$$Res[f, -2i+\sqrt{3}i] = \lim_{z\to -2i+\sqrt{3}i}(z+2i-\sqrt{3}i)\ \frac{2}{(z+2-\sqrt{3}i)(z+2i+\sqrt{3}i)}$$

$$= \frac{2}{-2i+\sqrt{3}i+2i+\sqrt{3}i} = \frac{1}{\sqrt{3}i}$$

$$\therefore \int_c = 2\pi i\left[\frac{1}{\sqrt{3}i}\right] = \frac{2\pi}{\sqrt{3}}$$

Example

Convert this integral to contour integral and evaluate it .

$$\int_0^{2\pi} \frac{1}{5-4\sin\theta}d\theta$$

H.W

Example

Convert this integral to contour integral and evaluate it .

$$\int_0^{2\pi} \frac{1}{1+\sin^2\theta}\,d\theta$$

Solution

$$\int_0^{2\pi} \frac{1}{1+\sin^2\theta}\,d\theta = \int_c \frac{1}{1+\frac{z^2-z^{-2}}{2i}}\,\frac{dz}{iz}$$

$$\int_c \frac{1}{iz+\frac{z^3-z^{-1}}{2}}\,dz = \int_c \frac{2}{2iz+z^3-z^{-1}}\,dz = \int_c \frac{2z}{z^4+2iz^2-1}\,dz = \int_c \frac{2z}{(z^2+i)^2}\,dz$$

$$\text{Sing. P.}\begin{cases} \to \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i \in C & , \ m=2 \\ \to -\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i \in C & , \ m=2 \end{cases}$$

$$\emptyset(z) = \lim_{z\to\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i}\left(z-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i\right)^2 \frac{1}{\left(z-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i\right)^2\left(z+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)^2} = \frac{1}{\left(\sqrt{2}+\sqrt{2}i\right)^2} \neq 0$$

$$Res\left[f,\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right] = \frac{1}{1!}\lim_{z\to\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i} \frac{d}{dz}\emptyset(z)$$

$$Res\left[f,\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right] = \lim_{z\to\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i} \frac{d}{dz}\frac{1}{\left(z+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)^2} = \lim_{z\to\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i} \frac{-2}{\left(z+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)^3}$$

$$Res\left[f,\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right] = \frac{-2}{\left(\sqrt{2}+\sqrt{2}i\right)^3}$$

$$\emptyset(z) = \lim_{z\to-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i}\left(z+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)^2 \frac{1}{\left(z-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i\right)^2\left(z+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)^2} = \frac{1}{\left(\sqrt{2}+\sqrt{2}i\right)^2} \neq 0$$

$$Res\left[f,\frac{-1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i\right] = \frac{1}{1!}\lim_{z\to\frac{-1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i} \frac{d}{dz}\emptyset(z)$$

$$Res\left[f,\frac{-1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i\right] = \lim_{z\to\frac{-1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i} \frac{d}{dz}\frac{1}{\left(z-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i\right)^2} = \lim_{z\to\frac{-1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i} \frac{-2}{\left(z-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i\right)^3}$$

$$Res\left[f, \frac{-1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right] = \frac{2}{\left(\sqrt{2}+\sqrt{2}i\right)^3}$$

$$\therefore \int_c = 2\pi i\left[\frac{-2}{\left(\sqrt{2}+\sqrt{2}i\right)^3} + \frac{2}{\left(\sqrt{2}+\sqrt{2}i\right)^3}\right] = 0$$

Example

Convert this integral to contour integral and evaluate it .

$$\int_0^{2\pi} \frac{d\theta}{\sqrt{2} - \cos\theta}$$

H.W

Example

Convert this integral to contour integral and evaluate it .

$$\int_0^{2\pi} \frac{d\theta}{5 + 3\cos\theta}$$

H.W