\}$

HW 7. *Construct a CFG for the language* $L = \{0^i 1^j$ *where i>=0, i<=j<=2i}*

HW 8. *Construct a CFG for the language* $L = \{b^{n+m} c^m d^n$ *where m=3, n=2, m>=1, n>=1}*

HW 9. *Construct a CFG for the language* $L = \{a^i b^{2i} c^j$ *where i>0, j>0}*

HW 10. *Construct a CFG for the language* $L = \{b^{2i} c^{j+1} d^j e^i b$ *where i, j>=0}*

HW 11. *Construct a CFG for the language* $L = \{b^m c^{3m} d e^n f^{2n} d^j$ *where m, n, j>=1}*

HW 12. *Construct a CFG for the language* $L = \{a^{2i+1}$ *where i>=0}*

HW 13. *Construct a CFG for the language* $L = \{a^i b^j c^{i+j}$ *where i, j>=1}*

HW 14. *Construct a CFG for the language* $L = \{a^n b^m c^j d^{k+2}$ *where m+n=j+k}*

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