5. $f(x) = \dfrac{x^3y - xy^3}{x^2+y^2}$; find f_y) and f_x

Solu

$$f_x = \frac{(x^2+y^2)\cdot(3x^2y - y^3) - (x^3y - xy^3)(2x)}{(x^2+y^2)^2}$$

$$= \frac{3x^4y - x^2y^3 + 3x^2y^3 - y^5 - 2x^4y + 2x^2y^3}{(x^2+y^2)^2}$$

$$= \frac{x^4y + 4x^2y^3 - y^5}{(x^2+y^2)^2}$$

$$f_y = \frac{(x^2+y^2)(x^3 - 3xy^2) - (x^3y - xy^3)(2y)}{(x^2+y^2)^2}$$

$$= \frac{x^5 - 3x^3y^2 + y^2x^3 - 3xy^4 - 2x^3y^2 + 2xy^4}{(x^2+y^2)^2}$$

$$= \frac{x^5 - 4x^3y^2 - xy^4}{(x^2+y^2)^2}$$

6. $f(x,y) = \sqrt{x^2+y^2}$, Find f_x and f_y ?

$$f_x = \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}\cdot 2x = \frac{x}{\sqrt{x^2+y^2}}$$

$$f_y = \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}\cdot 2y = \frac{y}{\sqrt{x^2+y^2}}$$

7. $f(x,y) = \sin^{-1}(2x+3y)$, find f_x and f_y ?

$$f_x = \frac{2}{\sqrt{1-(2x+3y)^2}}$$

$$f_y = \frac{2}{\sqrt{1-(2x+3y)^2}}$$

8. $f(x,y) = \ln(1+xy^2)$; find $f_x(1,-1)$; $f_y(1,-1)$

Solu
$$f_x = \frac{y^2}{1+xy^2} \quad ; \quad f_x(1,-1) = \frac{(-1)^2}{1+1\times(-1)^2} = \frac{1}{1+1} = \frac{1}{2}$$

$$f_y = \frac{2xy}{1+xy^2} \quad ; \quad f_y(1,-1) = \frac{2(1)(-1)}{1+(1)(-1)^2} = \frac{-2}{1+1} = -1$$

9. $f(x,y) = x^2 + xy^2$; Find: f_{xx} , f_{yy} , f_{xy} , f_{yx}

Solu

$f_x = 2x + y^2$; $f_{xy} = 2y$; $f_{xx} = 2$

$f_y = 2xy$; $f_{yx} = 2y$; $f_{yy} = 2x$

10. $f(x,y) = 3x^2 + y^2$; find f_{xx} , f_{yy} , f_{xy} , f_{yx}

Solu $f_x = 6x$; $f_{xx} = 6$; $f_{xy} = 0$

$f_y = 2y$; $f_{yy} = 2$; $f_{yx} = 0$

11. $f(x,y) = \ln(3x-5y)$

Find f_{xx}, f_{yy}, f_{xy}, f_{yx}

Solu

$$f_x = \frac{3}{3x-5y} \; ; \; f_{xx} = \frac{(3x-5y)(0)-3(3)}{(3x-5y)^2} = \frac{-9}{(3x-5y)^2}$$

$$f_y = \frac{-5}{3x-5y} \; ; \; f_{yy} = \frac{(3x-5y)(0)-(-5)(-5)}{(3x-5y)^2} = \frac{-25}{(3x-5y)^2}$$

$$f_{xy} = \frac{(3x-5y)(0)-(3)(-5)}{(3x-5y)^2} = \frac{15}{(3x-5y)^2}$$

$$f_{yx} = \frac{(3x-5y)(0)-(-5)(3)}{(3x-5y)^2} = \frac{15}{(3x-5y)^2}$$

12. $f(x,y) = \ln(2x+2y) + \tan(2x-2y)$

Solu

$$f_x = \frac{2}{2x+2y} + 2\sec^2(2x-2y) = \frac{1}{x+y} + 2\sec^2(2x-2y)$$

$$f_y = \frac{2}{2x+2y} - 2\sec^2(2x-2y) = \frac{1}{x+y} - 2\sec^2(2x-2y)$$

$$f_{xy} = \frac{(x+y)(0)-(1)(1)}{(x+y)^2} - 8\sec^2(2x-2y)\tan(2x-2y)$$

$$f_{yx} = \frac{(x+y)(0)-(1)(1)}{(x+y)^2} - 8\sec^2(2x-2y)\tan(2x-2y)$$

$$f_{xx} = \frac{(x+y)(0)-(1)(1)}{(x+y)^2} + 8\sec^2(2x-2y)\tan(2x-2y)$$

$$= \frac{-1}{(x+y)^2} + 8\sec^2(2x-2y)\tan(2x-2y)$$

$$f_{yy} = \frac{(x+y)(0)-(1)(1)}{(x+y)^2} + 8\sec^2(2x-2y)\tan(2x-2y)$$

$$= \frac{-1}{(x+y)^2} + 8\sec^2(2x-2y)\tan(2x-2y)$$

13. $f(x,y) = \cos 2x \; e^{-2y}$

prove $f_{xx} + f_{yy} = 0$ or $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$

Solu

$$f_x = -2\sin(2x)\cdot e^{-2y} + \cos(2x)\cdot(0)$$

$$= -2e^{-2y}\sin(2x)$$

$$f_{xx} = -2[e^{-2y}\cdot 2\cos(2x) + \sin(2x)(0)]$$

$$= -4e^{-2y}\cos(2x)$$

$$f_y = \cos(2x)\cdot e^{-2y}\cdot(-2) + e^{-2y}(0)$$

$$= -2e^{-2y}\cos(2x)$$

$$f_{yy} = -2[e^{-2y}(0) - \cos(2x)\cdot e^{-2y}(-2)]$$

$$f_{yy} = +4e^{-2y}\cos(2x)$$

$$f_{xx} + f_{yy} \overset{?}{=} 0$$

$$-4e^{-2y}\cos(2x) + 4e^{-2y}\cos(2x) = 0$$

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H.W.

① If the function:-

1. $f(x,y) = x\cos(xy)e^{xy} + \sin y$

2. $f(x,y) = y\cos(xy)e^{xy} + \sin x$

3. $f(x,y) = (x-y)\sin(x+y)$

4. $f(x,y) = y\cos^6(x^2)$

Find f_y and f_x

② $f(x,y) = \sin(xy^2) + \ln\left(\frac{x}{y}\right)$

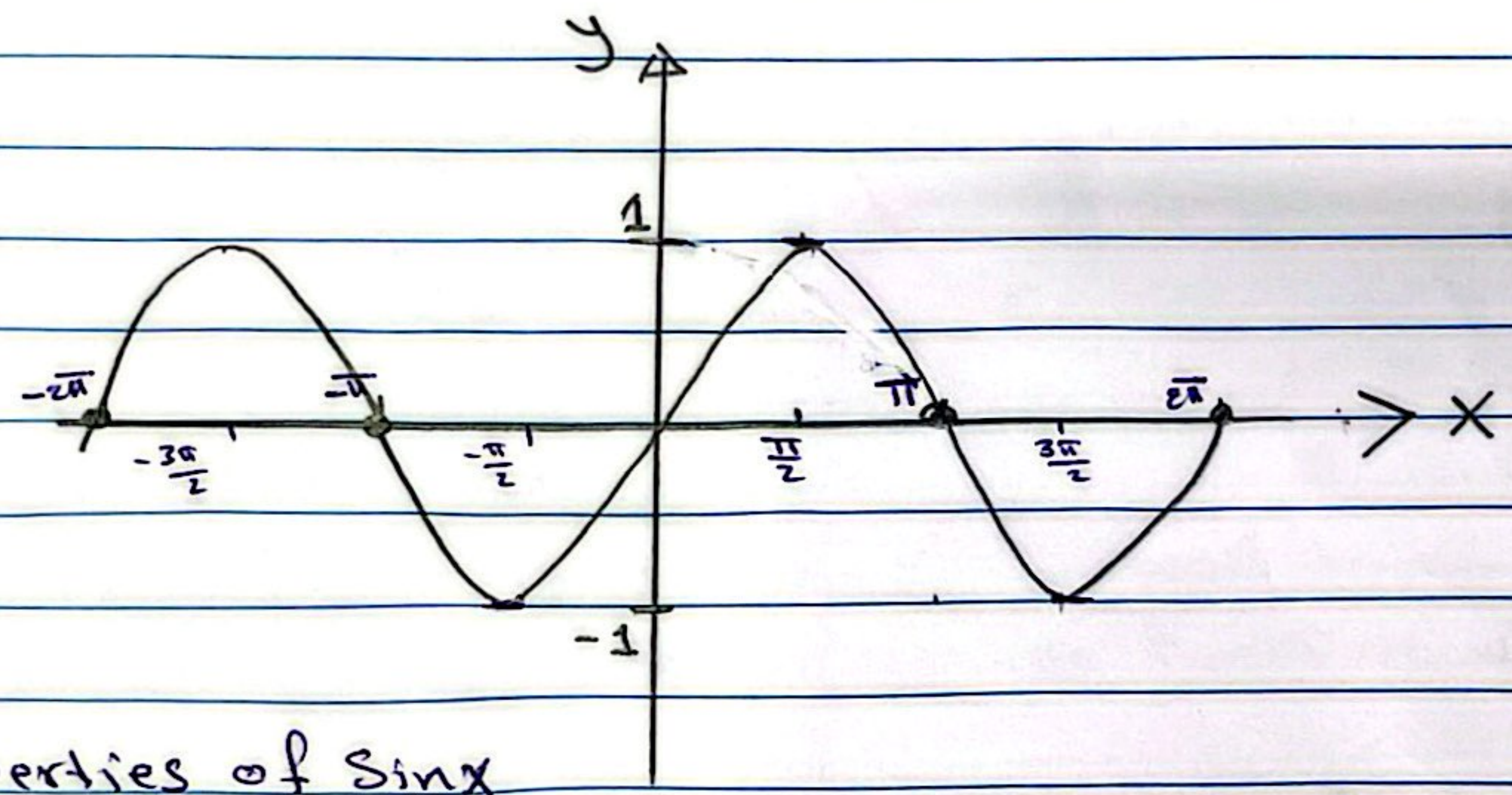
Find: $f_x, f_y, f_{xx}, f_{yy}, f_{xy}, f_{yx}$

The Trigonometric Functions :-

1. $f(x) = y = \sin x$

$D_y = \{ x : x \in R \} = (-\infty, \infty)$

$R_x = \{ y : y \in R \} = [-1, 1]$



The properties of $\sin x$

- $\sin(-x) = -\sin x$ — it is an odd function
- $\sin(x+2\pi) = \sin(x)$ — it is a periodic function with 2π

$\sin(\pi) = 0 \Rightarrow \sin(-\pi) = 0$

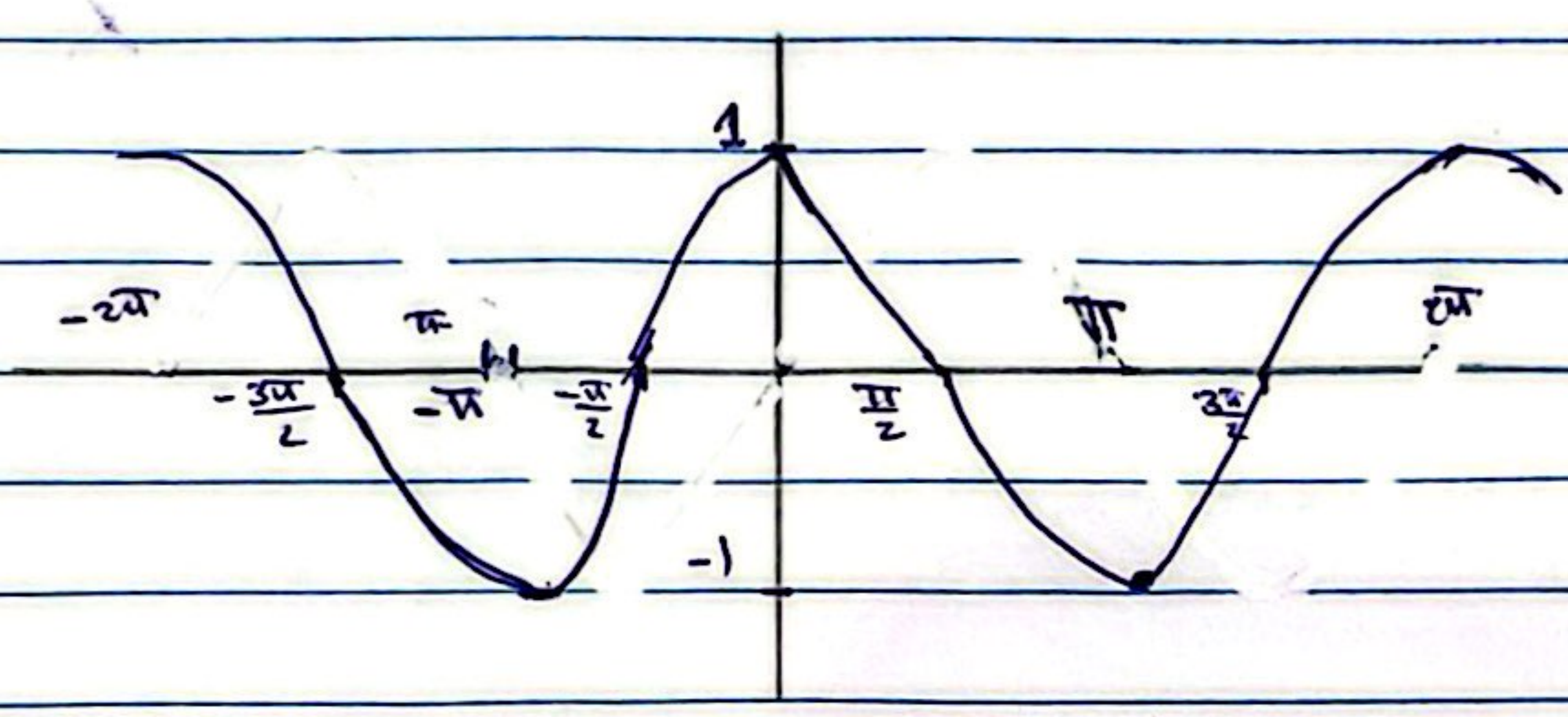
$\sin(\frac{\pi}{2}) = 1 \Rightarrow \sin(\frac{-\pi}{2}) = -1$

$$\frac{\partial \sin x}{\partial x} = \cos x$$

2. $f(x) = y = \cos x$

$D_y = \{x : x \in R\} = (-\infty, \infty)$

$R_x = \{y : y \in R\} = [-1, 1]$



The Properties of cosx

- $\cos(-x) = \cos x$ it is an even function
- $\cos(x + 2\pi) = \cos(x)$ it is a periodic function with 2π

$\cos(-\pi) = -1 \Rightarrow \cos(\pi) = 1$; $\frac{\partial \cos x}{dx} = -\sin x$

$\cos(\frac{-\pi}{2}) = 0 \Rightarrow \cos(\frac{\pi}{2}) = 0$

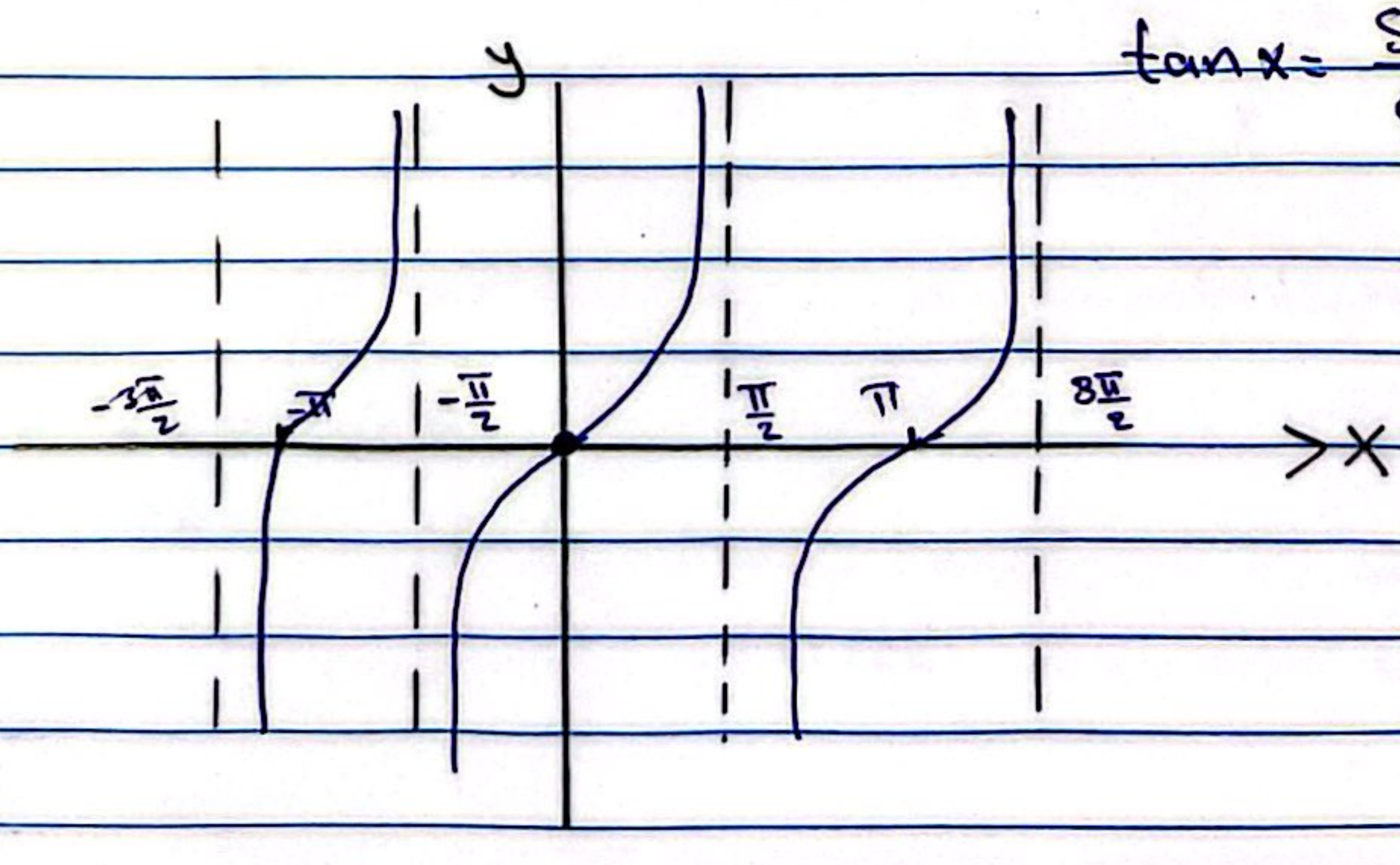
3. $f(x) = y = \tan x$

The domain of the function is all the real number, Except $\cos x = 0$

$D_y = \{x : x \in R, x \neq \frac{\pi}{2} + n\pi ; n = 0, \mp 1, \mp 2, \dots\}$

$R_x = R$

$\tan x = \frac{\sin x}{\cos x}$



The properties of $\tan x$

- $\tan(-x) = \tan x$ it is odd function
- $\tan(x+\pi) = \tan x$ it is periodic function with π

$\tan(-\pi) = 0 \Rightarrow \tan(\pi) = 0$; $\frac{\partial \tan x}{\partial x} = \sec^2 x$

$\tan(\frac{\pi}{4}) = 1 \Rightarrow \tan(-\frac{\pi}{4}) = -1$

$\tan(2\pi) = 0 \Rightarrow \tan(-2\pi) = 0$

4. $f(x) = y = \cot x$

The domain of the function is all the real numbers, Expect $\sin x = 0$; Such that

$D_y = \{x : x \in R,\ x \neq n\pi,\ n = 0, \mp 1, \mp 2, \dots\}$

$R_x = R$ the real numbers

$\cot x = \frac{\cos x}{\sin x}$

$\frac{\partial \cot x}{\partial x} = -\csc^2 x$

The properties of $\cot x$

- $\cot(-x) = -\cot(x)$ it is odd function
- $\cot(x+\pi) = \cot(x)$ it is apriodic function with π

5. $f(x) = y = \sec x = \frac{1}{\cos x}$; $\cos x \neq 0$

$D_y = \{x : x \in R ;\ x \neq \frac{\pi}{2} + n\pi ;\ n = 0, \mp 1, \mp 2, \dots\}$

$R_x = R$

$\frac{\partial \sec x}{\partial x} = \sec x \tan x$

The properties of $\sec x$

- $\sec(-x) = \sec(x)$ it is even function
- $\sec(2\pi + x) = \sec(x)$ it is a periodic function with 2π

6. $f(x) = y = \csc x = \frac{1}{\sin x}$; $\sin x \neq 0$

$D_y = \{x : x \in R ; x \neq n\pi ; n = 0, \mp 1, \mp 2, \dots\}$

$R_x = R$ $\qquad \frac{\partial \csc x}{\partial x} = -\csc x \cot x$

The properties of cscx

- $\csc(-x) = -\csc(x)$ it is odd function
- $\csc(x + 2\pi) = \csc(x)$ it is aperiodic function with 2π

Note

$\lim_{x \to 0} \frac{\sin x}{x} = 1$; $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

$\lim_{x \to 1} \frac{x - 1}{\ln x} = 1$; $\lim_{x \to 0} \left(\frac{e^x - 1}{x} \right) = 1$

$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e = 2.718$

$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$

$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta}$