Confidence Interval for ratio of two variances :-

المقصود ب (ratio) النسبة اي :-

$$\text{ratio} = \frac{\sigma_1^2}{\sigma_2^2} \quad \text{or} \quad \frac{\sigma_2^2}{\sigma_1^2}$$

$r_L < \frac{\sigma_1^2}{\sigma_2^2} < r_u \longleftarrow$ في الحالة الاولى

$r_L < \frac{\sigma_2^2}{\sigma_1^2} < r_u \longleftarrow$ في الحالة الثانية

~~Def.~~

$$F = \frac{\chi^2_{v_1}/v_1}{\chi^2_{v_2}/v_2} = F\text{-distribution}$$

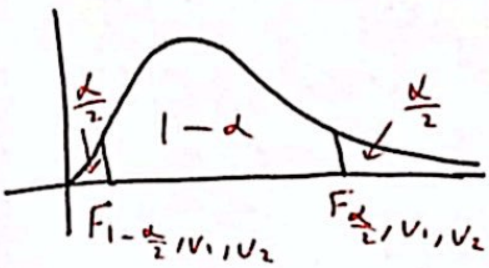
Now assume we have two populations with variances σ_1^2 and σ_2^2, and assume we took two samples with sizes n_1 and n_2 respectively, and we got s_1^2 and s_2^2. Then to construct $(1-\alpha)$ 100% C.I. for $\frac{\sigma_1^2}{\sigma_2^2}$ (or $\frac{\sigma_2^2}{\sigma_1^2}$), we know that

$$\frac{(n_1-1)S_1^2}{\sigma_1^2} \sim \chi^2_{(n_1-1)} \quad \text{and} \quad \frac{(n_2-1)S_2^2}{\sigma_2^2} \sim \chi^2_{n_2-1}$$

$$F = \frac{\chi^2_{n_1-1}/n_1-1}{\chi^2_{n_2-1}/n_2-1} = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} = \frac{S_1^2\,\sigma_2^2}{S_2^2\,\sigma_1^2}$$

and we know that

$$Pr\left(F_{1-\frac{\alpha}{2}, v_1, v_2} < F < F_{\frac{\alpha}{2}, v_1, v_2}\right) = 1-\alpha$$



عندئذٍ وبالتعويض عن قيمة F نحصل على

$$Pr\left[F_{1-\frac{\alpha}{2}, \overset{v_1}{n_1-1}, \overset{v_2}{n_2-1}} < \frac{S_1^2\,\sigma_2^2}{S_2^2\,\sigma_1^2} < F_{\frac{\alpha}{2}, \overset{v_1}{n_1-1}, \overset{v_2}{n_2-1}}\right) = 1-\alpha$$

وبضرب جميع حدود المتراجحة بـ $\frac{S_2^2}{S_1^2}$ نحصل على

$$Pr\left[\frac{S_2^2}{S_1^2} F_{1-\frac{\alpha}{2}, n_1-1, n_2-1} < \frac{\sigma_2^2}{\sigma_1^2} < \frac{S_2^2}{S_1^2} F_{\frac{\alpha}{2}, n_1-1, n_2-1}\right] = 1-\alpha$$

وبقلب المتراجحة البسط مقام والمقام بسط فيصبح المقدار الاصغر هو الاكبر والمقدار الاكبر هو الاصغر

$$P\left[\frac{S_1^2}{S_2^2}\,\frac{1}{F_{\frac{\alpha}{2}, n_1-1, n_2-1}} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{S_1^2}{S_2^2}\,\frac{1}{F_{1-\frac{\alpha}{2}, n_1-1, n_2-1}}\right] = 1-\alpha$$

$$\because F_{\frac{\alpha}{2}, n_1-1, n_2-1} = \frac{1}{F_{1-\frac{\alpha}{2}, n_2-1, n_1-1}}$$

or

$$F_{\frac{\alpha}{2}, n_2-1, n_1-1} = \frac{1}{F_{1-\frac{\alpha}{2}, n_1-1, n_2-1}}$$

خاصية الانعكاس لتوزيع F - نستخدم لانه قيمة $F_{1-\frac{\alpha}{2}}$ الجدولية غير موجودة في الجداول الاحصائية فنتحول الى $F_{\frac{\alpha}{2}}$ حسب خاصية الانعكاس

$$\Longrightarrow Pr\left[\frac{S_1^2}{S_2^2}\,\frac{1}{F_{\frac{\alpha}{2}, n_1-1, n_2-1}} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{S_1^2}{S_2^2} F_{\frac{\alpha}{2}, n_2-1, n_1-1}\right] = 1-\alpha$$

$\Longrightarrow$

$(1-\alpha)100\%$ C.I. for $\frac{\sigma_1^2}{\sigma_2^2}$ is :-

$$\boxed{\frac{S_1^2}{S_2^2}\,\frac{1}{F_{\frac{\alpha}{2}, n_1-1, n_2-1}} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{S_1^2}{S_2^2} F_{\frac{\alpha}{2}, n_2-1, n_1-1}}$$

and $(1-\alpha)100\%$ C.I. for $\frac{\sigma_1}{\sigma_2}$ is :-

$$\frac{S_1}{S_2}\sqrt{\frac{1}{F_{\frac{\alpha}{2}, n_1-1, n_2-1}}} < \frac{\sigma_1}{\sigma_2} < \frac{S_1}{S_2}\sqrt{F_{\frac{\alpha}{2}, n_2-1, n_1-1}}$$

Ex Let $X_1, X_2, \ldots, X_9$ and $Y_1, Y_2, \ldots, Y_{13}$ be two random samples from $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ respectively. The results of the two independent samples are: $S_1^2 = 36.7$, $S_2^2 = 128.41$. Construct 98% C.I. for $\frac{\sigma_1^2}{\sigma_2^2}$.

Solu:

$$F_{\frac{\alpha}{2}, n_1-1, n_2-1} = F_{0.01, 8, 12} = 4.50$$

$$F_{\frac{\alpha}{2}, n_2-1, n_1-1} = F_{0.01, 12, 8} = 5.67$$

$\Rightarrow$ 98% C.I. for $\frac{\sigma_1^2}{\sigma_2^2}$ is

$$\frac{S_1^2}{S_2^2} \frac{1}{F_{\frac{\alpha}{2}, n_1-1, n_2-1}} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{S_1^2}{S_2^2} F_{\frac{\alpha}{2}, n_2-1, n_1-1}$$

$\Rightarrow$

$$\frac{36.7}{128.41} \cdot \frac{1}{4.50} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{36.7}{128.41} \cdot 5.67$$

$$0.064 < \frac{\sigma_1^2}{\sigma_2^2} < 1.62$$

هذا يعني انه بثقة 98% ان القيمة الحقيقية للنسبة بين التباينين $\frac{\sigma_1^2}{\sigma_2^2}$ هي تقع ضمن الفترة $(0.064 , 1.62)$ وفي هذه الحالة لا نستطيع معرفة اي تباين اكبر لان الحد الادنى اقل من (1) والحد الاعلى اكبر من (1) .