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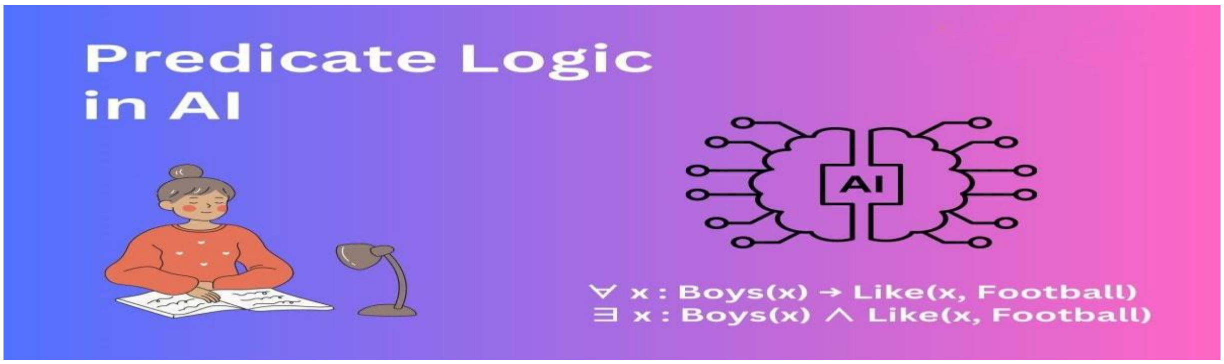
Lecture Two

Predicate Logic in AI (Artificial Intelligence)



المنطق الإسنادي في الذكاء الإصطناعي

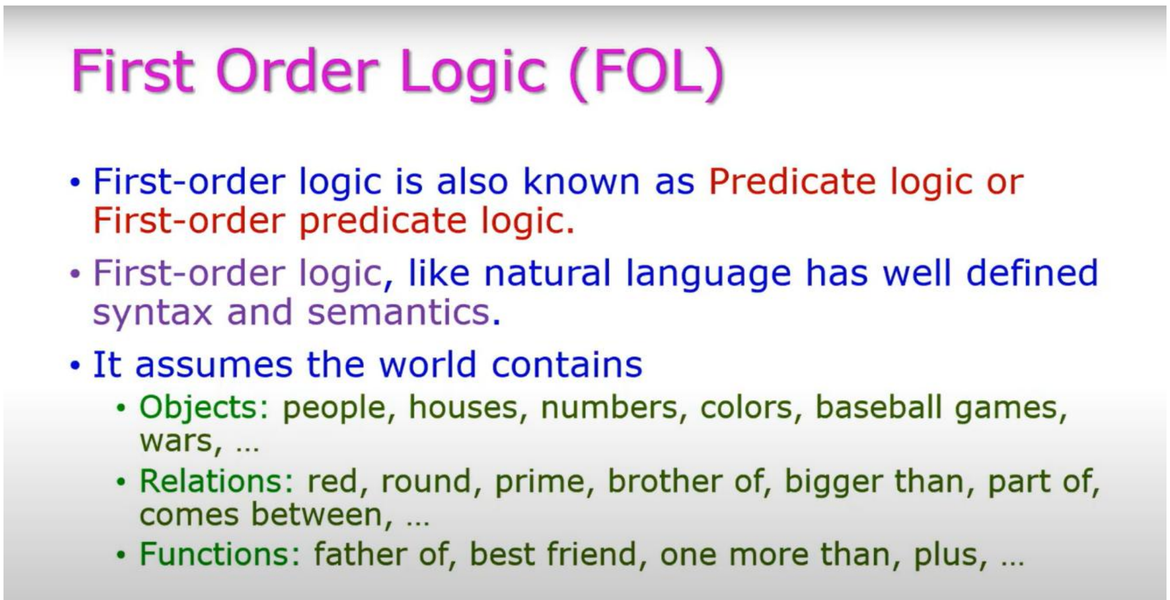
Introduction to Predicate Logic

In the world of Artificial Intelligence, the process of how and what a machine thinks plays a vital role in decision-making. There is a powerful tool that helps the computer to make the best decisions; this tool is known as predicate logic.

Predicate logic, also known as ***first-order logic*** **(FOL),** extends propositional logic by introducing **quantifiers and predicates**, allowing it to express statements about objects and their relationships. It is a foundational tool in mathematics, computer science, and philosophy for reasoning about formal systems. Predicate logic in AI is mainly used as a foundational framework for representation of knowledge and reasoning. Predicate logic in AI even embodies a set of systematic procedures that can prove that specific formulae can or can't be logically derived from others.

في عالم الذكاء الاصطناعي، تلعب عملية كيفية تفكير الآلة وما تفكر فيه دورًا حيويًا في اتخاذ القرار. هناك أداة قوية تساعد الكمبيوتر على اتخاذ أفضل القرارات؛ تُعرف هذه الأداة باسم المنطق الإسنادي.

يمتد المنطق الإسنادي، المعروف أيضًا باسم المنطق من الدرجة الأولى (FOL)، إلى منطق العبارات من خلال إدخال المحددات والمسندات، مما يسمح له بالتعبير عن بيانات حول الأشياء وعلاقاتها. إنها أداة أساسية في الرياضيات وعلوم الكمبيوتر والفلسفة للتفكير في الأنظمة الرسمية. يستخدم المنطق الإسنادي في الذكاء الاصطناعي بشكل أساسي كإطار أساسي لتمثيل المعرفة والمنطق. يجسد المنطق الإسنادي في الذكاء الاصطناعي مجموعة من الإجراءات المنهجية التي يمكن أن تثبت أنه يمكن أو لا يمكن اشتقاق صيغ معينة منطقيًا من صيغ أخرى.



First-Order Logic (FOL)

1. **Definition**:
   - First-order logic is also known as **Predicate Logic** or **First-order Predicate Logic**.
   - It extends **Propositional Logic** by introducing <u>**quantifiers**</u> and <u>**predicates**</u>, which allow us to express more complex statements about objects and their relationships.

2. **Syntax and Semantics**:
    - Like natural language, FOL has a **structured syntax** (rules for forming statements) and **semantics** (meaning of statements).
    - The syntax specifies how to form valid sentences, while semantics defines what these sentences mean.

بناء الجملة والدلالات:مثل اللغة الطبيعية، فإن لغة FOL لها بناء جملة منظم (قواعد تكوين العبارات) ودلالات (معنى العبارات) .يحدد بناء الجملة كيفية تكوين جمل صالحة، بينما تحدد الدلالات معنى هذه الجمل.

3. **Assumptions About the World**: FOL assumes that the world can be represented using three key components:

a. Objects:

- These are the basic entities in the domain of discourse.

الكيانات الأساسية في مجال الخطاب (التواصل المكتوب أو المنطوق أو المناقشة)

- Examples:
    - *People*: Alice, Bob
    - *Houses*: House1, House2
    - *Numbers*: 1, 2, 3
    - *Colors*: red, blue
    - *Events*: baseball games, wars

b. Relations:

- These describe how objects interact or relate to each other.
- Examples:
    - *Red(object)* means an object is red.
    - *Brother_of(X, Y)* denotes X is a brother of Y.
    - *Bigger_than(X, Y)* means X is bigger than Y.

- *Comes_between(A, B, C)* means A is between B and C.

c. Functions:

- Functions map objects to other objects.
- Examples:
  - *Father_of(X)* gives the father of X.
  - *Best_friend(X)* gives the best friend of X.
  - *One_more_than(3)* would return 4.
  - *Plus(2, 3)* would return 5.

Why Use FOL?

- **Expressiveness**: FOL allows us to represent complex statements and reason about objects and their interrelations.

  يسمح لنا FOL بتمثيل البيانات المعقدة والتفكير حول الكائنات وعلاقاتها المتبادلة.

- **Quantifiers**:
  - Universal Quantifier (∀): "For all."
  - Existential Quantifier (∃): "There exists." These help in expressing statements like "Every person has a mother" or "There exists someone who is a doctor."

Properties of FOL

خصائصFOL

1. Ability to Represent Facts

- FOL can express facts about **objects** and **relationships** within a universe of discourse.

يمكن للغة FOL التعبير عن الحقائق المتعلقة بالأشياء والعلاقات في عالم الخطاب.

- Example: "Alice is taller than Bob" can be represented as a fact using a predicate: Taller(Alice, Bob).

2. Representation of Laws and Rules

- FOL is not just about isolated facts; it can also represent **laws** and **rules** derived from the real world.
- Example: "If someone is a parent, then they have a child" could be written as: $\forall X(\text{Parent}(X) \rightarrow \exists Y(\text{ChildOf}(Y,X)))$

3. Utility in Multiple Disciplines

- FOL is widely used in:
  - **Mathematics**: For formal proofs and theorems.
  - **Philosophy**: To analyze and express logical arguments.
  - **Artificial Intelligence (AI)**: For reasoning and building knowledge-based systems.

4. Realistic Representation of Facts

- FOL allows for a more **nuanced** view of facts than simply assigning them as "true" or "false."

يسمح FOL برؤية أكثر دقة للحقائق بدلاً من مجرد تعيينها على أنها "حقيقية" أو "خاطئة."

- It can encode **relationships** and **conditions** that are closer to real-world complexities.

يمكنه ترميز العلاقات والظروف الأقرب إلى تعقيدات العالم الحقيقي.

4. Ontological Commitment

- **Ontology** refers to the study of existence. دراسة علم الوجود
- FOL makes a commitment to the existence of **objects** and their **relationships** in a domain.

يلتزم FOL بوجود الكائنات وعلاقاتها في مجال ما.

- Example: If we state **Loves(Alice, Bob)**, FOL assumes Alice and Bob **exist** and are part of the universe being modeled.

Why These Properties Matter:

- FOL is a powerful tool for modeling real-world systems and making logical inferences.

لماذا تعد هذه الخصائص مهمة:

FOL هي أداة قوية لنمذجة الأنظمة في العالم الحقيقي واستخلاص الاستنتاجات المنطقية.

- Its structured approach ensures clear and consistent representation of complex ideas.

يضمن نهجها المنظم تمثيلًا واضحًا ومتسقًا للأفكار المعقدة

Syntax of FOL: Basic Elements

• Constants	KingJohn, 2, NUS,...
• Predicates	Brother, >,...
• Functions	Sqrt, LeftLegOf, ...
• Variables	x, y, a, b,...
• Connectives	¬, ∧, ∨, ⇒, ⇔
• Equality	=
• Quantifiers	∀, ∃
• Sentences	Atom / complex sentences
• Atom	True / False / AP (atomic proposition)
• Complex sentence	(sentence) / connective sentence / ¬ sentence

Syntax of First-Order Logic (FOL) **and its basic components**

1. Constants

- **Definition**: These represent specific objects or entities in the domain.
- **Examples**:
  - **KingJohn** (a specific person)
  - **2** (a numeric constant)
  - **NUS** (a constant representing an entity like a university)

2. Predicates

- **Definition**: Predicates describe properties of objects or relationships between objects.
- **Examples**:
  - **Brother(x, y)** means "x is the brother of y."
  - > represents a comparison predicate for numbers.

3. Functions

- **Definition**: Functions map objects to other objects in the domain.
- **Examples**:
  - **Sqrt(x)** represents the square root of x.
  - **LeftLegOf(x)** refers to the left leg of x.

4. Variables

- **Definition**: These are placeholders for objects in the domain.
- **Examples**:
  - **x, y, a, b** can represent any object in the domain.

5. Connectives

- **Definition**: Logical operators used to form complex sentences.

- **Examples**:
  - ¬ (NOT)
  - ∧ (AND)
  - ∨ (OR)
  - ⇒ (IMPLIES)
  - ⇔ (IF AND ONLY IF)

6. Equality

- **Definition**: Used to express those two objects are the same.
- **Example**:
  - **x = y** means "x is equal to y."

7. Quantifiers

- **Definition**: Specify the scope of a statement over a domain.

تحديد نطاق الجملة في مجال معين.

- **Examples**:
  - ∀ (Universal Quantifier): "For all..."
  - ∃ (Existential Quantifier): "There exists..."

8. Sentences

- **Definition**: Well-formed expressions in FOL that are either atomic or complex.

تعبيرات جيدة التكوين في FOL والتي تكون إما بسيطة أو معقدة.

- **Types**:
  - **Atom**: Basic statements or propositions (e.g., *P(x)*).

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- **Complex Sentence**: Formed using connectives and quantifiers **(**e.g., **¬P(x)** ∨ **Q(y)).**

9. Atom

- **Definition**: The simplest form of a logical statement.
- **Examples**:
    - **True, False**, or **atomic propositions** (e.g., **Brother(John, James)**).

10. Complex Sentence

- **Definition**: Sentences made by combining simpler sentences using logical connectives.
- **Examples**:
    - **¬(P(x))**: The negation of an atomic sentence.
    - **(P(x) ∧ Q(y))**: A sentence combining two propositions with AND.

Summary:

The syntax of FOL consists of these core components, allowing us to build structured logical expressions to represent and reason about the world accurately. Each plays a distinct role in forming meaningful logical sentences.

ان بناء جملة FOL يتالف من هذه المكونات الأساسية، مما يسمح لنا ببناء تعبيرات منطقية منظمة لتمثيل العالم والاستدلال عليه بدقة. يلعب كل منها دورًا مميزًا في تكوين جمل منطقية ذات معنى.

Atomic sentences

- Atomic sentence = *predicate* (*term*$_1$,…,*term*$_n$) or *term*$_1$ = *term*$_2$
- Term = *function* (*term*$_1$,…,*term*$_n$) or *constant* or *variable*

- Example.
- *Brother(KingJohn,RichardTheLionheart)*
- *> (Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn)))*

1. Atomic Sentence

- An **atomic sentence** is the simplest type of statement in logic. It cannot be broken down into smaller logical components.

الجملة البسيطة هي أبسط أنواع العبارات في المنطق. ولا يمكن تقسيمها إلى مكونات منطقية أصغر.

- There are two forms of atomic sentences:
    1. **Predicate(term$_1$, ..., term$_n$):** A predicate is a property or relationship applied to one or more "terms."

    2. **term$_1$ = term$_2$:** An equality statement asserting that two terms are the same

2. Term

- A **term** is a building block of atomic sentences. It can represent:

1. A **function**: Something like **Function(term$_1$, ..., term$_n$)** that maps input terms to an output.
2. A **constant**: A fixed object (e.g., "KingJohn" or "RichardTheLionheart").
3. **variable**: A placeholder that can represent different objects.

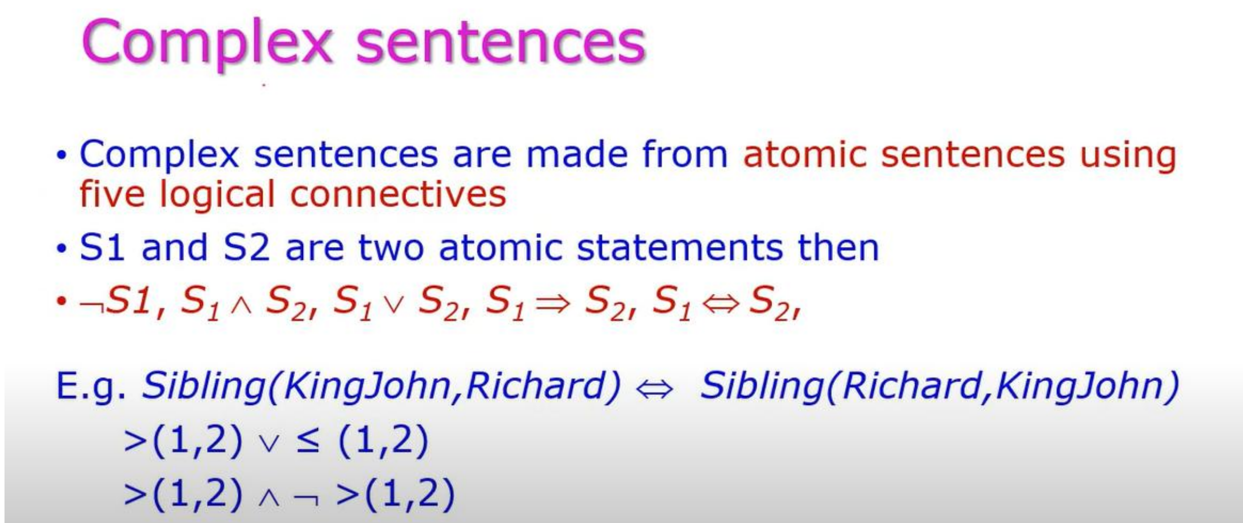
3. Examples

a. Brother(KingJohn, RichardTheLionheart)

- This is an atomic sentence where:
  - **Brother** is the **predicate** (relationship).
  - **KingJohn** and **RichardTheLionheart** are **terms** (constants).
- It expresses the relationship that "King John is the brother of Richard the Lionheart."

b. > **(Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn)))**

- This is a more complex atomic sentence involving a **comparison** (>).
  - **Length** is a **function** that calculates the length of something.
  - **LeftLegOf(Richard)** and **LeftLegOf(KingJohn)** are also functions, referring to the left legs of Richard and King John.
- This sentence states that the length of Richard's left leg is greater than the length of King John's left leg.



Complex sentences in logic, which are built from **atomic sentences** using logical connectives

1. Complex Sentences

- Complex sentences are logical expressions that combine **atomic sentences** (the basic building blocks of logic) using **five logical connectives**.

الجمل المعقدة هي تعبيرات منطقية تجمع الجمل البسيطة (اللبنات الأساسية للمنطق) باستخدام خمسة روابط منطقية.

2. Logical Connectives

These connectives define how atomic sentences are combined:

1. $\neg \mathbf{S_1}$: Negation (NOT S_1)
    - Negates the truth of the atomic sentence S_1.
2. $\mathbf{S_1 \wedge S_2}$: Conjunction (S_1 AND S_2)
    - True if both S_1 and S_2 are true.

3. **$S_1 \lor S_2$**: Disjunction (S_1 OR S_2)
   - True if at least one of S_1 or S_2 is true.
4. **$S_1 \Rightarrow S_2$**: Implication (IF S_1 THEN S_2)
   - True if either S_1 is false or S_2 is true.
5. **$S_1 \Leftrightarrow S_2$**: Biconditional (S_1 IF AND ONLY IF S_2)
   - True if S_1 and S_2 have the same truth value.

3. Examples

a. Sibling(KingJohn, Richard) ⇔ Sibling(Richard, KingJohn)

- This uses the biconditional connective (⇔), meaning "King John is a sibling of Richard if and only if Richard is a sibling of King John."
- It expresses the symmetry of the sibling relationship.

b. >(1,2) ∨ ≤(1,2)

- This is a **disjunction (∨)**, meaning "either 1 is greater than 2, or 1 is less than or equal to 2."
- At least one of these must be true.

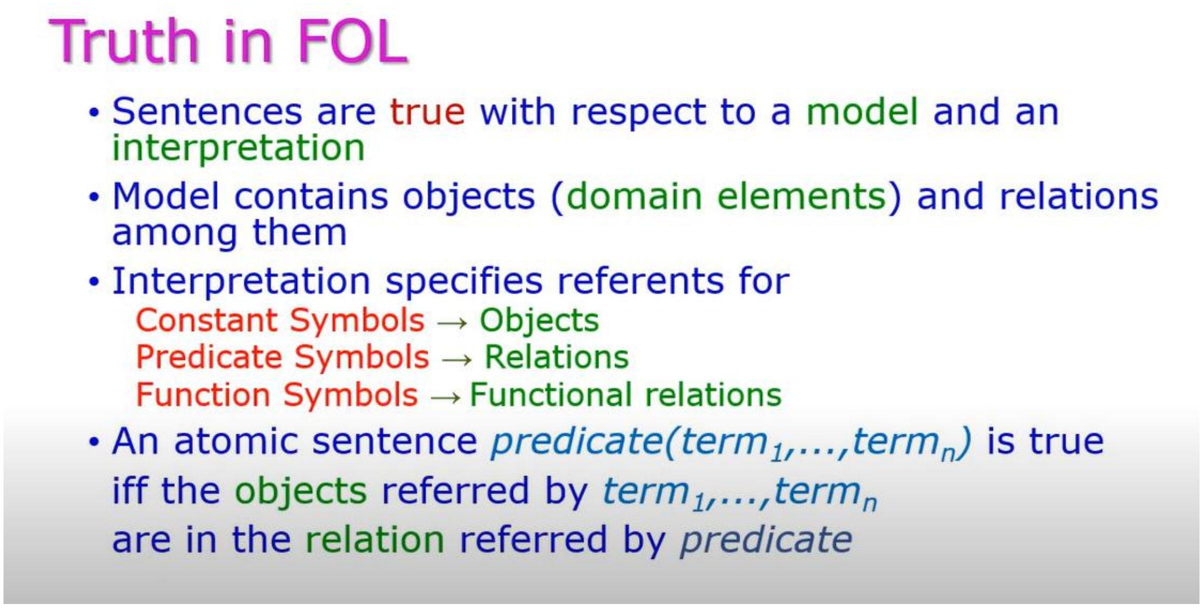
c. >(1,2) ∧ ¬>(1,2)

- This is a **conjunction (∧)** with negation (¬).
- It means "1 is greater than 2 AND 1 is NOT greater than 2."
- This is a logical **contradiction** because both cannot be true at the same time.

Summary

Complex sentences combine atomic ones using logical connectives to create more detailed expressions. Each connective specifies how the sentences interact logically.

تجمع الجمل المعقدة بين الجمل البسيطة باستخدام أدوات الربط المنطقية لإنشاء تعبيرات أكثر تفصيلاً. تحدد كل أداة ربط كيفية تفاعل الجمل منطقيًا.



1. Truth in FOL

الحقيقة في منطق الاسناد FOL

- Sentences in First-Order Logic (FOL) are considered **true** only when evaluated against:

تعتبر الجمل في منطق الترتيب الأول (FOL) صحيحة فقط عند تقييمها مقابل:

1. A **model**: This defines the objects (elements of the domain) and their relationships.
2. An **interpretation**: This specifies how symbols in the logic correspond to objects, relations, or functions in the model.

1. النموذج: يحدد الكائنات (عناصر المجال) وعلاقاتها.
2. التفسير: يحدد كيف تتوافق الرموز في المنطق مع الكائنات أو العلاقات أو الوظائف في النموذج.

2. Model

- A **model** includes:
    1. A set of **objects** (called domain elements).
    2. The **relationships** between these objects.

3. Interpretation

- An interpretation maps the symbols in FOL to specific meanings within the model:
    1. **Constant Symbols** → Objects:
       A constant symbol refers to a specific object in the model (e.g., "KingJohn" might refer to an actual person).
    2. **Predicate Symbols** → Relations:
       A predicate defines a relationship between objects (e.g., "Sibling" represents the sibling relationship).
    3. **Function Symbols** → Functional Relations:
       A function maps one or more objects to another object (e.g., "LeftLegOf" maps a person to their left leg).

4. Atomic Sentence Truth

- An atomic sentence of the form **predicate($term_1$, ..., $term_n$)** is **true** if:
    - The objects referred to by $term_1$, ..., $term_n$ are in the **relation** specified by the predicate.
    - Example:
        - If the atomic sentence is **Sibling(KingJohn, Richard):**
            - It is true if, in the model, King John and Richard are in the sibling relationship.

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Summary

In FOL, sentences aren't inherently "true" or "false." Their truth depends on:

1. The **model**, which defines the objects and relationships.
2. The **interpretation**, which assigns meanings to the logical symbols.

في لغةFOL ، لا تكون الجمل "صحيحة" أو "خاطئة" بطبيعتها. يعتمد صدقها على:

النموذج، الذي يحدد الكائنات والعلاقات. التفسير، الذي يعين المعاني للرموز المنطقية.

Example: Model and Interpretation

1. The Model

Our model represents a family:

- **Objects** (Domain):
  {Alice, Bob, Charlie}
- **Relationships**:
  - **Parent(x, y)**: "x is a parent of y"
  - **Sibling(x, y)**: "x is a sibling of y"

2. Interpretation

The interpretation assigns specific meanings to the symbols:

- **Constant Symbols**:
  - **a** → Alice
  - **b** → Bob
  - **c** → Charlie
- **Predicate Symbols**:
  - **Parent(x, y)** → True if "x is a parent of y" in our model
  - **Sibling(x, y)** → True if "x is a sibling of y" in our model
- **Function Symbols**:
  None for this example.

Truth of Atomic Sentences

1. **Sentence**: **Parent(a, b)**
   - Meaning: "Alice is a parent of Bob."
   - Truth: This is **true** if our model contains the relationship Parent(Alice, Bob).
2. **Sentence**: **Sibling(b, c)**
   - Meaning: "Bob is a sibling of Charlie."
   - Truth: This is **true** if our model contains the relationship Sibling(Bob, Charlie).
3. **Sentence**: **Parent(c, b)**
   - Meaning: "Charlie is a parent of Bob."
   - Truth: This is **false** if the model does not include the relationship Parent(Charlie, Bob).

Adding a Complex Sentence

We combine atomic sentences with logical connectives:

Sentence:

Sibling(b, c) ∧ ¬Parent(c, b)

- Meaning:
  - "Bob is a sibling of Charlie **and** Charlie is not a parent of Bob."
- Truth:
  - This is **true** if both conditions hold in the model:
    1. **Sibling(b, c)** is true (Bob and Charlie are siblings).
    2. **Parent(c, b)** is false (Charlie is not a parent of Bob).

Summary

- The **truth** of any sentence depends on:
    1. The **model**: Does it have the specified objects and relationships?
    2. The **interpretation**: How are symbols assigned meaning in the model?
- By evaluating whether these match, we determine whether a sentence is **true** or **false**.

تعتمد حقيقة أي جملة على:
النموذج: هل يحتوي على الكائنات والعلاقات المحددة؟
التفسير: كيف يتم تعيين معنى الرموز في النموذج؟
من خلال تقييم ما إذا كانت هذه الرموز متطابقة، نحدد ما إذا كانت الجملة صحيحة أم خاطئة.

Models for FOL: Example

- Richard the Lionheart was a King of England from 1189 to 1199;
- his younger brother was the evil King John, who ruled from 1199 to 1215;
- The left legs of Richard and John were different;
- and John had a crown (Because he was king).

Richard the Lionheart and John:
The statements describe historical facts about King Richard I of England (1189–1199) and his younger brother King John (1199–1215). These facts would be translated into FOL as predicates and relationships between entities.

Example <u>predicates</u>:

<u>King</u>(Richard, England, 1189, 1199) — Richard was King of England from 1189 to 1199.

<u>YoungerBrother</u>(John, Richard) — John is Richard's younger brother.

<u>King</u>(John, England, 1199, 1215) — John was King of England from 1199 to 1215.

The left legs of Richard and John were different. This might be expressed in FOL as:

Different(LeftLeg(Richard), LeftLeg(John)).

John had a crown because he was king:

HasCrown(John) if **King**(John).

يوضح هذا المثال كيف يمكن استخدام FOL لنمذجة الكيانات والعلاقات والخصائص في العالم الحقيقي بشكل منهجي. يتم تمثيل كل حقيقة كتعبيرات منطقية لإنشاء "نموذج" رسمي.