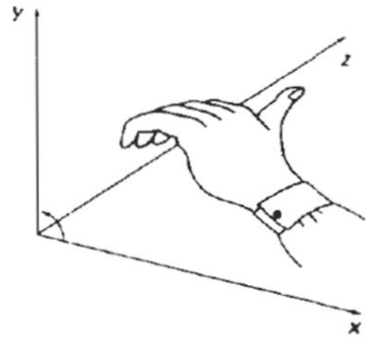


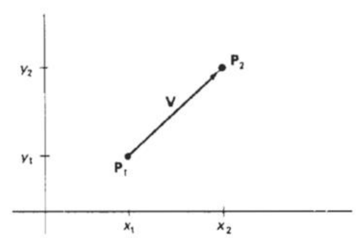
Left-handed Cartesian coordinate system superimposed on the surface of a video monitor.

Points and Vectors

There is a fundamental difference between the concept of a point and that of a vector. A point is a position specified with coordinate values in some reference frame, so that the distance from the origin depends on the choice of reference frame.

A vector, on the other hand, is defined as the difference between two point positions. Thus, for a two-dimensional vector (the figure in below), we have:

$$\mathbf{V} = \mathbf{P}_2 - \mathbf{P}_1$$
$$= (x_2 - x_1, y_2 - y_1)$$
$$= (V_x, V_y)$$



Vector V in the *xy* plane of a Cartesian reference frame.

where the Cartesian components (or Cartesian elements) V_x and V_y are the projections of **V** onto the *x* and *y* axes. We can describe a vector as a directed line segment that has two fundamental properties: magnitude and direction. For a two-dimensional vector, we calculate vector magnitude using the Pythagorean Theorem:

$$|\mathbf{V}| = \sqrt{V_x^2 + V_y^2}$$

The direction for this two-dimensional vector can be given in terms of the angular displacement from the *x* axis as:

$$\alpha = \tan^{-1}\left(\frac{V_y}{V_x}\right)$$

Line Representation

A straight line can be represented by a slope intercept equation as:

$$Y = mx+b$$

If two endpoints of the line are specified at positions (x_1,y_1) and (x_2,y_2), the values of the slope *m* and intercept *b* can be determined as:

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{And} \qquad b = y_1 - mx_1$$

The slope *m* is the change in height (y_2- y_1) divided by the change in the width ($x_2 - x_1$) for two points on the line. The intercept *b* is the height at which the line crosses the *y*-axis.

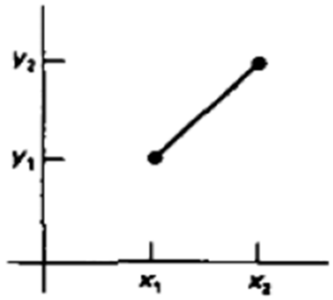
If Δx and Δy are the intervals corresponding to x and y respectively for a line, then for given interval Δx, we can calculate Δy:

$$\Delta y = m\Delta x$$

Similarly for given interval Δy, Δx can be calculated as:

$$\Delta x = \frac{\Delta y}{m}$$

The following figure shows line drawn between points (x_1,y_1) and (x_2,y_2) in Cartesian coordinate:



Line drawn between points (x_1,y_1) and (x_2,y_2) in Cartesian coordinate

Examples :

Write an equation in slope-intercept form of the line that passes through the points (2,2) and (3,4).

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \frac{4-2}{3-2} \qquad m = \frac{2}{1} \qquad m=2$$

Use the slope-intercept form of a line and the values we know to solve for '*b*', which is the *y*-intercept We can plug in 2 for *m*, and we take one of our ordered pairs and plug it in for *x* and *y* we solve:

$$Y = mx + b \qquad Y = 2x + b \qquad 4 = 2(3) + b \qquad 4 = 6 + b \qquad b = 4 - 6 \qquad b = -2$$

'b' is equal to -2, our *y*-intercept Plug the values for slope and *y*-intercept into the slope-intercept form to find the equation of our line: $y = 2x - 2$

Write the slope-intercept form of the equation of the line that passes through the two points.

1. (2, 3), (6, 11) $y = 2x - 1$
2. (1, -7), (3, -15) $y = -4x - 3$

Write an equation of the line in point-slope form that passes through the point and has the given slope. Then rewrite the equation in slope-intercept form.

3. $(-1, 1)$, $m = \frac{2}{3}$

$$y = \frac{2}{3}x + \frac{5}{3}$$

4. $(6, -3)$, $m = -\frac{1}{2}$

$$y = -\frac{1}{2}x$$

Write the equation in standard form of the line that passes through the two points.

5. $(5, 8)$, $(3, 2)$

$$3x - y = 7$$

6. $(-4, -5)$, $(-2, 5)$

$$5x - y = -15$$