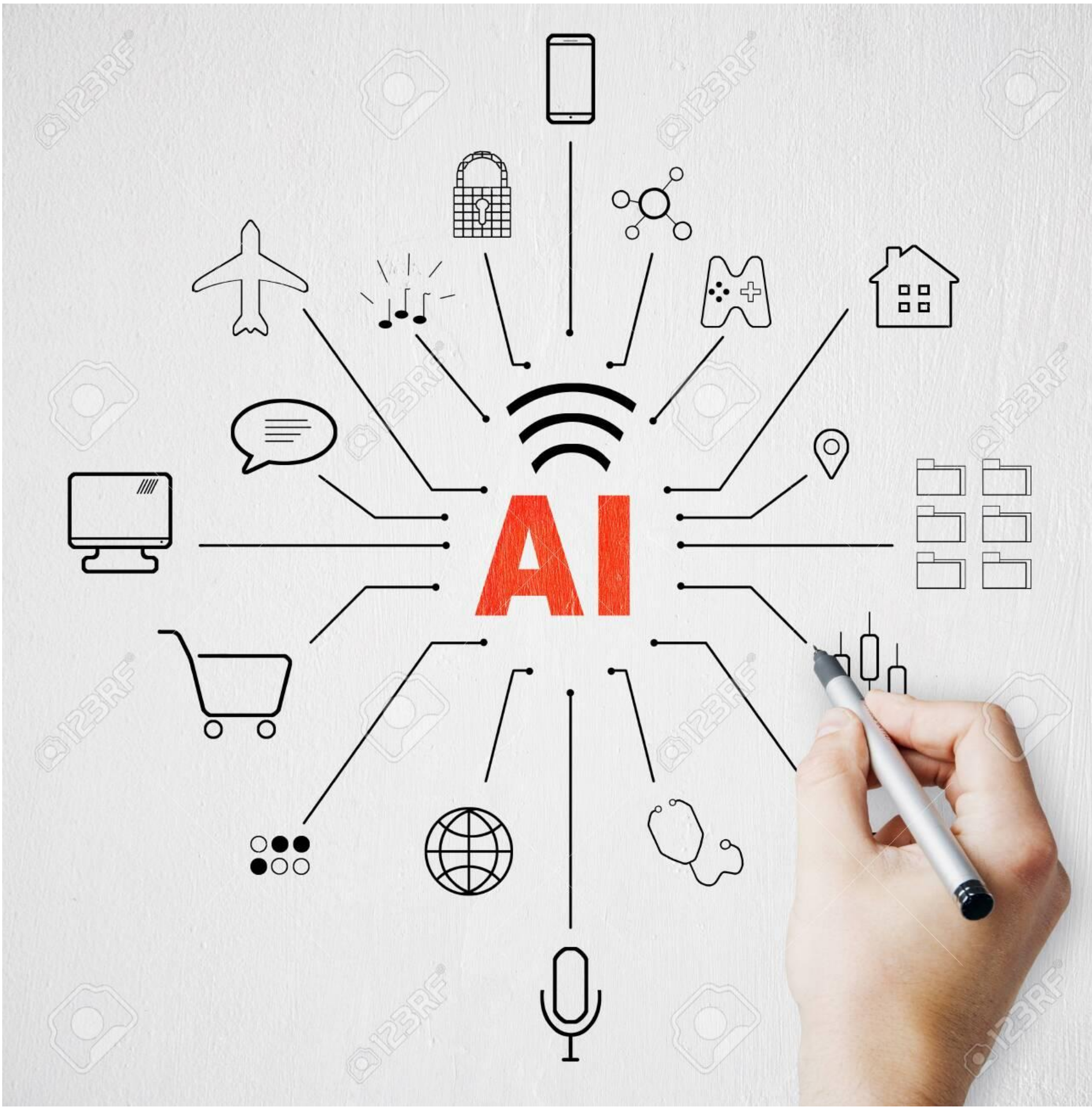
Dr. Luma Al-Saffar
Dept. of Artificial Intelligence



المنهاج الاسبوعي النظري

Week	Material Covered (Weekly Syllabus)
Week 1	Introduction, Propositional Logic and Predicate Logic
Week 2	First-Order- Predicate
Week 3	Production rules and Problem Characteristics
Week 4	Search Strategies (Problem state space and search space)
Week 5	Search Strategies (Problem Solving)
Week 6	Search Strategies (Blind Search)
Week 7	Search Strategies (Search Space Problems)
Week 8	Search Strategies (Monkey & Banana)
Week 9	Search Strategies (8puzzle, 2-jug)
Week 10	Forward & Backward
Week 11	Forward & Backward
Week 12	Matching
Week 13	Prolog (Terms)
Week 14	Prolog (List)
Week 15	Prolog (String)
Week 16	Final Exam

References:

Carter, M. (2007). *Minds and computers: An introduction to the philosophy of artificial intelligence*. Edinburgh University Press.

Lecture One

Propositional logic in Artificial intelligence

منطق العبارات في الذكاء الاصطناعي

Introduction:

Propositional logic (PL), also known as **sentential logic**, is a branch of logic that deals with propositions that can be true or false. In **Artificial Intelligence (AI)**, propositional logic forms a foundational framework for representing knowledge and reasoning about it. It allows us to express facts about the world and derive conclusions from them through formal reasoning. Understanding propositional logic is critical for building automated reasoning agents, expert systems, and even more advanced AI models. Propositional logic is the simplest form of logic where propositions make all the statements. A proposition is a declarative statement which is either true or false. It is a technique of knowledge representation in logical and mathematical form. Formal reasoning is a type of logical analysis that helps to construct valid arguments. It is an important tool used by scientists to evaluate the truthfulness of theories and hypotheses. There are two main types of formal reasoning: deductive and inductive.

منطق العبارات (PL) ، والمعروف أيضًا باسم المنطق الجملي، هو فرع من المنطق يتعامل مع العبارات التي يمكن أن تكون صحيحة أو خاطئة. في الذكاء الاصطناعي (AI) ، يشكل منطق العبارات إطارًا أساسيًا لتمثيل المعرفة والاستدلال عليها. إنه يسمح لنا بالتعبير عن الحقائق حول العالم واستخلاص النتائج منها من خلال

الاستدلال الحقيقي. يعد فهم منطق العبارات أمرًا بالغ الأهمية لبناء وكالات الاستدلال الآلي والانظمة الخبيرة وحتى نماذج الذكاء الاصطناعي الأكثر تقدمًا. منطق العبارات هو أبسط أشكال المنطق الذي به تعمل العبارات جميع الجمل. العبارات هي جمل تصريحية اما ان تكون صحيحة او خاطئة. إنها تقنية لتمثيل المعرفة في شكل منطقي ورياضي. الاستدلال الحقيقي هو نوع من التحليل المنطقي الذي يساعد في بناء حجج صحيحة.

إنها أداة مهمة يستخدمها العلماء لتقييم صدق النظريات والفرضيات. هناك نوعان رئيسيان من الاستدلال الحقيقي: الاستنتاجي والاستقرائي.

Propositional logic provides the AI programs with a well-defined language for describing and reasoning about qualitative aspects of a system. Using their words, phrases, and sentences, we can represent and reason about properties and relationships in the world.

يوفر منطق العبارات لبرامج الذكاء الاصطناعي لغة محددة جيدًا لوصف الجوانب النوعية للنظام والاستدلال عليها. باستخدام كلماتهم وعباراتهم وجملهم، يمكننا تمثيل الخصائص والعلاقات في العالم والاستدلال عليها.

We are not connected with the <u>truth</u> of statements, but rather with their <u>validity</u>.

نحن لا نرتبط بحقيقة البيانات، بل بصلاحيتها.

All lemons are blue. <u>Not true</u> but it is <u>valid</u>

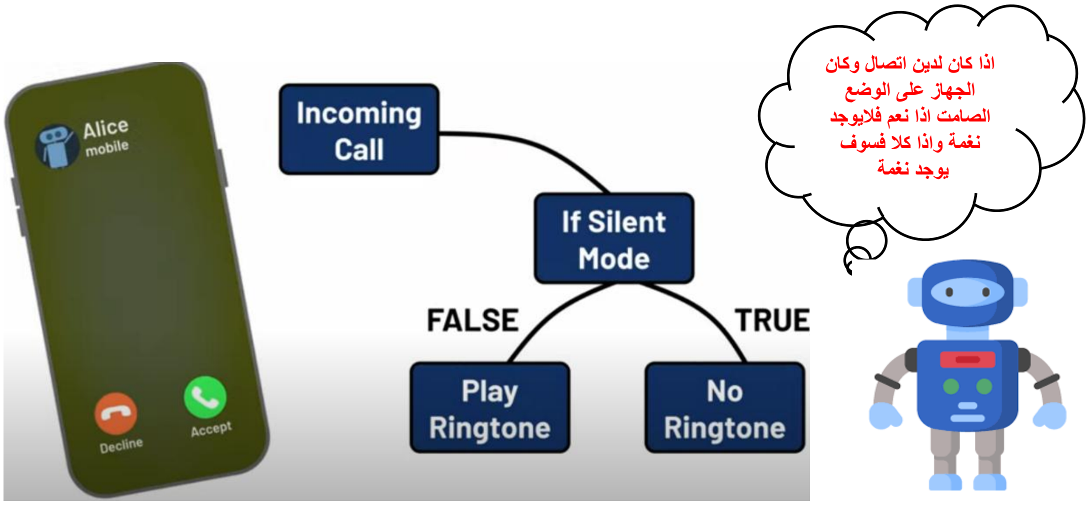
Mary is a Lemon. <u>Not true</u> but it is <u>valid</u>

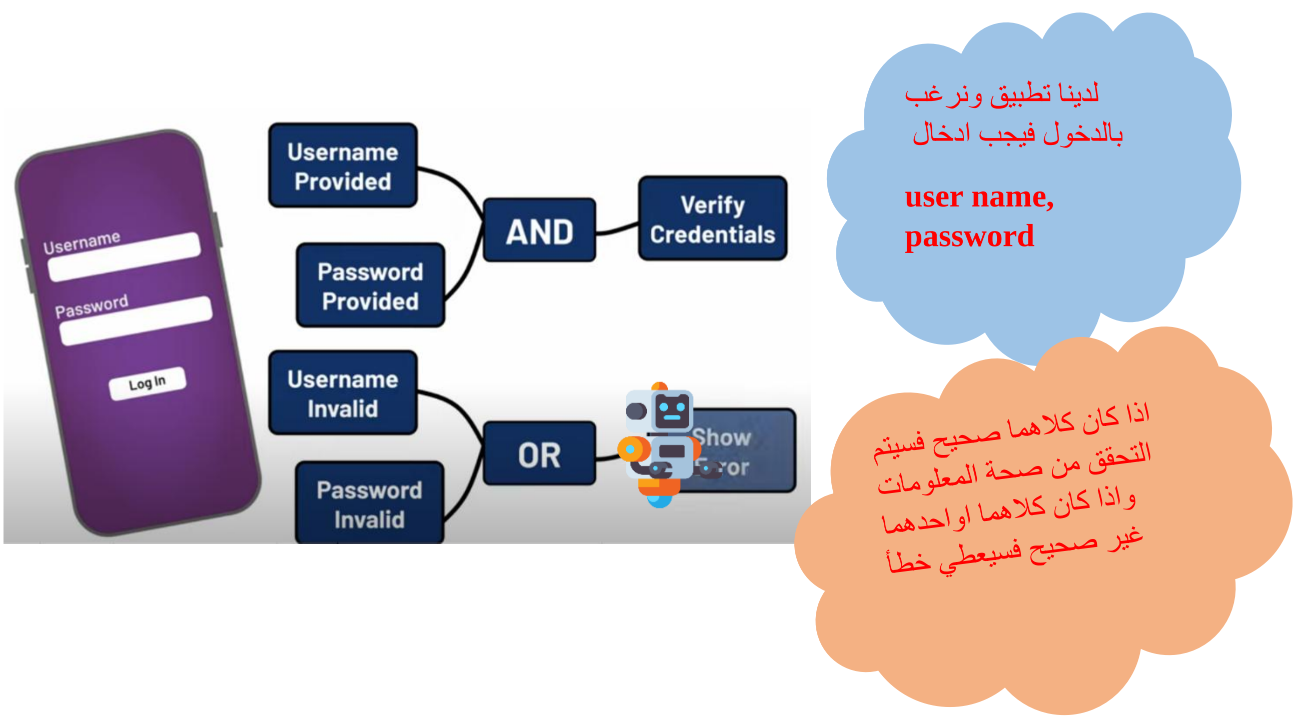
Therefore, Mary is blue. استنتجنا بانه ماري زرقاء جملة غير صحيحة ولكنها مثبتة وليست منفية فتعتبر حقيقة

Propositional logic works with statements, or propositions, that are either **True (T)** or **False (F)**. These propositions are combined using logical operators to form more complex expressions.

Dr. Luma Al-Saffar
Dept. of Artificial Intelligence

يعمل منطق العبارات مع العبارات التي تكون إما صحيحة (T) أو خاطئة (F). يتم دمج هذه العبارات باستخدام عوامل منطقية لتكوين تعبيرات أكثر تعقيدًا.





Examples of Propositions

	Statement	
1.	It is Sunday.	True proposition
2.	The Sun rises from West (False proposition).	False proposition
3.	3+3= 7.	False proposition
4.	5 is a prime number.	True proposition
5.	Baghdad is the capital of Iraq.	True proposition
6.	There is no red banana.	True proposition
7.	The water is wet.	True proposition
8.	P.	True proposition
9.	Q.	True proposition

Examples of not Propositions

1. What time is it?
2. X is greater than 2.
3. Is the girl happy?
4. Y-6.
5. Z=6.
6. 2 + x =10.
7. It is good.
8. What a beautiful morning!
9. Get up and do your exercise.
10. Are you busy?
11. He is the tallest person in this class.
12. The number x is an integer.

Propositional symbols

Propositional symbols: P, Q, R, S, …. .

Truth symbols: - True or False.

Connectives symbols: $\wedge$, $\vee$, ~ , $\leftrightarrow$, $\rightarrow$ and…. .

If the sky is cloudy then it will rain

Cloudy $\rightarrow$ rain, cloudy $\leftrightarrow$ rain

Proposition: is a statement or its negation or a group of statements and/or their negations, connected by AND, OR, and IF-Then operators.

- It is hot, the sky is cloudy. It is hot, and the sky is cloudy.
- It is hot $\rightarrow$ the sky is cloudy.

Tautologies in propositional logic

Ex: P1= the sky is cloudy.

P2 = it will rain.

P3 = if the sky is cloudy then it will rain.

P3 = P1 $\rightarrow$ P2

P1 & P2 represent the premise and conclusion respectively for the if-then clause.

يمثل P1 و P2 المقدمة والاستنتاج لجملة if-then على التوالي.

Sentences in the propositional logic

1. Every propositional symbol and truth symbol is a sentence.
2. The negation of a sentence is a sentence.
3. The conjunction, or (and), of two sentences, is a sentence. Ex. $P \wedge Q$; P & Q called conjuncts.
4. Disjunction, or (or), of two sentences, is a sentence. Ex. $P \vee Q$; P & Q called disjuncts.
5. The implication of one sentence from another is a sentence.
6. $P \rightarrow Q$, P is the premise or antecedent and Q is the conclusion or consequent.
7. The equivalence of two sentences is a sentence.
8. The bracts ( ) and [ ] are used to group symbols into sub-expressions.

Dr. Luma Al-Saffar
Dept. of Artificial Intelligence

Syntax of propositional logic:

The syntax of propositional logic defines the allowable sentences for the knowledge representation. There are two types of Propositions:

1. **Atomic Propositions**
2. **Compound propositions**

- **Atomic Propositions:** Atomic propositions are the simple propositions. It consists of a single proposition symbol. These are the sentences which must be either true or false.

Example:

1. a) 2+2 is 4, it is an atomic proposition as it is a **true** fact.
2. b) "The Sun is cold" is also a proposition as it is a **false** fact.

- **Compound proposition:** Compound propositions are constructed by combining simpler or atomic propositions, using parenthesis and logical connectives.

Example:

1. a) "It is raining today, and street is wet."
2. b) "Ankit is a doctor, and his clinic is in Mumbai."

Translating between English and propositional

Sentence $\longrightarrow$ *Atomic Sentence* | *Complex Sentence*

Atomic Sentence $\longrightarrow$ *True* | *False* | *P* | *Q* | *R* …

Complex Sentence $\longrightarrow$ (*Sentence*) | [*Sentence*]

| ¬ *Sentence*

| *Sentence* ∧ *Sentence*

Dr. Luma Al-Saffar
Dept. of Artificial Intelligence

| *Sentence* $\vee$ *Sentence*

| *Sentence* $\Longrightarrow$ *Sentence*

| *Sentence* $\Longleftrightarrow$ *Sentence*

Operator precedence: $\neg, \wedge, \vee, \Longrightarrow, \Longleftrightarrow$

Notes:

1. To replace Atomic or simple sentence directly by appropriate simple
   Ex: it is rain $\longrightarrow$ R

2. Complex sentences which are contain two or more simple sentences joined by AND, OR, IF-then. Divide it to individual sentences as in 1 above.
   Ex: the sky is cloudy, replace it by C. Replace it is rain by R. Then the proposition become $C \wedge R$

3. The sentence contains (NOT), simple work as (1) or (2) and put ~ before it.
   EX: today is not Saturday, replace it by ~ S

Examples1:

1. It is raining and it is Tuesday. $R \wedge T$
2. It is raining in New York. R(N)
3. It is raining in New York, and I'm either getting sick or just very tired. $R(N) \wedge (S(I) \vee (T(I))$.
4. It is not raining in New York. ~ R(N)
5. I'm either not well or just very tired. ~ $W(I) \vee T(I)$
6. If it is raining then I will get wet. $R \Longrightarrow W(I)$

7. Whenever he eats sandwiches that have pickles in them, he ends up either asleep at his desk or singing loud songs.
   S(y) means that y is a sandwiche.
   E(x,y) means that x (the man) eats y (the sandwiche).
   P(y) means that y (the sandwiche) has pickles in it.
   A(X) means that x ends up asleep at his desk.
   Ss(x,z) means that x (the man) sings z (songs).
   L(z) means that z (the songs) are loud.
   $S(y) \land P(y) \land E(x,y) \Rightarrow A(X)) \lor (Ss(x,z) \land L(z))$

Examples2:

1. If it is Saturday, we go fishing. $P \Rightarrow Q$
   It is Saturday. P
   Therefore, we go fishing. Q
2. If it is Saturday, we go fishing. $P \Rightarrow Q$
   We don’t go for fishing. $\sim Q$
   Therefore, it is not Saturday. $\sim P$
3. If we win the game, we will get much money. If we have money we will go on a trip to China. Therefore, if we win the game we will go on a trip to China.
   $P \Rightarrow Q$
   $Q \Rightarrow R$
   $P \Rightarrow R$

Following are some basic facts about propositional logic:

1. Propositional logic is also called Boolean logic as it works on 0 and 1.
2. In propositional logic, we use symbolic variables to represent the logic, and we can use any symbol to represent a proposition, such as A, B, C, P, Q, R, etc.
4. Propositions can be either true or false, but they cannot be both.
5. Propositional logic consists of an object, relations or functions, and logical connectives.
   These connectives are also called logical operators.

6. The propositions and connectives are the basic elements of the propositional logic.
7. Connectives can be said as a logical operator which connects two sentences.
8. A proposition formula that is always true is called tautology, and it is also called a valid sentence.
9. A proposition formula that is always false is called Contradiction.
10. A proposition formula which has both true and false values is called
11. Statements that are questions, commands, or opinions are not propositions such as "Where is Rohini", "How are you", and "What is your name", are not propositions.

Properties of Operators

- **Commutativity:**
  - $P \land Q = Q \land P$, or
  - $P \lor Q = Q \lor P$.
- **Associativity:**
  - $(P \land Q) \land R = P \land (Q \land R)$,
  - $(P \lor Q) \lor R = P \lor (Q \lor R)$
- **Identity element:**
  - $P \land \text{True} = P$,
  - $P \lor \text{True} = \text{True}$.
- **Distributive:**
  - $P \land (Q \lor R) = (P \land Q) \lor (P \land R)$.
  - $P \lor (Q \land R) = (P \lor Q) \land (P \lor R)$.
- **DE Morgan's Law:**
  - $\neg (P \land Q) = (\neg P) \lor (\neg Q)$
  - $\neg (P \lor Q) = (\neg P) \land (\neg Q)$.
- **Double-negation elimination:**
  - $\neg (\neg P) = P$.
- $P \lor P = P$

  $P \land P = P$

Dr. Luma Al-Saffar
Dept. of Artificial Intelligence

$P \longrightarrow Q = \neg P \vee Q$

Logical Connectives:

Logical connectives are used to connect two simpler propositions or representing a sentence logically. We can create compound propositions with the help of logical connectives. There are mainly five connectives, which are given as follows:

1. **Negation:** A sentence such as ¬ P is called negation of P. A literal can be either Positive literal or negative literal.
2. **Conjunction:** A sentence which has **∧** connective such as, **P ∧ Q** is called a conjunction.
   Example: Rohan is intelligent and hardworking. It can be written as,
   P= Rohan is intelligent,
   Q= Rohan is hardworking. → P∧ Q.
3. **Disjunction:** A sentence which has ∨ connective, such as **P ∨ Q**. is called disjunction, where P and Q are the propositions.
   Example: "Ritika is a doctor or Engineer",
   Here P= Ritika is Doctor. Q= Ritika is Doctor, so we can write it as **P ∨ Q**.
4. **Implication:** A sentence such as P → Q, is called an implication. Implications are also known as if-then rules. It can be represented as
   If it is raining, then the street is wet.
   Let P= It is raining, and Q= Street is wet, so it is represented as P → Q
5. **Biconditional:** A sentence such as **P⇔ Q is a Biconditional sentence, example If I am breathing, then I am alive**
   P= I am breathing, Q= I am alive, it can be represented as P ⇔ Q.

Following is the summarized table for Propositional Logic Connectives:

Connective symbols	Word	Technical term	Example
∧	AND	Conjunction	A ∧ B
∨	OR	Disjunction	A ∨ B
→	Implies	Implication	A → B
⇔	If and only if	Biconditional	A⇔ B
¬ or ~	Not	Negation	¬ A or ¬ B

Truth Table:

In propositional logic, we need to know the truth values of propositions in all possible scenarios. We can combine all the possible combination with logical connectives, and the representation of these combinations in a tabular format is called Truth table: A **truth table** lists all possible truth values of propositions and their combinations through operators. It's a fundamental tool for evaluating logical expressions. Following are the truth table for all logical connectives:

For Negation:

P	¬ P
True	**False**
False	**True**

For Conjunction:

P	Q	P∧ Q
True	True	**True**
True	False	**False**
False	True	**False**
False	False	**False**

For disjunction:

P	Q	P V Q.
True	True	**True**
False	True	**True**
True	False	**True**
False	False	**False**

For Implication:

P	Q	P→ Q
True	True	**True**
True	False	**False**
False	True	**True**
False	False	**True**

For Biconditional:

P	Q	P⇔ Q
True	True	**True**
True	False	**False**
False	True	**False**
False	False	**True**

Truth table with three propositions:

We can build a proposition composing three propositions P, Q, and R. This truth table is made-up of 8n Tuples as we have taken three proposition symbols.

P	Q	R	¬R	P∨Q	P∨Q→¬R
True	True	True	False	True	False
True	True	False	True	True	True
True	False	True	False	True	False
True	False	False	True	True	True
False	True	True	False	True	False
False	True	False	True	True	True
False	False	True	False	False	True
False	False	False	True	False	True

Precedence of connectives:

Just like arithmetic operators, there is a precedence order for propositional connectors or logical operators. This order should be followed while evaluating a propositional problem. Following is the list of the precedence order for operators:

Precedence	Operators
First Precedence	Parenthesis
Second Precedence	Negation
Third Precedence	Conjunction(AND)
Fourth Precedence	Disjunction(OR)
Fifth Precedence	Implication
Six Precedence	Biconditional

Let take two propositions A and B, so for logical equivalence, we can write it as A⇔B. In below truth table we can see that column for ¬A∨ B and A→B, are identical hence A is Equivalent to B

A	B	¬A	¬A∨ B	A→B
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Applications in AI

In AI, propositional logic helps to represent **facts** and **rules** about the world, enabling **reasoning** over them. Some of the critical AI applications include:

1. **Knowledge Representation**:
   - Propositional logic provides a formal way to encode facts and rules. For instance:
     - "If the light is off (¬L), then it is dark (D)" → ¬L → D.
2. **Inference in AI Systems**:
   - AI systems use inference rules to deduce new facts from known ones.
   - **Modus Ponens**: If P → Q and P is true, then Q must also be true.
     - Example: If "It is raining" (P) implies "The ground is wet" (Q), and we know it is raining, we can infer the ground is wet.
3. **Automated Theorem Proving**:
   - Propositional logic forms the basis for many **automated theorem proving** systems, which can verify the correctness of propositions using logic rules.
4. **Expert Systems**:
   - Early AI systems, known as **expert systems**, rely heavily on propositional logic to encode domain-specific knowledge and apply rules to provide solutions in fields like medicine or finance.
5. **Boolean Satisfiability Problem (SAT)**:

- SAT solvers, which determine whether there is a set of truth assignments to variables that make a propositional logic formula true, are widely used in AI for tasks like **planning**, **optimization**, and **circuit design**.
- **Example**: Is there a way to assign values to P, Q, and R such that

  $(P \lor \neg Q) \land (\neg P \lor R)$ is true?

Limitations of Propositional logic:

While propositional logic is powerful for simple reasoning tasks, it has limitations:

Lack of expressiveness: Propositional logic can only deal with facts that are either true or false. It cannot express more complex relationships, such as quantifying over objects (this is where **predicate logic** comes in). In propositional logic, we cannot describe statements in terms of their properties or logical relationships. Propositional logic has limited expressive power. It cannot represent relations like **all, some, or none** with propositional logic. Example:

- **All the girls are intelligent.**
- **Some apples are sweet.**

Scalability: For large, real-world problems, propositional logic can become unwieldy due to the sheer number of propositions required to represent different states.

No representation of uncertainty: It does not handle uncertainty. More advanced AI systems use **probabilistic reasoning** (e.g., Bayesian Networks) when uncertainty is involved.

Extensions and Alternatives to Propositional Logic

To overcome its limitations, AI often relies on extensions of propositional logic or completely different approaches:

- **First-Order Logic (Predicate Logic)**: Allows reasoning about objects and their properties, enabling more complex expressions. It includes variables, functions, and quantifiers like ∀ (for all) and ∃ (there exists).
- **Probabilistic Logic**: Incorporates uncertainty into reasoning. This is crucial for AI applications in fields like **natural language processing** and **robotics**.
- **Modal Logic**: Adds modalities like necessity and possibility, helping with reasoning about beliefs, knowledge, and obligations.

Reasoning Techniques in Propositional Logic

AI uses several reasoning techniques based on propositional logic to infer new knowledge from existing facts:

1. **Resolution**:
    - A rule of inference used in **automated theorem proving**. It involves combining two clauses to eliminate a literal and deduce new information.
    - Example: From $(P \lor Q)$ and $(\neg Q \lor R)$, we can deduce $(P \lor R)$.
2. **Forward and Backward Chaining**:
    - **Forward chaining** starts from known facts and applies inference rules to derive conclusions.
    - **Backward chaining** works backward from a goal to check if existing facts support it.
3. **Model Checking**:
    - Given a logic formula and a model (a particular assignment of truth values), model checking verifies whether the formula holds true under that model. It is crucial for verifying software and hardware correctness.

Conclusion

Propositional logic is an essential building block in the realm of AI, providing a solid foundation for knowledge representation, automated reasoning, and problem-solving. While it has limitations, understanding propositional logic allows AI practitioners to delve into more advanced topics like predicate logic, probabilistic reasoning, and automated decision-making systems.

In practical AI systems, propositional logic is often just one tool in a more extensive toolkit that also includes machine learning, reasoning under uncertainty, and complex decision-making frameworks. Nonetheless, mastering propositional logic is crucial to understanding the fundamental principles of AI reasoning systems.

Key Takeaways:

1. **Propositional logic** involves statements that can be true or false and uses logical operators to combine them.
2. It is foundational for **knowledge representation** and **reasoning** in AI.
3. While **limited in expressiveness**, propositional logic powers important AI tools like SAT solvers and expert systems.
4. Extensions like **first-order logic** provide more complex reasoning capabilities for real-world AI applications.