$$+\frac{2}{5}e^{2x}\left[\frac{e^{-2x}(-2\sin x-\cos x)}{5}+\frac{1}{5}\right]$$

$$\therefore y_p(x)=\frac{3\sin x}{25}+\frac{4\cos x}{25}-\frac{e^{x}}{4}+\frac{e^{-3x}}{100}+\frac{2}{25}e^{2x}$$

باهمال الحدود التي تظهر في الحل المتمم نجد

$$y_p(x)=\frac{3}{25}\sin x+\frac{4}{25}\cos x$$

$$y(x)=y_c(x)+y_p(x)$$

$$y(x)=c_1e^{x}+c_2e^{-3x}+c_3e^{2x}+\frac{3}{25}\sin x+\frac{4}{25}\cos x$$

Exercise By the method of variation of parameters to solve the following D.E.

1. $y''+4y'+4y=\dfrac{e^{-2x}}{x^2}$, $x\neq 0$, $x_0=1$
2. $y''+2y'+y=x^2.e^{-x}$
3. $y''+y=\tan x$
4. $y'''+y''=e^{x}$
5. $y'''-2y''+y'-2y=12\sin 2x-4x$
6. $y'''-y''+y'-y=x^3+5+2e^{x}-\cos x$
7. $y'''+4y'+5y=10$

Linear Differential systems الأنظمة التفاضلية الخطية

In general, Linear differential system of first order is the system of the for:

$$\frac{dx_1}{dt} = a_{11}(t)\,x_1 + a_{12}(t)\,x_2 + \cdots + a_{1n}(t)\,x_n + b_1(t)$$

$$\frac{dx_2}{dt} = a_{21}(t)\,x_1 + a_{22}(t)\,x_2 + \cdots + a_{2n}(t)\,x_n + b_2(t)$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$\frac{dx_n}{dt} = a_{n1}(t)\,x_1 + a_{n2}(t)\,x_2 + \cdots + a_{nn}(t)\,x_n + b_n(t)$$

اختصار

or more compactly written

$$\frac{dx_i}{dt} = \sum_{j=1}^{n} a_{ij}(t)\,x_j + b_i(t) \quad , \quad i = 1, \ldots, n$$

If $b_i(t) = 0$, then D. system is homogenous

If $b_i(t) \neq 0$, then D. system is non-homogenous

If a_{ij} , $i,j = 1, \ldots, n$ constant, then the D. system is called
ذات المعاملات الثابتة
D. system with constant coefficients.

In vector matrix notation, the D. system become:

$$\frac{d\underline{x}}{dt} = A(t)\underline{x} + b(t), \text{ where } \underline{x} = (x_1, x_2, \ldots, x_n)$$

$$b(t) = (b_1(t), b_2(t), \ldots, b_n(t)) \quad , \quad A(t) = \begin{bmatrix} a_{11}(t) & a_{12}(t) & \cdots & a_{1n}(t) \\ a_{21}(t) & a_{22}(t) & \cdots & a_{2n}(t) \\ \vdots & \vdots & & \vdots \\ a_{n1}(t) & \cdots & \cdots & a_{nn}(t) \end{bmatrix}$$

Definition (7): The name of Cauchy problem for system (I.v.p) is gives to the problem of finding the solution $x_1 = x_1(t), x_2 = x_2(t), \dots, x_n = x_n(t)$ of this system satisfying the initial conditions $x_1(t_0) = x_1^0, x_2(t_0) = x_2^0, \dots x_n(t_0) = x_n^0$, where $t_0, x_1^0, x_2^0, \dots, x_n^0$ being the given number.

Homogenous Differential system الانظمة التفاضلية المتجانسة

Given D. system $Y' = AY$ where A is the matrix $n \times n$ and Y non zero vector such that

$$AY = rY \quad ---- (1)$$

قيمة مميزة
r called eigen value for A
متجه مميز
Y called eigen vector for A

we can rewrite (1) as following:

$$\begin{aligned}
a_{11}y_1 + a_{12}y_2 + \dots + a_{1n}y_n &= ry_1 \\
a_{21}y_1 + a_{22}y_2 + \dots + a_{2n}y_n &= ry_2 \\
\vdots \qquad \vdots \qquad & \vdots \qquad \vdots \\
a_{n1}y_1 + a_{n2}y_2 + \dots + a_{nn}y_n &= ry_n
\end{aligned}$$

or

$$\left.\begin{aligned}
(a_{11} - r)y_1 + a_{12}y_2 + \dots + a_{1n}y_n &= 0 \\
a_{21}y_1 + (a_{22} - r)y_2 + \dots + a_{2n}y_n &= 0 \\
&\vdots \\
a_{n1}y_1 + a_{n2}y_2 + \dots + (a_{nn} - r)y_n &= 0
\end{aligned}\right\} \dots (2)$$

we note $y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$ are non zero solution of (2) iff

$$\begin{vmatrix} a_{11}-r & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22}-r & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn}-r \end{vmatrix} = 0 \quad \cdots (3)$$

the eq.(3) Called characteristic eq. of A, and we can write it of the form:

$$P_n(r) = |A - r\underset{\text{مصفوفة الوحدة}}{E}| = 0 \quad \cdots (4)$$

where E is Identity matrix. eq.(4) has n of roots $r_1, r_2, \ldots, r_n$ which is eigen value of matrix A. the eigen Vector V_i we can evalute from the eq.

$$(A - r_i E) V_i = 0 \quad , \text{ where}$$

$$V_i = \begin{bmatrix} V_{i1} \\ V_{i2} \\ \vdots \\ V_{in} \end{bmatrix} , \quad 0 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} , \quad E = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

Example(21): Find eigen values and eigen vectors for the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$$

Solution: the characteristic eq. for A is

$$P_2(r) = \begin{vmatrix} 1-r & 2 \\ 3 & 2-r \end{vmatrix} = r^2 - 3r - 4 = 0$$

$r_1 = -1$, $r_2 = 4$ eigen values

To find eigen vector for $r_1 = -1$, substituting $r = -1$ in eq. (4), then:

$$(A - r_1 E) v_1 = \begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \left.\begin{array}{l} 2v_{11} + 2v_{12} = 0 \\ 3v_{11} + 3v_{12} = 0 \end{array}\right\} \Rightarrow v_{12} = -v_{11}$$

هذه المعادلة لها عدد غير منتهٍ من الحلول. لذلك يمكن ان نختار قيمة لاحدهما ومنها نجد القيمة الاخرى، فاذا افترضنا $v_{11} = 1$ فان $v_{12} = -1$ وعليه فان احد المتجهات الذاتية المقابلة لـ $r_1 = -1$ هو

$$v_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

وبنفس الطريقة يمكن حساب متجه خاص v_2 المقابل لـ $r_2 = 4$

substituting $r_2 = 4$ in eq. (4) then:

$$(A - r_2 E) v_2 = \begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{array}{l} -3v_{21} + 2v_{22} = 0 \\ 3v_{21} - 2v_{22} = 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 3v_{21} = 2v_{22} \\ 3v_{21} = 2v_{22} \end{array}\right\} \Rightarrow v_{22} = \frac{3}{2} v_{21}$$

$$v_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

نختار $v_{21} = 2$ وعليه $v_{22} = 3$ وعليه المتجه الخاص