② Tests Concerning Differences between two means :-

① σ_1^2 and σ_2^2 are known

If we take two independent random samples of size n_1 and n_2 from two normal populations. If we want to test the null hypothesis

$H_0 : \mu_1 - \mu_2 = \delta_0$

against one of the following alternatives

① $H_1 : \mu_1 - \mu_2 \neq \delta_0 \Longrightarrow C.R : |Z| \geq Z_{\frac{\alpha}{2}}$

② $H_1 : \mu_1 - \mu_2 < \delta_0 \Longrightarrow C.R : Z < -Z_\alpha$

③ $H_1 : \mu_1 - \mu_2 > \delta_0 \Longrightarrow C.R : Z > Z_\alpha$

where

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - \delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

② σ_1^2 and σ_2^2 are unknown and $n_1 \geq 30$, $n_2 \geq 30$

In this case even if the population variances are unknown , We can also use the Z-test as above , but using s_1 and s_2 instead of σ_1 and σ_2

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - \delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

③ If σ_1^2 and σ_2^2 unknown and $n_1 < 30$, $n_2 < 30$

In this case we suppose that $(\sigma_1^2 = \sigma_2^2 = \sigma^2)$ and we use t-test , i.e

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - \delta_0}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Where $S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}$

C.R: ① $|t_{cal}| \geq t_{\frac{\alpha}{2}, n_1+n_2-2}$

② $t < -t_{\alpha, n_1+n_2-2}$

③ $t > t_{\alpha, n_1+n_2-2}$

Ex الدهون

Suppose that the fat of two kinds foods are being measured (تقيست). If 50-cans (علب) of the first kind had an average fat $(\bar{x}_1 = 2.61)$ grams , with s.d. $(S_1 = 0.12)$ grams , While 40-cans of the 2nd kind had an average $(\bar{x}_2 = 2.38)$ grams with s.d. $(S_2 = 0.14)$ grams. Test the null hypothesis

$H_0: \mu_1 - \mu_2 = 0.2$

$H_1: \mu_1 - \mu_2 \neq 0.2$

using critical region of size $\alpha = 0.05$

solu:

$H_0: \mu_1 - \mu_2 = \delta_0 = 0.2$

$H_1: \mu_1 - \mu_2 \neq 0.2$

$C.R: |Z| \geq Z_{\frac{\alpha}{2}} = Z_{0.025} = 1.96$

where

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - \delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$\bar{x}_1 = 2.61$, $\bar{x}_2 = 2.38$, $n_1 = 50$, $n_2 = 40$, $S_1 = 0.12$

$S_2 = 0.14$

$$Z = \frac{(2.61 - 2.38) - 0.2}{\sqrt{\frac{(0.12)^2}{50} + \frac{(0.14)^2}{40}}} = 1.08 < 1.96$$

we accept H_0

This means that $\mu_1 > \mu_2$ or the average fat in first kind food is greater than the average fat in the 2nd kind.

Ex:

صواريخ المدى قصيرة

If 8 short range rockets of one kind have a mean target error of $(\bar{x}_1 = 98)$ feet, with s.d. $(S_1 = 18)$ feet, while 10-short range rockets of 2nd kind have a mean target error of $(\bar{x}_2 = 76)$ feet with s.d $(S_2 = 15)$ feet. Test the hypothesis

هدف قدم

$H_0: \mu_1 - \mu_2 = 15$

$H_1: \mu_1 - \mu_2 > 15$

using level of significant $\alpha = 0.05$

Solu:

$H_0: \mu_1 - \mu_2 = \delta_0 = 15$

$H_1: \mu_1 - \mu_2 > 15$

$C.R: t_{cal} \geq t_{\alpha, n_1+n_2-2} = t_{0.05, 16} = 1.746$

$$t_{cal} = \frac{(\bar{x}_1 - \bar{x}_2) - \delta_0}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} = \frac{7(18)^2 + 9(15)^2}{16}$$

$\Longrightarrow S_p = 16.38$

$$t_{cal} = \frac{(98-76)-15}{(6.38)(0.474)} = 0.901 < 1.746$$

we accept H_0, this means that $\mu_1 > \mu_2$ and the difference between μ_1, μ_2 is (15) feet.