22. Sale Prices for Houses The average sales price of new one-family houses in the Midwest is \$250,000 and in the South is \$253,400. A random sample of 40 houses in each region was examined with the following results. At the 0.05 level of significance can it be concluded that the difference in mean sales price for the two regions is greater than \$3400?

	South	Midwest
Sample size	40	40
Sample mean	261,500	248,200
Population standard deviation	10,500	12,000

Source: *New York Times Almanac.*

23. Average Earnings for College Graduates The average earnings of year-round full-time workers with bachelor's degrees or more is \$88,641 for men and \$58,000 for women—a difference of slightly over \$30,000 a year. One hundred of each were sampled, resulting in a sample mean of \$90,200 for men, and the population standard deviation is \$15,000, and a mean of \$57,800 for women, and the population standard deviation is \$12,800. At the 0.01 level of significance can it be concluded that the difference in means is not \$30,000?

Source: *New York Times Almanac.*

Technology *Step by Step*

TI-83 Plus or TI-84 Plus Step by Step

Hypothesis Test for the Difference Between Two Means and *z* Distribution (Data)

1. Enter the data values into L_1 and L_2.
2. Press **STAT** and move the cursor to TESTS.
3. Press **3** for 2-SampZTest.
4. Move the cursor to Data and press **ENTER.**
5. Type in the appropriate values.
6. Move the cursor to the appropriate alternative hypothesis and press **ENTER.**
7. Move the cursor to Calculate and press **ENTER.**

Hypothesis Test for the Difference Between Two Means and *z* Distribution (Statistics)

1. Press **STAT** and move the cursor to TESTS.
2. Press **3** for 2-SampZTest.
3. Move the cursor to Stats and press **ENTER.**
4. Type in the appropriate values.
5. Move the cursor to the appropriate alternative hypothesis and press **ENTER.**
6. Move the cursor to Calculate and press **ENTER.**

Confidence Interval for the Difference Between Two Means and *z* Distribution (Data)

1. Enter the data values into L_1 and L_2.
2. Press **STAT** and move the cursor to TESTS.
3. Press **9** for 2-SampZInt.
4. Move the cursor to Data and press **ENTER.**
5. Type in the appropriate values.
6. Move the cursor to Calculate and press **ENTER.**

Confidence Interval for the Difference Between Two Means and *z* Distribution (Statistics)

1. Press **STAT** and move the cursor to TESTS.
2. Press **9** for 2-SampZInt.
3. Move the cursor to Stats and press **ENTER.**
4. Type in the appropriate values.
5. Move the cursor to Calculate and press **ENTER.**

Excel
Step by Step

z Test for the Difference Between Two Means

Excel has a two-sample *z* test included in the Data Analysis Add-in. To perform a *z* test for the difference between the means of two populations, given two independent samples, do this:

1. Enter the first sample data set into column A.
2. Enter the second sample data set into column B.
3. If the population variances are not known but $n \geq 30$ for both samples, use the formulas =VAR(A1:A*n*) and =VAR(B1:B*n*), where A*n* and B*n* are the last cells with data in each column, to find the variances of the sample data sets.
4. Select the Data tab from the toolbar. Then select Data Analysis.
5. In the Analysis Tools box, select *z* test: Two sample for Means.
6. Type the ranges for the data in columns A and B and type a value (usually 0) for the Hypothesized Mean Difference.
7. If the population variances are known, type them for Variable 1 and Variable 2. Otherwise, use the sample variances obtained in step 3.
8. Specify the confidence level Alpha.
9. Specify a location for the output, and click [OK].

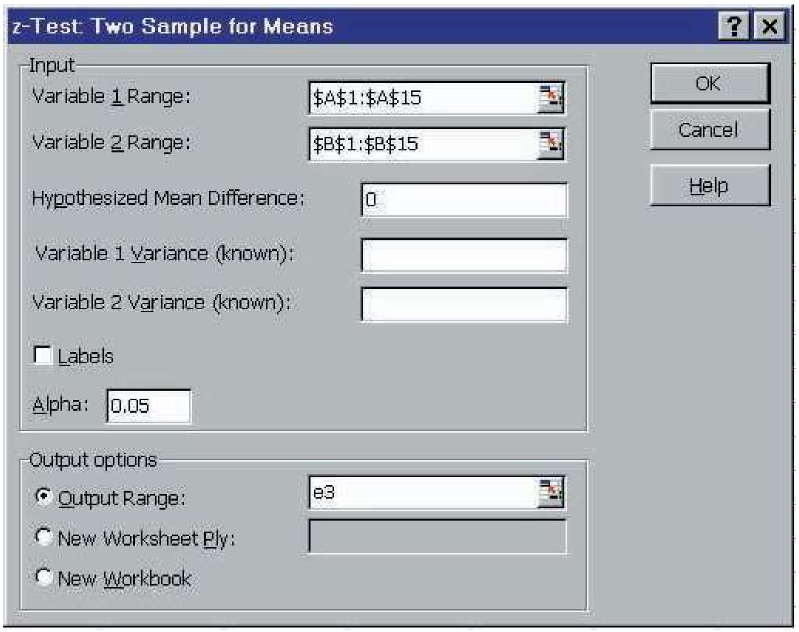
Example XL9–1

Test the claim that the two population means are equal, using the sample data provided here, at $\alpha = 0.05$. Assume the population variances are $\sigma_A^2 = 10.067$ and $\sigma_B^2 = 7.067$.

Set A	10	2	15	18	13	15	16	14	18	12	15	15	14	18	16
Set B	5	8	10	9	9	11	12	16	8	8	9	10	11	7	6

The two-sample *z* test dialog box is shown (before the variances are entered); the results appear in the table that Excel generates. Note that the *P*-value and critical *z* value are provided for both the one-tailed test and the two-tailed test. The *P*-values here are expressed in scientific notation: 7.09045E-06 = 7.09045×10^{-6} = 0.00000709045. Because this value is less than 0.05, we reject the null hypothesis and conclude that the population means are not equal.

Two-Sample *z* Test Dialog Box



z-Test: Two Sample for Means		
	Variable 1	*Variable 2*
Mean	14.06666667	9.266666667
Known Variance	10.067	7.067
Observations	15	15
Hypothesized Mean Difference	0	
z	4.491149228	
P(Z<=z) one-tail	3.54522E-06	
z Critical one-tail	1.644853	
P(Z<=z) two-tail	7.09045E-06	
z Critical two-tail	1.959961082	

9–2 Testing the Difference Between Two Means of Independent Samples: Using the *t* Test

Objective 2

Test the difference between two means for independent samples, using the *t* test.

In Section 9–1, the *z* test was used to test the difference between two means when the population standard deviations were known and the variables were normally or approximately normally distributed, or when both sample sizes were greater than or equal to 30. In many situations, however, these conditions cannot be met—that is, the population standard deviations are not known. In these cases, a *t* test is used to test the difference between means when the two samples are independent and when the samples are taken from two normally or approximately normally distributed populations. Samples are **independent samples** when they are not related. Also it will be assumed that the variances are not equal.

Formula for the *t* Test—For Testing the Difference Between Two Means—Independent Samples

Variances are assumed to be unequal

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

where the degrees of freedom are equal to the smaller of $n_1 - 1$ or $n_2 - 1$.

The formula

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

follows the format of

$$\text{Test value} = \frac{(\text{observed value}) - (\text{expected value})}{\text{standard error}}$$

where $\bar{X}_1 - \bar{X}_2$ is the observed difference between sample means and where the expected value $\mu_1 - \mu_2$ is equal to zero when no difference between population means is hypothesized. The denominator $\sqrt{s_1^2/n_1 + s_2^2/n_2}$ is the standard error of the difference between two means. Since mathematical derivation of the standard error is somewhat complicated, it will be omitted here.

Assumptions for the *t* Test for Two Independent Means When σ_1 and σ_2 Are Unknown

1. The samples are random samples.
2. The sample data are independent of one another.
3. When the sample sizes are less than 30, the populations must be normally or approximately normally distributed.

Example 9–4

Farm Sizes

The average size of a farm in Indiana County, Pennsylvania, is 191 acres. The average size of a farm in Greene County, Pennsylvania, is 199 acres. Assume the data were obtained from two samples with standard deviations of 38 and 12 acres, respectively, and sample sizes of 8 and 10, respectively. Can it be concluded at $\alpha = 0.05$ that the average size of the farms in the two counties is different? Assume the populations are normally distributed.

Source: *Pittsburgh Tribune-Review.*

Solution

Step 1 State the hypotheses and identify the claim for the means.

$$H_0: \mu_1 = \mu_2 \quad \text{and} \quad H_1: \mu_1 \neq \mu_2 \text{ (claim)}$$

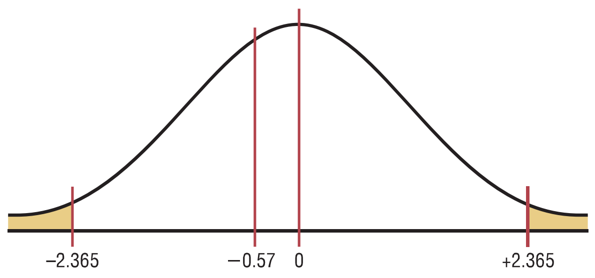
Step 2 Find the critical values. Since the test is two-tailed, since $\alpha = 0.05$, and since the variances are unequal, the degrees of freedom are the smaller of $n_1 - 1$ or $n_2 - 1$. In this case, the degrees of freedom are $8 - 1 = 7$. Hence, from Table F, the critical values are $+2.365$ and -2.365.

Step 3 Compute the test value. Since the variances are unequal, use the first formula.

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \frac{(191 - 199) - 0}{\sqrt{\dfrac{38^2}{8} + \dfrac{12^2}{10}}} = -0.57$$

Step 4 Make the decision. Do not reject the null hypothesis, since $-0.57 > -2.365$. See Figure 9–5.

Figure 9–5
Critical and Test Values for Example 9–4



Step 5 Summarize the results. There is not enough evidence to support the claim that the average size of the farms is different.

When raw data are given in the exercises, use your calculator or the formulas in Chapter 3 to find the means and variances for the data sets. Then follow the procedures shown in this section to test the hypotheses.

Confidence intervals can also be found for the difference between two means with this formula:

Confidence Intervals for the Difference of Two Means: Independent Samples

Variances assumed to be unequal:

$$(\bar{X}_1 - \bar{X}_2) - t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

d.f. = smaller value of $n_1 - 1$ or $n_2 - 1$

Example 9–5

Find the 95% confidence interval for the data in Example 9–4.

Solution

Substitute in the formula.

$$(\bar{X}_1 - \bar{X}_2) - t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2$$

$$< (\bar{X}_1 - \bar{X}_2) + t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(191 - 199) - 2.365\sqrt{\frac{38^2}{8} + \frac{12^2}{10}} < \mu_1 - \mu_2$$

$$< (191 - 199) + 2.365\sqrt{\frac{38^2}{8} + \frac{12^2}{10}}$$

$$-41.02 < \mu_1 - \mu_2 < 25.02$$

Since 0 is contained in the interval, the decision is to not reject the null hypothesis H_0: $\mu_1 = \mu_2$.

In many statistical software packages, a different method is used to compute the degrees of freedom for this t test. They are determined by the formula

$$\text{d.f.} = \frac{(s_1^2/n_1 + s_2^2/n_2)^2}{(s_1^2/n_1)^2/(n_1 - 1) + (s_2^2/n_2)^2/(n_2 - 1)}$$

This formula will not be used in this textbook.

There are actually two different options for the use of t tests. *One option is used when the variances of the populations are not equal, and the other option is used when the variances are equal.* To determine whether two sample variances are equal, the researcher can use an F test, as shown in Section 9–5.

When the variances are assumed to be equal, this formula is used and

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

follows the format of

$$\text{Test value} = \frac{(\text{observed value}) - (\text{expected value})}{\text{standard error}}$$

For the numerator, the terms are the same as in the previously given formula. However, a note of explanation is needed for the denominator of the second test statistic. Since both populations are assumed to have the same variance, the standard error is computed with what is called a pooled estimate of the variance. A **pooled estimate of the variance** is a weighted average of the variance using the two sample variances and the *degrees of freedom* of each variance as the weights. Again, since the algebraic derivation of the standard error is somewhat complicated, it is omitted.

Note, however, that not all statisticians are in agreement about using the F test before using the t test. Some believe that conducting the F and t tests at the same level of significance will change the overall level of significance of the t test. Their reasons are beyond the scope of this textbook. Because of this, we will assume that $\sigma_1 \neq \sigma_2$ in this textbook.

Applying the Concepts 9–2

Too Long on the Telephone

A company collects data on the lengths of telephone calls made by employees in two different divisions. The mean and standard deviation for the sales division are 10.26 and 8.56, respectively. The mean and standard deviation for the shipping and receiving division are 6.93 and 4.93, respectively. A hypothesis test was run, and the computer output follows.

Degrees of freedom = 56
Confidence interval limits = −0.18979, 6.84979
Test statistic t = 1.89566
Critical value t = −2.0037, 2.0037
P-value = 0.06317
Significance level = 0.05

1. Are the samples independent or dependent?
2. Which number from the output is compared to the significance level to check if the null hypothesis should be rejected?
3. Which number from the output gives the probability of a type I error that is calculated from the sample data?
4. Was a right-, left-, or two-tailed test done? Why?
5. What are your conclusions?
6. What would your conclusions be if the level of significance were initially set at 0.10?

See page 531 for the answers.

Exercises 9–2

For these exercises, perform each of these steps. Assume that all variables are normally or approximately normally distributed.

a. State the hypotheses and identify the claim.
b. Find the critical value(s).
c. Compute the test value.
d. Make the decision.
e. Summarize the results.

Use the traditional method of hypothesis testing unless otherwise specified.

For these exercises assume the variances are unequal.

1. **Bestseller Books** The mean for the number of weeks 15 *New York Times* hard-cover fiction books spent on the bestseller list is 22 weeks. The standard deviation is 6.17 weeks. The mean for the number of weeks 15 *New York Times* hard-cover nonfiction books spent on the list is 28 weeks. The standard deviation is 13.2 weeks. At $\alpha = 0.10$, can we conclude that there is a difference in the mean times for the number of weeks the books were on the bestseller lists?
2. **Tax-Exempt Properties** A tax collector wishes to see if the mean values of the tax-exempt properties are different for two cities. The values of the tax-exempt properties for the two samples are shown. The data are given in millions of dollars. A $\alpha = 0.05$, is there enough evidence to support the tax collector's claim that the means are different?

City A				City B			
113	22	14	8	82	11	5	15
25	23	23	30	295	50	12	9
44	11	19	7	12	68	81	2
31	19	5	2	20	16	4	5

3. **Noise Levels in Hospitals** The mean noise level of 20 areas designated as "casualty doors" was 63.1 dBA, and the standard deviation is 4.1 dBA. The mean noise level for 24 areas designated as operating theaters was 56.3 dBA, and the standard deviation was 7.5 dBA. At $\alpha = 0.05$, can it be concluded that there is a difference in the means?

4. **Ages of Gamblers** The mean age of a sample of 25 people who were playing the slot machines is 48.7 years, and the standard deviation is 6.8 years. The mean age of a sample of 35 people who were playing roulette is 55.3 with a standard deviation of 3.2 years. Can it be concluded at $\alpha = 0.05$ that the mean age of those playing the slot machines is less than those playing roulette?

5. **Carbohydrates in Candies** The number of grams of carbohydrates contained in 1-ounce servings of randomly selected chocolate and nonchocolate candy is listed here. Is there sufficient evidence to conclude that the difference in the means is significant? Use $\alpha = 0.10$.

Chocolate:	29	25	17	36	41	25	32	29
	38	34	24	27	29			
Nonchocolate:	41	41	37	29	30	38	39	10
	29	55	29					

Source: *The Doctor's Pocket Calorie, Fat, and Carbohydrate Counter.*

6. **Teacher Salaries** A researcher claims that the mean of the salaries of elementary school teachers is greater than the mean of the salaries of secondary school teachers in a large school district. The mean of the salaries of a sample of 26 elementary school teachers is $48,256, and the sample standard deviation is $3,912.40. The mean of the salaries of a sample of 24 secondary school teachers is $45,633. The standard deviation is $5,533. At $\alpha = 0.05$, can it be concluded that the mean of the salaries of the elementary school teachers is greater than the mean of the salaries of the secondary school teachers? Use the *P*-value method.

7. **Weights of Running Shoes** The weights in ounces of a sample of running shoes for men and women are shown. Test the claim that the means are different. Use the *P*-value method with $\alpha = 0.05$.

Men		Women		
10.4	12.6	10.6	10.2	8.8
11.1	14.7	9.6	9.5	9.5
10.8	12.9	10.1	11.2	9.3
11.7	13.3	9.4	10.3	9.5
12.8	14.5	9.8	10.3	11.0

8. **Weights of Vacuum Cleaners** Upright vacuum cleaners have either a hard body type or a soft body type. Shown are the weights in pounds of a sample of each type. At $\alpha = 0.05$, can it be concluded that the means of the weights are different?

Hard body types				Soft body types			
21	17	17	20	24	13	11	13
16	17	15	20	12	15		
23	16	17	17				
13	15	16	18				
18							

9. Find the 95% confidence interval for the difference of the means in Exercise 3 of this section.

$3.066 < \mu_1 - \mu_2 < 10.534$

10. Find the 95% confidence interval for the difference of the means in Exercise 8 of this section.

$-2.481 < \mu_1 - \mu_2 < 7.971$

11. **Hours Spent Watching Television** According to Nielsen Media Research, children (ages 2–11) spend an average of 21 hours 30 minutes watching television per week while teens (ages 12–17) spend an average of 20 hours 40 minutes. Based on the sample statistics obtained below, is there sufficient evidence to conclude a difference in average television watching times between the two groups? Use $\alpha = 0.01$.

	Children	Teens
Sample mean	22.45	18.50
Sample variance	16.4	18.2
Sample size	15	15

Source: *Time Almanac.*

12. **NFL Salaries** An agent claims that there is no difference between the pay of safeties and linebackers in the NFL. A survey of 15 safeties found an average salary of $501,580, and a survey of 15 linebackers found an average salary of $513,360. If the standard deviation in the first sample is $20,000 and the standard deviation in the second sample is $18,000, is the agent correct? Use $\alpha = 0.05$.

Source: NFL Players Assn./*USA TODAY.*

13. **Cyber School Enrollment** The data show the number of students attending cyber charter schools in Allegheny County and the number of students attending cyber schools in counties surrounding Allegheny County. At $\alpha = 0.01$ is there enough evidence to support the claim that the average number of students in school districts in Allegheny County who attend cyber schools is greater than those who attend cyber schools in school districts outside Allegheny County? Give a factor that should be considered in interpreting this answer.

Allegheny County						Outside Allegheny County					
25	75	38	41	27	32	57	25	38	14	10	29

Source: *Pittsburgh Tribune-Review.*

14. Ages of Homes Whiting, Indiana, leads the "Top 100 Cities with the Oldest Houses" list with the average age of houses being 66.4 years. Farther down the list resides Franklin, Pennsylvania, with an average house age of 59.4 years. Researchers selected a random sample of 20 houses in each city and obtained the following statistics. At $\alpha = 0.05$, can it be concluded that the houses in Whiting are older? Use the *P*-value method.

	Whiting	Franklin
Mean age	62.1 years	55.6 years
Standard deviation	5.4 years	3.9 years

Source: www.city-data.com

15. Hospital Stays for Maternity Patients Health Care Knowledge Systems reported that an insured woman spends on average 2.3 days in the hospital for a routine childbirth, while an uninsured woman spends on average 1.9 days. Assume two samples of 16 women each were used in both samples. The standard deviation of the first sample is equal to 0.6 day, and the standard deviation of the second sample is 0.3 day. At $\alpha = 0.01$, test the claim that the means are equal. Find the 99% confidence interval for the differences of the means. Use the *P*-value method.

Source: Michael D. Shook and Robert L. Shook, *The Book of Odds.*

16. Hockey's Highest Scorers The number of points held by a sample of the NHL's highest scorers for both the Eastern Conference and the Western Conference is shown below. At $\alpha = 0.05$, can it be concluded that there is a difference in means based on these data?

Eastern Conference				Western Conference			
83	60	75	58	77	59	72	58
78	59	70	58	37	57	66	55
62	61	59		61			

Source: www.foxsports.com

17. Medical School Enrollments A random sample of enrollments from medical schools that specialize in research and from those that are noted for primary care is listed. Find the 90% confidence interval for the difference in the means. $9.87 < \mu_1 - \mu_2 < 219.6$

Research				Primary care			
474	577	605	663	783	605	427	728
783	467	670	414	546	474	371	107
813	443	565	696	442	587	293	277
692	694	277	419	662	555	527	320
884							

Source: *U.S. News & World Report Best Graduate Schools.*

18. Out-of-State Tuitions The out-of-state tuitions (in dollars) for random samples of both public and private four-year colleges in a New England state are listed. Find the 95% confidence interval for the difference in the means.

Private		Public	
13,600	13,495	7,050	9,000
16,590	17,300	6,450	9,758
23,400	12,500	7,050	7,871
		16,100	

Source: *New York Times Almanac.* $\$1789.70 < \mu_1 - \mu_2 < \$12,425.41$

Technology *Step by Step*

MINITAB
Step by Step

Test the Difference Between Two Means: Independent Samples*

MINITAB will calculate the test statistic and *P*-value for differences between the means for two populations when the population standard deviations are unknown.

For Example 9–2, is the average number of sports for men higher than the average number for women?

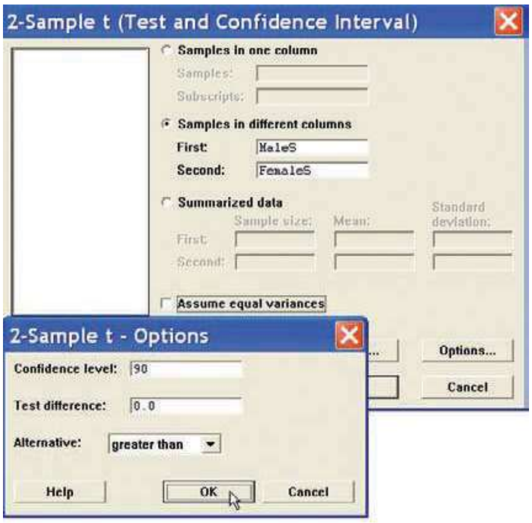
1. Enter the data for Example 9–2 into C1 and C2. Name the columns **MaleS** and **FemaleS.**
2. Select **Stat>Basic Statistics>2-Sample t.**
3. Click the button for Samples in different columns.

There is one sample in each column.

4. Click in the box for First:. Double-click C1 MaleS in the list.

*MINITAB does not calculate a *z* test statistic. This statistic can be used instead.

5. Click in the box for Second:, then double-click C2 FemaleS in the list. Do not check the box for Assume equal variances. MINITAB will use the large sample formula. The completed dialog box is shown.
6. Click [Options].
   a) Type in **90** for the Confidence level and **0** for the Test mean.
   b) Select greater than for the Alternative. This option affects the *P*-value. It must be correct.
7. Click [OK] twice. Since the *P*-value is greater than the significance level, $0.172 > 0.1$, do not reject the null hypothesis.



Two-Sample t-Test and CI: MaleS, FemaleS

```
Two-sample  t  for  MaleS  vs FemaleS
          N   Mean  StDev  SE Mean
MaleS    50   8.56   3.26     0.46
FemaleS  50   7.94   3.27     0.46
Difference = mu (MaleS) - mu (FemaleS)
Estimate for difference: 0.620000
90% lower bound for difference: -0.221962
t-Test of difference = 0 (vs >): t-Value = 0.95 P-Value = 0.172 DF = 97
```

TI-83 Plus or TI-84 Plus
Step by Step

Hypothesis Test for the Difference Between Two Means and *t* Distribution (Statistics)

1. Press **STAT** and move the cursor to TESTS.
2. Press **4** for 2-SampTTest.
3. Move the cursor to Stats and press **ENTER.**
4. Type in the appropriate values.
5. Move the cursor to the appropriate alternative hypothesis and press **ENTER.**
6. On the line for Pooled, move the cursor to No (standard deviations are assumed not equal) and press **ENTER.**
7. Move the cursor to Calculate and press **ENTER.**

Confidence Interval for the Difference Between Two Means and *t* Distribution (Data)

1. Enter the data values into L_1 and L_2.
2. Press **STAT** and move the cursor to TESTS.
3. Press **0** for 2-SampTInt.
4. Move the cursor to Data and press **ENTER.**
5. Type in the appropriate values.
6. On the line for Pooled, move the cursor to No (standard deviations are assumed not equal) and press **ENTER.**
7. Move the cursor to Calculate and press **ENTER.**

Confidence Interval for the Difference Between Two Means and *t* Distribution (Statistics)

1. Press **STAT** and move the cursor to TESTS.
2. Press **0** for 2-SampTInt.

3. Move the cursor to Stats and press **ENTER.**
4. Type in the appropriate values.
5. On the line for Pooled, move the cursor to No (standard deviations are assumed not equal) and press **ENTER.**
6. Move the cursor to Calculate and press **ENTER.**

Excel
Step by Step

Testing the Difference Between Two Means: Independent Samples

Excel has a two-sample *t* test included in the Data Analysis Add-in. The following example shows how to perform a *t* test for the difference between two means.

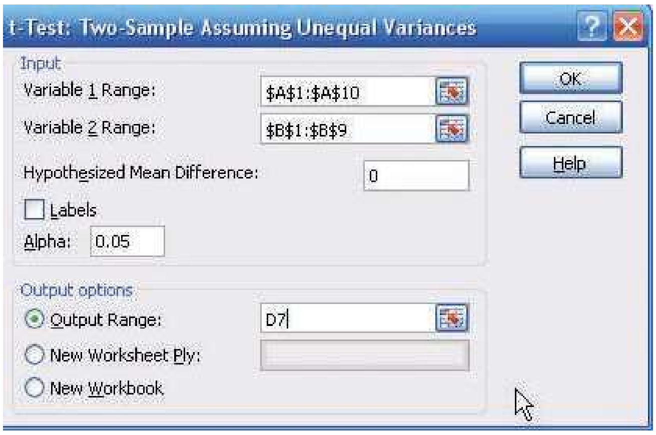
Example XL9–2

Test the claim that there is no difference between population means based on these sample data. Assume the population variances are not equal. Use $\alpha = 0.05$.

Set A	32	38	37	36	36	34	39	36	37	42
Set B	30	36	35	36	31	34	37	33	32	

1. Enter the 10-number data set A into column A.
2. Enter the 9-number data set B into column B.
3. Select the Data tab from the toolbar. Then select Data Analysis.
4. In the Data Analysis box, under Analysis Tools select *t*-test: Two-Sample Assuming Unequal Variances, and click [OK].
5. In Input, type in the Variable 1 Range: **A1:A10** and the Variable 2 Range: **B1:B9.**
6. Type **0** for the Hypothesized Mean Difference.
7. Type **0.05** for Alpha.
8. In Output options, type D9 for the Output Range, then click [OK].

Two-Sample *t* Test in Excel



t-Test: Two-Sample Assuming Unequal Variances		
	Variable 1	*Variable 2*
Mean	36.7	33.77777778
Variance	7.344444444	5.944444444
Observations	10	9
Hypothesized Mean Difference	0	
df	17	
t Stat	2.474205364	
P(T<=t) one-tail	0.012095	
t Critical one-tail	1.739606716	
P(T<=t) two-tail	0.024189999	
t Critical two-tail	2.109815559	

Note: You may need to increase the column width to see all the results. To do this:

1. Highlight the columns D, E, and F.
2. Select **Format>AutoFit** Column Width.

The output reports both one- and two-tailed *P*-values.

9–3

Testing the Difference Between Two Means: Dependent Samples

Objective 3

Test the difference between two means for dependent samples.

In Section 9–2, the t test was used to compare two sample means when the samples were independent. In this section, a different version of the t test is explained. This version is used when the samples are dependent. Samples are considered to be **dependent samples** when the subjects are paired or matched in some way.

For example, suppose a medical researcher wants to see whether a drug will affect the reaction time of its users. To test this hypothesis, the researcher must pretest the subjects in the sample first. That is, they are given a test to ascertain their normal reaction times. Then after taking the drug, the subjects are tested again, using a posttest. Finally, the means of the two tests are compared to see whether there is a difference. Since the same subjects are used in both cases, the samples are *related;* subjects scoring high on the pretest will generally score high on the posttest, even after consuming the drug. Likewise, those scoring lower on the pretest will tend to score lower on the posttest. To take this effect into account, the researcher employs a t test, using the differences between the pretest values and the posttest values. Thus only the gain or loss in values is compared.

Here are some other examples of dependent samples. A researcher may want to design an SAT preparation course to help students raise their test scores the second time they take the SAT. Hence, the differences between the two exams are compared. A medical specialist may want to see whether a new counseling program will help subjects lose weight. Therefore, the preweights of the subjects will be compared with the postweights.

Besides samples in which the same subjects are used in a pre-post situation, there are other cases where the samples are considered dependent. For example, students might be matched or paired according to some variable that is pertinent to the study; then one student is assigned to one group, and the other student is assigned to a second group. For instance, in a study involving learning, students can be selected and paired according to their IQs. That is, two students with the same IQ will be paired. Then one will be assigned to one sample group (which might receive instruction by computers), and the other student will be assigned to another sample group (which might receive instruction by the lecture discussion method). These assignments will be done randomly. Since a student's IQ is important to learning, it is a variable that should be controlled. By matching subjects on IQ, the researcher can eliminate the variable's influence, for the most part. Matching, then, helps to reduce type II error by eliminating extraneous variables.

Two notes of caution should be mentioned. First, when subjects are matched according to one variable, the matching process does not eliminate the influence of other variables. Matching students according to IQ does not account for their mathematical ability or their familiarity with computers. Since not all variables influencing a study can be controlled, it is up to the researcher to determine which variables should be used in matching. Second, when the same subjects are used for a pre-post study, sometimes the knowledge that they are participating in a study can influence the results. For example, if people are placed in a special program, they may be more highly motivated to succeed simply because they have been selected to participate; the program itself may have little effect on their success.

When the samples are dependent, a special t test for dependent means is used. This test employs the difference in values of the matched pairs. The hypotheses are as follows:

Two-tailed	Left-tailed	Right-tailed
H_0: $\mu_D = 0$	H_0: $\mu_D = 0$	H_0: $\mu_D = 0$
H_1: $\mu_D \neq 0$	H_1: $\mu_D < 0$	H_1: $\mu_D > 0$

where μ_D is the symbol for the expected mean of the difference of the matched pairs. The general procedure for finding the test value involves several steps.

First, find the differences of the values of the pairs of data.

$$D = X_1 - X_2$$

Second, find the mean $\overline{D}$ of the differences, using the formula

$$\overline{D} = \frac{\Sigma D}{n}$$

where n is the number of data pairs. Third, find the standard deviation s_D of the differences, using the formula

$$s_D = \sqrt{\frac{n\Sigma D^2 - (\Sigma D)^2}{n(n-1)}}$$

Fourth, find the estimated standard error $s_{\overline{D}}$ of the differences, which is

$$s_{\overline{D}} = \frac{s_D}{\sqrt{n}}$$

Finally, find the test value, using the formula

$$t = \frac{\overline{D} - \mu_D}{s_D/\sqrt{n}} \qquad \text{with d.f.} = n - 1$$

The formula in the final step follows the basic format of

$$\text{Test value} = \frac{(\text{observed value}) - (\text{expected value})}{\text{standard error}}$$

where the observed value is the mean of the differences. The expected value μ_D is zero if the hypothesis is $\mu_D = 0$. The standard error of the difference is the standard deviation of the difference, divided by the square root of the sample size. Both populations must be normally or approximately normally distributed. Example 9–6 illustrates the hypothesis-testing procedure in detail.

Assumptions for the *t* Test for Two Means When the Samples Are Dependent

1. The sample or samples are random.
2. The sample data are dependent.
3. When the sample size or sample sizes are less than 30, the population or populations must be normally or approximately normally distributed.

Example 9–6

Bank Deposits

A sample of nine local banks shows their deposits (in billions of dollars) 3 years ago and their deposits (in billions of dollars) today. At $\alpha = 0.05$, can it be concluded that the average in deposits for the banks is greater today than it was 3 years ago? Use $\alpha = 0.05$.

Source: SNL Financial.

Bank	1	2	3	4	5	6	7	8	9
3 years ago	11.42	8.41	3.98	7.37	2.28	1.10	1.00	0.9	1.35
Today	16.69	9.44	6.53	5.58	2.92	1.88	1.78	1.5	1.22