Estimation Theory نظرية التقدير

Estimation Theory consists of :-

1) <u>Point Estimate</u> : Yield تعطي one value for parameter, which is our best guess تخمين.

2) <u>Interval Estimate</u> : Yield a range $(\underline{\theta}, \bar{\theta})$, which we think includes the true but unknown value of a parameter.

Methods of point Estimate :-

1- Moments method. طريقة العزوم

2- Least squares method. طريقة المربعات الصغرى

3- Maximum Likelihood method طريقة الامكان الاعظم

4- Bayesian Method. طريقة بيز

* Moments Method:

Let $\{X_i\}, i = 1, 2, \ldots, n$ be a r.s. drawn from a population which has a p.d.f. $f(x;\theta)$ where the parameter $\theta = \{\theta_1, \theta_2, \ldots, \theta_k\}$ is unknown. Our object is to estimate the parameter θ. The Concept of the method of moments depends on the idea of estimating the population moments by the sample moments. Therefore, depending on the number of parameters, we form a system of equation, that is:-

$$\hat{m}_j = m_j(\theta_1, \dots, \theta_k) \qquad j = 1, 2, \dots, k$$

عزم العينة من الرتبة j

Where $\hat{m}_j = \frac{1}{n} \sum_{i=1}^{n} X_i^j$ be the jth sample moment,

عزم المجتمع من الرتبة j

and $m_j = E X_i^j$ be the jth population moment.

The solution of this system of equations will be the estimation of the parameter $\theta = \{\theta_1, \dots, \theta_k\}$

* **Example:**

Let $\{X_i\}$, $i = 1, 2, \dots, n$ be a r.s. from the exponential density function as follows:

$$f(x;\theta) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}} & x > 0 \\ 0 & \text{o.w.} \end{cases}$$

use the method of moments to estimate θ.

solu:

The method of moments equation is:-

$$\hat{m}_1 = m_1(\theta)$$

$$\frac{1}{n} \sum_{i=1}^{n} x_i = E(x) \implies \bar{X} = \theta$$

Hence $\bar{X}$ is the estimator of θ.

* **Example:**

Let $X_1, X_2, \dots, X_n$ be a r.s. from Bernoulli dist.

$$f(x;\theta) = \begin{cases} \theta^x (1-\theta)^{1-x} & x = 0, 1 \\ 0 & \text{o.w.} \end{cases}$$

use the method of moments to estimate θ.

Solu:

$\hat{m}_1 = m_1(\theta)$

$\frac{1}{n}\sum_{i=1}^{n} x_i = EX$

$\bar{X} = \theta$

$\therefore$ $\bar{X}$ is an estimator for the parameter θ using moments method.

*** Example:**

Let $x_1, x_2, \dots, x_n$ be a r.s. from the normal population with mean μ and variance σ^2. Estimate the parameters μ and σ^2 by the method of moments.

Solu:

There are two parameters μ and σ^2, therefore, we will have a system of two equations as follows:-

$\hat{m}_j = m_j(\mu, \sigma^2) \qquad j = 1, 2$

$\hat{m}_1 = m_1(\mu, \sigma^2)$ ---- (1)

$\frac{1}{n}\sum_{i=1}^{n} x_i = EX$

$\bar{X} = EX = \mu \implies \bar{X} = \hat{\mu}$ ($\therefore \bar{X}$ is the estimator for μ).

$\hat{m}_2 = m_2(\mu, \sigma^2)$ ---- (2)

$\frac{1}{n}\sum_{i=1}^{n} x_i^2 = EX^2$

$= var(X) + (EX)^2$

$\Rightarrow \frac{1}{n}\sum_{i=1}^{n} x_i^2 = \sigma^2 + \mu^2$

وبالتعويض عن قيمة μ^2 بالقيمة التقديرية التي حصلنا عليها من المعادلة ① يكون :-

$$\frac{1}{n}\sum_{i=1}^{n} x_i^2 = \sigma^2 + \bar{X}^2$$

ولايجاد القيمة التقديرية للمعلمة σ^2 سوف نجري التبسيطات التالية ونجعلها في طرف واحد

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \bar{X}^2$$

$$= \frac{1}{n}\left[\sum_{i=1}^{n} x_i^2 - n\bar{X}^2\right]$$

$$= \frac{1}{n}\left[\sum_{i=1}^{n} x_i^2 - 2n\bar{X}^2 + n\bar{X}^2\right]$$ نجعل المقدار مربع كامل

$$= \frac{1}{n}\left[\sum_{i=1}^{n} x_i^2 - 2n\bar{X}\frac{\sum x_i}{n} + n\bar{X}^2\right]$$

$$= \frac{1}{n}\left[\sum_{i=1}^{n} x_i^2 - 2\bar{X}\sum_{i=1}^{n} x_i + n\bar{X}^2\right]$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left[x_i^2 - 2\bar{X}x_i + \bar{X}^2\right]$$

$$\Rightarrow \hat{\sigma^2} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{X})^2$$

$\therefore \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{X})^2$ is the estimator for the parameter σ^2 using moments method.