دالة القوى

Power Function :-

The power function, of a hypothesis $H_0: \theta = \theta_0$ against an alternative hypothesis, is that functi denoted by $k(\theta)$, which yields the probability of rejecting H_0, given θ is true

$$k(\theta) = p(\text{reject. } H_0: \theta = \theta_0, \text{ given } \theta \text{ is true})$$
$$= p(\text{rejecting } H_0 \mid \theta = \text{true})$$

If $\theta = \theta_0$ (i.e under H_0)

$$\Rightarrow k(\theta_0) = p(\text{rejecting } H_0 \mid \theta = \theta_0)$$
$$= \alpha$$
$$= p(\text{type I error})$$
$$= p(\text{rejecting } H_0 \mid H_0 = \text{true})$$

If $H_1: \theta = \theta_1 \Rightarrow k(\theta_1)$ is

$$k(\theta_1) = p(\text{rejecting } H_0 \mid \theta = \theta_1)$$
$$= p(\text{rejecting } H_0 \mid H_1 \text{ true})$$
$$= 1 - p(\text{accepting } H_0 \mid H_1 = \text{true})$$
$$= 1 - p(\text{accepting } H_0 \mid \theta = \theta_1)$$
$$= 1 - p(\text{type II error})$$
$$= 1 - \beta$$

$$\Rightarrow \beta = 1 - k(\theta_1)$$

$$\Rightarrow k(\theta) = \begin{cases} \alpha & \text{under } H_0 \\ 1-\beta & \text{under } H_1 \end{cases}$$

Operating Characteristic function (OC - function)

The OC - function is defined by

$OC(\theta) = p(\text{accepting } H_0 \mid \theta \text{ true})$

under $H_0 : \theta = \theta_0$

$OC(\theta_0) = p(\text{accepting } H_0 \mid \theta = \theta_0)$

$= 1 - p(\text{rejecting } H_0 \mid H_0 \text{ true})$

$= 1 - \alpha$

under $H_1 : \theta = \theta_1$

$\Rightarrow OC(\theta) = p(\text{accepting } H_0 \mid \theta = \theta_1)$

$= p(\text{accepting } H_0 \mid H_1 = \text{true})$

$= \beta$

$$\Rightarrow OC(\theta) = \begin{cases} 1 - \alpha & \text{under } H_0 \\ \beta & \text{under } H_1 \end{cases}$$

This mean the relation between $k(\theta)$ and $OC(\theta)$ is

$\Rightarrow OC(\theta) = 1 - k(\theta)$

Ex:

Let X have the p.d.f $f(x;\theta) = \begin{cases} \theta^x (1-\theta)^{1-x} & x=0,1 \\ 0 & \text{o.w.} \end{cases}$

To test the simple hypothesis $H_0: \theta = \frac{1}{4}$ against the alternative composite hypothesis $H_1: \theta < \frac{1}{4}$ suppose that the C.R. is :-

$C.R = \{X_1, X_2, \ldots, X_{10} : \sum_{i=1}^{10} X_i \leq 1\}$, Find

a) The power function, $K(\theta)$

b) The significant level α

c) The power of this test at $\theta = \frac{1}{16}$

d) The operating Characteristic function of the test.

e) P(type II error), at $\theta = \frac{1}{16}$

Solu

since $X \sim b(1,\theta)$ or $Ber(\theta)$

Hence $Y = \sum_{i=1}^{10} X_i \sim b(10,\theta)$ (by using m.g.f)

$\therefore f(y;\theta) = C_y^{10} \theta^y (1-\theta)^{10-y} \qquad y = 0,1,2,\ldots,10$

$$\begin{aligned} \therefore K(\theta) &= p(\text{rejecting } H_0 \mid \theta = \text{true}) \\ &= p(Y = \sum_{i=1}^{10} X_i \leq 1 ; \theta) \\ &= C_0^{10} (1-\theta)^{10} + C_1^{10} \theta (1-\theta)^9 \\ &= (1-\theta)^{10} + 10(1-\theta)^9 \theta \\ &= (1-\theta)^9 [1-\theta+10\theta] \\ &= (1-\theta)^9 (1+9\theta) \qquad 0 \leq \theta < \tfrac{1}{4} \end{aligned}$$

b) $\alpha = k(\theta_0) = (1+\frac{9}{4})(\frac{3}{4})^9 = \frac{13}{4}(\frac{3}{4})^9$

c) power of the test $= k(\frac{1}{16}) = (1+\frac{9}{16})(1-\frac{1}{16})^9$

$$= \frac{25}{16}(\frac{15}{16})^9$$

d) $OC(\theta) = 1-k(\theta) = 1-(1+9\theta)(1-\theta)^9$

e) $\beta = 1-k(\frac{1}{16}) = 1-(\frac{25}{16})(\frac{15}{16})^9$

Ex. suppose that $X_1, X_2, \dots, X_n$ is a r.s. from $N(\theta, 1)$

To test the simple hypothesis $H_0: \theta = 0$ against the simple alternative hypothesis $H_1: \theta = 1$, the C.R. is :

C.R. $= \{X_1, X_2, \dots, X_n ; \bar{X} \geq k\}$. Find n and k such that $\alpha = \beta = 0.01$

Solu

$\alpha = p(\text{rejecting } H_0 \mid H_0 \text{ true})$

$= p(\bar{X} \geq k \mid \theta = 0)$ — نحولها الى Z

$\bar{X} \sim N(\theta, \frac{\sigma^2}{n})$

But $\bar{X} \sim N(\theta, \frac{1}{n})$

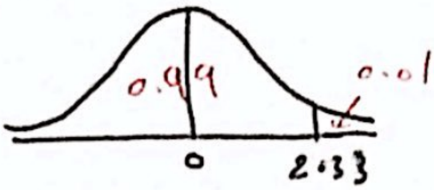
$\therefore$ under H_0

$\bar{X} \sim N(0, \frac{1}{n})$

There fore, $\alpha = p\left(\frac{\bar{X}-0}{\frac{1}{\sqrt{n}}} \geq \frac{k-0}{\frac{1}{\sqrt{n}}}\right)$

$$\Rightarrow 0.01 = P\left(Z \geq \frac{k-0}{\frac{1}{\sqrt{n}}}\right)$$

$$0.01 = 1 - P\left(Z < \frac{k}{\frac{1}{\sqrt{n}}}\right)$$



$$\Rightarrow 0.99 = P(Z < \sqrt{n}\,k)$$

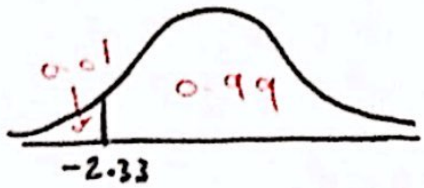
$$\therefore \sqrt{n}\,k = 2.33 \quad \text{----} \ ①$$

Also

$$\beta = P(\text{accepting } H_0 \mid H_1 \text{ true})$$

$$= P(\bar{X} < k \mid \theta = 1)$$

$$0.01 = P\left(\frac{\bar{X}-1}{\frac{1}{\sqrt{n}}} < \frac{k-1}{\frac{1}{\sqrt{n}}}\right)$$



$$0.01 = P(Z < \sqrt{n}(k-1))$$

$$\Rightarrow \sqrt{n}(k-1) = -2.33 \quad \text{----} \ ②$$

On solving ① and ②, for n we find it to be, $n = 22$ and $k = 0.5$

التحقيق

$$\because \sqrt{n}\,k = 2.33 \Rightarrow \sqrt{n} = \frac{2.33}{k}$$

من المعادلة ②

$$\frac{2.33}{k}(k-1) = -2.33 \Rightarrow 2.33k - 2.33 = -2.33k$$

$$\Rightarrow 4.66\,k = 2.33 \Rightarrow k = \frac{2.33}{4.66} = \boxed{0.5}$$

من المعادلة ①

$$\sqrt{n}\,\frac{1}{2} = 2.33 \Rightarrow \sqrt{n} = 4.66 \xRightarrow{\text{بالتربيع}} \boxed{n = 22}$$

يجب أنه تكون n عدد صحيح لأنها تمثل حجم العينة
وإذا كانت عدد فيه كسر نقرب إلى أقرب عدد صحيح

Ex نفاذ/استنزاف تمرين

The consumption of electricity in a small township is assumed to be exponentially distributed with parameter θ. Determine the size of type I and type II errors if $H_0: \theta = 1000$ k.w. is tested against the alternative $H_1: \theta = 2000$ and if the test criterion is as follows: select any day at random. If the consumption on that day is 4000 k.w. or more reject H_0, otherwise accept H_0.

Solu:

$$f(x;\theta) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}} & , x>0 , \theta>0 \\ 0 & \text{o.w.} \end{cases}$$

$$\begin{aligned} \alpha &= p(\text{rejecting } H_0 \mid H_0 \text{ true}) \\ &= p(X \geq 4000 \mid \theta_0 = 1000) \\ &= \int_{4000}^{\infty} \frac{1}{1000} e^{-\frac{x}{1000}} \, dx = -e^{-\frac{x}{1000}} \Big|_{4000}^{\infty} \\ &= -(0 - e^{-\frac{4000}{1000}}) = e^{-4} \end{aligned}$$

and

$$\begin{aligned} \beta &= p(\text{accepting } H_0 \mid H_1 \text{ true}) \\ &= p(X < 4000 \mid \theta_1 = 2000) \\ &= \int_0^{4000} \frac{1}{2000} e^{-\frac{x}{2000}} \, dx = -e^{-\frac{x}{2000}} \Big|_0^{4000} \\ &= -(e^{-2} - 1) \\ &= 1 - e^{-2} \end{aligned}$$

Ex

The same as in above example except that the test criterion is follows

Select any two days at random, if the consumption of electricity of there two days is 7000 k.w. or more reject H_0, otherwise accept H_0. Find the power function $K(\theta)$, α, β.

In another words the test to be used is defined by taking the critical region to be:

$$C.R: \{(x_1, x_2) : x_1 + x_2 \geq 7000\}$$

Solu

$$f(x;\theta) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}} & \theta > 0 \ , \ x > 0 \\ 0 & o.w. \end{cases}$$

The power function $K(\theta) = p(\text{rejecting } H_0 \mid \theta = \text{true})$

$$= p(X_1 + X_2 \geq 7000 \mid \theta)$$

$$= 1 - p(X_1 + X_2 < 7000 \mid \theta)$$

$X_1 + X_2 < 7000$

$X_1 < 7000 - X_2$

$0 < X_2 < 7000$

$$= 1 - \int_0^{7000} \int_0^{7000 - x_2} \frac{1}{\theta^2} e^{-\frac{x_1 + x_2}{\theta}} \, dx_1 \, dx_2$$

$$= 1 - \int_0^{7000} \frac{1}{\theta} e^{-\frac{x_2}{\theta}} \left[\int_0^{7000 - x_2} \frac{1}{\theta} e^{-\frac{x_1}{\theta}} \, dx_1 \right] dx_2$$

$$= 1 - \int_0^{7000} \frac{1}{\theta} e^{-\frac{x_2}{\theta}} \left[-e^{-\frac{x_1}{\theta}} \Big|_0^{7000 - x_2} \right] dx_2$$

$$= 1 - \int_0^{7000} \frac{1}{\theta} e^{-\frac{x_2}{\theta}} \left[-e^{\frac{x_2 - 7000}{\theta}} + 1 \right] dx_2$$

$$= 1 - \int_0^{7000} \frac{1}{\theta} e^{-\frac{x_2}{\theta}} \left[1 - e^{\frac{x_2}{\theta}} \cdot e^{-\frac{7000}{\theta}} \right] dx_2$$

$$= 1 - \left[\int_0^{7000} \frac{1}{\theta} e^{-\frac{x_2}{\theta}} - e^{-\frac{7000}{\theta}} \int_0^{7000} \frac{1}{\theta} e^{-\frac{x_2}{\theta}} \cdot e^{\frac{x_2}{\theta}} dx_2 \right]$$

$$= 1 - \left[- e^{-\frac{x_2}{\theta}} \Big|_0^{7000} - e^{-\frac{7000}{\theta}} \frac{1}{\theta} \int_0^{7000} dx_2 \right.$$

$$= 1 - \left[- e^{-\frac{7000}{\theta}} + 1 - \frac{1}{\theta} e^{-\frac{7000}{\theta}} x_2 \Big|_0^{7000} \right]$$

$$= 1 - \left[1 - e^{-\frac{7000}{\theta}} - \frac{1}{\theta} e^{-\frac{7000}{\theta}} (7000) \right]$$

$$= 1 - 1 + e^{-\frac{7000}{\theta}} + \frac{1}{\theta} (7000) e^{-\frac{7000}{\theta}}$$

$$k(\theta) = e^{-\frac{7000}{\theta}} \left[1 + \frac{7000}{\theta} \right]$$

The size of type I error $= k(\theta_0) = \alpha$

$$= e^{-\frac{7000}{1000}} \left(1 + \frac{7000}{1000} \right)$$

$$= e^{-7} (1 + 7)$$

$$= 8 e^{-7}$$

and $\beta = 1 - k(\theta_1)$

$$= 1 - \left[e^{-\frac{7000}{2000}} \left(1 + \frac{7000}{2000} \right) \right]$$

$$= 1 - \left[e^{-\frac{7}{2}} \left(1 + \frac{7}{2} \right) \right]$$

$$= 1 - 4.5 e^{-3.5}$$

Most powerful Critical Region (MPCR)
or Best C.R. (B.C.R)

$\because\ k(\theta) = 1-\beta =$ power of the test (under H_1)

This means the C.R. which maximize $(1-\beta)$, is called most powerful or best region, for fixed α.

$\min \beta = \max(1-\beta)$

Assume we want to test the null hypothesis

$H_0: \theta = 0.90$

$H_1: \theta = 0.70$, Let $n = 25$

and acceptance region $X \geq 18 = 18, 19, 20, \dots, 25$

$\Rightarrow$ rejection region $=$ C.R. $= 0, 1, 2, \dots, 17$

Let $\alpha = 0.05$ (constant)

Then

$\beta = P(X = 18, 19, \dots, 25 \mid \theta = 0.70)$

نستطيع تغيير المنطقة الحرجة الى ان نجعل β اصغر ما يمكن
بعد تثبيت قيمة $\alpha = 0.05$ في هذه الحالة سوف نحصل
على ما يسمى ب

Most powerful Critical region.

Ex

Assume a r.s. of size n taken from $N(\theta,1)$. Then find the value of k, which make $\bar{X} > k$ a C.R. of size $\alpha = 0.05$ for testing

$H_0: \theta = \theta_0$

$H_1: \theta = \theta_1$ (where $\theta_1 > \theta_0$)

solu:

$Z_{0.05} = 1.6449$

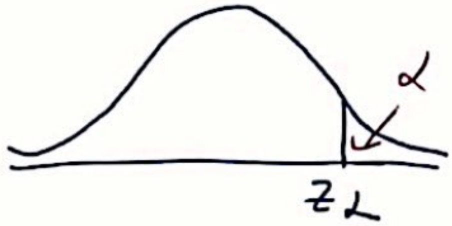
نحتاج الى ايجاد قيمة k التي تجعل المنطقة الحرجة عند 1.6449

$$\because \alpha = p(\text{rejecting } H_0 \mid H_0 \text{ true})$$

$$= p(\bar{X} > k \,;\, H_0)$$

$$= 1 - p(\bar{X} < k \,;\, H_0)$$

$$= 1 - p\left(\frac{\bar{X} - \theta_0}{\frac{1}{\sqrt{n}}} < \frac{k - \theta_0}{\frac{1}{\sqrt{n}}}\right)$$



$$\alpha = 1 - p\left(Z < \frac{k - \theta_0}{\frac{1}{\sqrt{n}}}\right)$$

$$\alpha = 1 - p(Z < Z_\alpha)$$

$$\Rightarrow Z_\alpha = \frac{k - \theta_0}{\frac{1}{\sqrt{n}}}$$

$$\Rightarrow k = \theta_0 + \frac{Z_\alpha}{\sqrt{n}} \quad \Rightarrow k = \theta_0 + \frac{1.6449}{\sqrt{n}}$$

$\therefore$ C.R. is

$$\bar{X} > \theta_0 + \frac{1.6449}{\sqrt{n}}$$

لذلك يمكن معرفة المنطقة الحرجة بعد معرفة قيمة كل من n و θ_0 .

Construction of (MPCR): (best critical region)

تكوين منطقة مرفوضة ذات اكبر قوة اختبار

We use Neman pearson Theorem:-

If C is a C.R. of size α, and k is a constant, such that

$$\frac{L_0}{L_1} \begin{cases} \leq k & \text{inside } C \\ > k & \text{outside } C \end{cases}$$

k is positive number

Then C is a MPCR of size α, for testing

$H_0: \theta = \theta_0$

$H_1: \theta = \theta_1 \quad (\theta_1 > \theta_0)$

where:

$$L_0 = L(\theta_0, x_1, x_2, \dots, x_n) = \prod_{i=1}^{n} f(x_i; \theta_0)$$

$$L_1 = L(\theta_1, x_1, x_2, \dots, x_n) = \prod_{i=1}^{n} f(x_i; \theta_1)$$

Ex:

Suppose that $x_1, x_2, \dots, x_{25}$ is a r.s. from $N(\theta, 1)$. Find a BCR of size 0.05 to test $H_0: \theta = 0$ against $H_1: \theta = 2$

Solu:

$$f(x_i; \theta) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x_i - \theta)^2} \qquad i = 1, 2, \dots, 25$$

$$L(\theta) = (2\pi)^{-\frac{25}{2}} e^{-\frac{1}{2}\sum_{i=1}^{25}(x_i - \theta)^2}$$

$$\frac{L(\theta_0)}{L(\theta_1)} = \frac{(2\pi)^{-\frac{25}{2}} e^{-\frac{1}{2}\sum_{i=1}^{25} x_i^2}}{(2\pi)^{-\frac{25}{2}} e^{-\frac{1}{2}\sum_{i=1}^{25}(x_i-2)^2}} \leq k$$

$$\Rightarrow e^{-\frac{1}{2}\sum_{i=1}^{25} x_i^2 + \frac{1}{2}\sum_{i=1}^{25}(x_i-2)^2} \leq k$$

$$\Rightarrow e^{-\frac{1}{2}\left[\sum_{i=1}^{25} x_i^2 - \left(\sum_{i=1}^{25} x_i^2 - 4\sum_{i=1}^{25} x_i + 25(4)\right)\right]} \leq k$$

$$\Rightarrow e^{-\frac{1}{2}\left(4\sum_{i=1}^{25} x_i - (25)4\right)} \leq k$$

$$\Rightarrow e^{-2\sum_{i=1}^{25} x_i + (25)(2)} \leq k$$

$$\Rightarrow -2\sum_{i=1}^{25} x_i + 50 \leq \ln k$$

$$\Rightarrow -2\sum_{i=1}^{25} x_i \leq \ln k - 50$$

$$\Rightarrow \sum_{i=1}^{25} x_i \geq -\frac{1}{2}\ln k + 25$$

بالقسمة على (25)

$$\Rightarrow \bar{X} \geq -\frac{1}{50}\ln k + 1$$

$\therefore$ The C.R. is:-

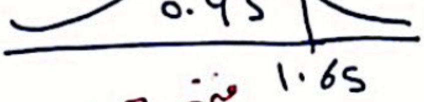
$$C = \{x_1, x_2, \dots, x_{25} ; \bar{X} \geq k_0\}$$

But $\alpha = P(\bar{X} \geq k_0 ; H_0)$, where $\bar{X} \sim N(\theta, \frac{1}{25})$

Therefore

$$0.05 = P\left(Z \geq \frac{k_0 - 0}{\frac{1}{5}}\right)$$

$$0.95 = P(Z \leq 5k_0)$$



قيمة Z عند مستوى معنوية 0.05

$$\therefore 5k_0 = 1.65 \Rightarrow k_0 = 0.33$$

And so, the Best C.R. is :-

$C = \{x_1, x_2, \ldots, x_{25} \; ; \; \bar{x} > 0.33\}$

Ex:

Let $x_1, x_2, \ldots, x_{15}$ denote a r.s. from $N(0, \theta)$. Find a best critical region of size $\alpha = 0.05$ to test $H_0 : \theta = 3$, against the alternative Composite hypothesis $H_1 : \theta > 3$

Solu:

$$f(x) = \frac{1}{\sqrt{2\pi\theta}} e^{-\frac{1}{2\theta} x_i^2}$$

$$L(\theta ; x_1, x_2, \ldots, x_{15}) = \left(\frac{1}{2\pi\theta}\right)^{\frac{15}{2}} e^{-\frac{\sum x_i^2}{2\theta}}$$

$$\frac{L(\theta_0)}{L(\theta_1)} \leq k \implies B.C.R$$

$$\frac{\left(\frac{1}{2\pi(3)}\right)^{\frac{15}{2}} e^{-\frac{\sum x_i^2}{2(3)}}}{\left(\frac{1}{2\pi\theta_1}\right)^{\frac{15}{2}} e^{-\frac{\sum x_i^2}{2\theta_1}}} \leq k \quad , \text{where } \theta_1 > 3$$

$$\frac{\left(\frac{1}{3}\right)^{\frac{15}{2}} e^{-\frac{\sum x_i^2}{2(3)}}}{\left(\frac{1}{\theta_1}\right)^{\frac{15}{2}} e^{-\frac{\sum x_i^2}{2\theta_1}}} \leq k$$

$$\Rightarrow \left(\frac{\theta_1}{3}\right)^{\frac{15}{2}} e^{-\frac{1}{2} \sum x_i^2 \left(\frac{1}{3} - \frac{1}{\theta_1}\right)} \leq k$$

$$\Rightarrow \left(\frac{\theta_1}{3}\right)^{\frac{15}{2}} e^{-\frac{1}{2} \sum_{i=1}^{15} x_i^2 \left(\frac{\theta_1 - 3}{3\theta_1}\right)} \leq k$$

$$\Rightarrow \left(\frac{\theta_1}{3}\right)^{\frac{15}{2}} e^{-\frac{(\theta_1-3)}{6\theta_1}\sum_{i=1}^{15} x_i^2} \leq k$$

$$\Rightarrow e^{\frac{-(\theta_1-3)}{6\theta_1}\sum_{i=1}^{n} x_i^2} \leq k \left(\frac{3}{\theta_1}\right)^{\frac{15}{2}}$$

بأخذ ال ln للطرفين

$$\frac{-(\theta_1-3)}{6\theta_1}\sum_{i=1}^{15} x_i^2 \leq \ln k + \frac{15}{2}\ln\left(\frac{3}{\theta_1}\right)$$

$$\sum_{i=1}^{n} x_i^2 \geq \left[\ln k + \frac{15}{2}\ln\frac{3}{\theta_1}\right]\left(\frac{-6\theta_1}{\theta_1-3}\right) = k_0$$

$$\Rightarrow \sum x_i^2 \geq k_0$$

Therefore, the set $C = \{x_1, x_2, \dots, x_n ; \sum_{i=1}^{15} x_i^2 \geq k_0\}$ is the best critical region for this test

But $\alpha = p(\text{rejecting } H_0 \mid H_0 \text{ true})$

$$= p\{\sum_{i=1}^{15} x_i^2 \geq k_0 ; H_0 : \theta = 3\}$$

$$\Rightarrow 0.05 = p\left(\sum_{i=1}^{15} x_i^2 \geq k_0 ; H_0\right)$$

هدفنا هو ايجاد قيمة k_0 حتى نجد منها قيمة K ولهذا ليه

recalling that $\frac{\sum_{i=1}^{15} x_i^2}{\theta}$ has a Chi-square dist. with 15 degrees of freedom

Therefore,

$$0.05 = p\left(\frac{\sum x_i^2}{3} \geq \frac{k_0}{3}\right) \quad \text{where } \frac{\sum_{i=1}^{15} x_i^2}{3} \sim \chi^2_{(15)}$$

using tables, $\frac{k_0}{3} = 24.996 \simeq 25$

$$\Rightarrow k_0 = 75$$

Hence, the BCR is , $C=\{x_1,x_2,\dots,x_{15} ; \sum_{i=1}^{15} x_i^2 \geq 75\}$ for testing the simple hypothesis $H_0:\theta=3$ against the simple hypothesis $H_1:\theta=\theta_1$.

Accordingly $C=\{x_1,x_2,\dots,x_{15} ; \sum_{i=1}^{15} x_i^2 \geq 75\}$ is a uniformly most powerful critical region of size 0.05 for testing $H_0:\theta=3$ against $H_1:\theta>3$.

H.W.

Let $x_1,x_2,\dots,x_n$ be a r.s. of an exponential r.v. To test $H_0:\lambda=\lambda_0$ against $H_1:\lambda\neq\lambda_0$

Find B.C.R. of size α

Solu:

$$f(x;\lambda)=\frac{1}{\lambda}e^{-\frac{x}{\lambda}} \qquad x>0 \quad ,\lambda>0$$

$$L(\lambda)=\left(\frac{1}{\lambda}\right)^n e^{-\frac{\sum x_i}{\lambda}}$$

Let λ_1 be a number not equal to λ_0, then

$$\frac{L(\lambda_0)}{L(\lambda_1)}=\frac{\left(\frac{1}{\lambda_0}\right)^n e^{-\frac{\sum x_i}{\lambda_0}}}{\left(\frac{1}{\lambda_1}\right)^n e^{-\frac{\sum x_i}{\lambda_1}}} \leq k$$

$$\Rightarrow \left(\frac{\lambda_1}{\lambda_0}\right)^n e^{\sum x_i\left(\frac{1}{\lambda_1}-\frac{1}{\lambda_0}\right)} \leq k$$

$$\Rightarrow e^{\sum x_i\left(\frac{\lambda_0-\lambda_1}{\lambda_1\lambda_0}\right)} \leq k\left(\frac{\lambda_0}{\lambda_1}\right)^n$$ بأخذ الـ ln للطرفين

$$\sum x_i\left(\frac{\lambda_0-\lambda_1}{\lambda_1\lambda_0}\right) \leq \ln k + n\ln\left(\frac{\lambda_0}{\lambda_1}\right)$$

$$\frac{\sum x_i}{n}\left(\frac{\lambda_0-\lambda_1}{\lambda_1\lambda_0}\right) \leq \frac{\ln k}{n} + \ln\left(\frac{\lambda_0}{\lambda_1}\right)$$ بالقسمة على n

$$\bar{X} \leq \frac{\lambda_1\lambda_0\left(\frac{\ln k}{n} + \ln\left(\frac{\lambda_0}{\lambda_1}\right)\right)}{\lambda_0-\lambda_1} = k_0 \quad \text{if } \lambda_1 > \lambda_0 \quad \text{--- (1)}$$

$$> \quad = k_0 \quad \text{if } \lambda_1 < \lambda_0 \quad \text{--- (2)}$$

التعبير الأول يمثل BCR لاختبار الفرضية

$H_0: \lambda = \lambda_0$

$H_1: \lambda = \lambda_1$

if $\lambda_1 > \lambda_0$

والتعبير الثاني يمثل BCR لاختبار الفرضية

$H_0: \lambda = \lambda_0$

$H_1: \lambda = \lambda_1$

if $\lambda_1 < \lambda_0$

نلاحظ أن الفرضية البديلة يجب أن تكون مركبة معين

اما $H_1: \lambda > \lambda_0$ or $H_1: \lambda < \lambda_0$

وفي هذه الحالة لا يوجد BCR وحيدة للمسألة.