Tests Concerning Variances :-

Some times, we assume that variances of a population is equal a constant ① (σ_0^2), and sometimes we assume that two populations have equal variances ② $(\sigma_1^2 = \sigma_2^2)$. In this section we will try to test the above assumptions :-

In case ①, we try to test the null hypothesis $H_0: \sigma^2 = \sigma_0^2$ against one of the alternatives

① $\sigma^2 \neq \sigma_0^2$ ② $\sigma^2 > \sigma_0^2$ ③ $\sigma^2 < \sigma_0^2$

Critical Region

C.R : ① Two sided test $\chi^2_{cal} \geq \chi^2_{1-\frac{\alpha}{2}, n-1}$

or $\chi^2_{cal} \leq \chi^2_{\frac{\alpha}{2}, n-1}$

② $\chi^2_{cal} \geq \chi^2_{1-\alpha, n-1}$

③ $\chi^2_{cal} \leq \chi^2_{\alpha, n-1}$

where $\chi^2_{cal} = \dfrac{(n-1) S^2}{\sigma_0^2}$

where S^2 is the variance of the sample

$$S^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{X})^2}{n-1}$$

Ex.

In a random sample, the weights of 24-items have a standard deviation of (23.8) pounds. Assuming that the weights constitute a r.s. from normal population. Test the null hypothesis that $\sigma^2 = 250$ pounds against the alternative $\sigma^2 \neq 250$ pounds, use $\alpha = 0.01$, $\alpha = 0.05$

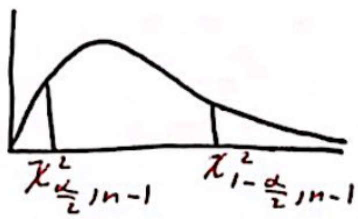
Soln:

$H_0 : \sigma^2 = 250$

$H_1 : \sigma^2 \neq 250$

C.R : $\chi^2_{cal} \geq \chi^2_{1-\frac{\alpha}{2}, n-1}$ or $\chi^2_{cal} \leq \chi^2_{\frac{\alpha}{2}, n-1}$

where $\chi^2_{cal} = \dfrac{(n-1) S^2}{\sigma_0^2}$



$n = 24$, $S = 23.8$, $\alpha = 0.01$

$\chi^2_{1-\frac{\alpha}{2}, n-1} = \chi^2_{0.995, 23} = 44.181$

$\chi^2_{\frac{\alpha}{2}, n-1} = \chi^2_{0.005, 23} = 9.260$

$\chi^2_{cal} = \dfrac{(23)(23.8)^2}{250} = 20.84$

$\because \chi^2_{cal} \ngtr \chi^2_{1-\frac{\alpha}{2}, n-1} = 44.181$ and $\chi^2 \nless \chi^2_{\frac{\alpha}{2}, n-1} = 9.260$

Therefore we cannot reject H_0.

This means we accept H_0.

In case ②

② Assumption of equality of variances :-

لدينا فرضية العدم

$$H_0: \sigma_1^2 = \sigma_2^2$$

والفرضيات البديلة

① $H_1: \sigma_1^2 \neq \sigma_2^2$

or ② $H_1: \sigma_1^2 > \sigma_2^2$

or ③ $H_1: \sigma_1^2 < \sigma_2^2$

المختبر الاحصائي هو F ويحسب كالاتي

$$F = \frac{\chi^2_{v_1}/v_1}{\chi^2_{v_2}/v_2}$$

حيث أن

$$\chi^2_{v_1} = \frac{(n_1-1)S_1^2}{\sigma_1^2} \quad , \quad \chi^2_{v_2} = \frac{(n_2-1)S_2^2}{\sigma_2^2}$$

وان v_1 و v_2 تمثلان درجة الحرية الخاصة بالمجتمع الاول والثاني على التوالي اي أن

$$v_1 = n_1 - 1$$
$$v_2 = n_2 - 1$$

$$\Rightarrow F = \frac{(n_1-1)S_1^2/\sigma_1^2(n_1-1)}{(n_2-1)S_2^2/\sigma_2^2(n_2-1)} = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2}$$

وتحت فرضية العدم التي تفترض تجانس تبايني المجتمعين اي $\sigma_1^2 = \sigma_2^2$ يكون :

under Ho we have

$$F = \frac{S_1^2}{S_2^2} \quad \text{because } \sigma_1^2 = \sigma_2^2$$

Critical region :- (C.R.)

$H_0: \sigma_1^2 = \sigma_2^2$
$H_1: \sigma_1^2 \neq \sigma_2^2$ ← عندما تكون الفرضية

① $F_{cal} = \dfrac{S_1^2}{S_2^2} \geq F_{\frac{\alpha}{2}, n_1-1, n_2-1}$ if $S_1^2 > S_2^2$

or $F_{cal} = \dfrac{S_2^2}{S_1^2} \geq F_{\frac{\alpha}{2}, n_2-1, n_1-1}$ if $S_2^2 > S_1^2$

في حالة الفرضية في اتجاهين أي $\neq$

②

$H_0: \sigma_1^2 = \sigma_2^2$
$H_1: \sigma_1^2 > \sigma_2^2$ عندما تكون الفرضية

$$F_{cal} = \frac{S_1^2}{S_2^2} \geq F_{\alpha, n_1-1, n_2-1}$$

③

$H_0: \sigma_1^2 = \sigma_2^2$
$H_1: \sigma_1^2 < \sigma_2^2$ عندما تكون الفرضية

$$F_{cal} = \frac{S_2^2}{S_1^2} \geq F_{\alpha, n_2-1, n_1-1}$$

Ex:

In comparing the variability (التوتر) tensile (قوة) strength of two kinds of (حديد البناء) Instruction steel, an equipments (الاجهزة) (اعطت) yielded the following resnlts

$n_1 = 13$, $S_1^2 = 19.2$, $n_2 = 16$, $S_2^2 = 3.5$

where the units of the measurements are :- $(1000)\,kg/inch^2$. Assuming that the measurements (مأخوذة) constitute independent r.s. from two-normal populations . Test the null hypothesis $H_0: \sigma_1^2 = \sigma_2^2$ against $H_1: \sigma_1^2 \neq \sigma_2^2$, use $\alpha = 0.02$

solu:

$H_0: \sigma_1^2 = \sigma_2^2$

$H_1: \sigma_1^2 \neq \sigma_2^2$

C.R:

$$S_1^2 = 19.2 > S_2^2 = 3.5$$

$$\Rightarrow C.R: F_{cal} = \frac{S_1^2}{S_2^2} \geq F_{\frac{\alpha}{2}, n_1-1, n_2-1} = F_{0.01, 12, 15} = 3.67$$

$$F_{cal} = \frac{19.2}{3.5} = 5.49 > 3.67$$

$\therefore$ we reject H_0. ← القرار

Ex: An instructor (مدرس) has two classes A and B in particular (بالخصوص) subject class A has 16-students while class B has 25 students. On the same examination, although there was no significant difference in mean grading (يصنف المعدل). Class A had a s.d. (انحراف معياري) of (9), while class B had a s.d (انحراف معياري) of (12). Can we conclude at $\alpha = 0.01$ that the variability (الاختلاف) of class B is greater than that in class A.

solu:

$H_0: \sigma_1^2 = \sigma_2^2$

$H_1: \sigma_1^2 < \sigma_2^2$

$$C.R: F = \frac{S_2^2}{S_1^2} > F_{\alpha, n_2-1, n_1-1} = F_{0.01, 24, 15} = 3.29$$

$$F = \frac{144}{81} = 1.77 \not> 3.29$$

$\therefore$ We accept H_0 (نقبل فرضية العدم). This means there is no significant difference between variances of class A and class B. This means any (أي) difference (فرق) in the variances due chance (خطاء عشوائية)