Logical Design

Lecture 2: Conversion between Number Systems

Produced By: Dr. Haleema Essa

2023-2024

1- Conversion from Binary to Decimal

A number with base 2 is converted into a number with base 10. Each binary digit here is multiplied by a decreasing power of 2. Let us see the examples:

Example1: Convert $(11011)_2$ to decimal number

We need to multiply each binary digit with the decreasing power of 2. That is;

$1\times2^4+1\times2^3+0\text{x}2^2+1\times2^1+1\times2^0$

$=16+8+0+2+1 =27$

Therefore, $(11011)_2 = (27)_{10}$

Example2: Convert $(10110)_2$ to decimal number

$10110_2 =$

$1 \text{ x } 2^4 + 0 \text{ x } 2^3 + 1 \text{ x } 2^2 + 1 \text{ x } 2^1 + 0 \text{ x } 2^0$

$16 + 0 + 4 + 2 + 0 = (22)_{10}$

EX:

$1011110_2 =(?)_{10}$

Binary no:	1	0	1	1	1	1	0
	2^6	2^5	2^4	2^3	2^2	2^1	2^0

$1\text{x } 2^6 + 0\text{x } 2^5 + 1\text{x } 2^4 + 1 \text{ x } 2^3 + 1 \text{ x } 2^2 + 1 \text{ x } 2^1 + 0 \text{ x } 2^0$

$64+0+16 + 8 + 4 + 2 + 0 = (94)_{10}$

H.W: Convert

$101010011_2 = (?)_{10}$

$111000101_2 = (?)_{10}$

2- Octal to Decimal Conversion

A number with base 8 is converted into a number with base 10. Each digit of the octal number here is multiplied by a decreasing power of 8.

Example: Convert $(121)_8$ into the equivalent decimal number

Solution: Given $(121)_8$ is an octal number. Here, we have to multiply each octal digit with the decreasing power of 8, such as:

$1\times8^2+2\times8^1+1\times8^0$

$64+16+1= (81)_{10}$

H.W: Convert

$(2058)_8 = (?)_{10}$

$(318)_8 = (?)_{10}$

3- Hexadecimal to Decimal Conversion

Each digit of a hexadecimal number here is multiplied by a decreasing power of 16.

Example: Convert $(12)_{16}$ into a decimal number.

Solution: Given $(12)_{16}$, Multiply each digit with a decreasing power of 16 to obtain an equivalent decimal number.

$1\times16^1+2\times16^0$

$16+2 = (18)_{10}$

H.W: Convert

$(1AF)_{16} = (?)_{10}$

$(5AC)_{16} = (?)_{10}$

$(29B)_{16} = (?)_{10}$

$(386)_{16} = (?)_{10}$

➢ *Number System Relationship*

HEXADECIMAL	DECIMAL	OCTAL	BINARY
0	0	0	0000
1	1	1	0001
2	2	2	0010
3	3	3	0011
4	4	4	0100
5	5	5	0101
6	6	6	0110
7	7	7	0111
8	8	10	1000
9	9	11	1001
A	10	12	1010
B	11	13	1011
C	12	14	1100
D	13	15	1101
E	14	16	1110
F	15	17	1111

Decimal Number System to Other Bases Conversion

1- Decimal to Binary Conversion

To convert a decimal number into an equivalent binary number we have to divide the original number system by 2 until the quotient is 0, when no more division is possible. The remainder obtained is counted for the required number in the order of LSB (Least significant bit) to MSB (most significant bit).

Example: Convert $(26)_{10}$ into a binary number.

Solution: Given $(26)_{10}$ is a decimal number.

Divide 26 by 2

26/2 = 13 Remainder →0 (LSB)

13/2 = 6 Remainder →1

6/2 = 3 Remainder →0
3/2 = 1 Remainder →1
½ = 0 Remainder →1 (MSB)
Hence, the equivalent binary number is $(11010)_2$

H.W: Convert

$(113)_{10} = (?)_2$

$(209)_{10} = (?)_2$

2- Decimal to Octal Conversion

The decimal number is required to be divided by 8 until the quotient is 0. Then, in the same way, we count the remainder from LSB to MSB to get the equivalent octal number.

Example: Convert $(65)_{10}$ into an octal number.

Solution: Given $(65)_{10}$ is a decimal number.

Divide by 8

65/8 = 8 Remainder →1 (LSB)

8/8 = 1 Remainder →0

⅛ = 0 Remainder →1 (MSB)

Hence, the equivalent octal number is $(101)_8$

H.W: Convert

$(952)_{10} = (?)_8$

$(259)_{10} = (?)_8$

3- Decimal to Hexadecimal Conversion

The given decimal number here is divided by 16 to get the equivalent hex. The division of the number continues until we get the quotient 0.

Example: Convert $(127)_{10}$ to a hexadecimal number.

Solution: Given $(127)_{10}$ is a decimal number.

Divide by 16

127/16 = 7 Remainder →15
7/16 = 0 Remainder → 7
In the hexadecimal number system, alphabet F is considered as 15.

Hence, $(127)_{10}$ is equivalent to $(7F)_{16}$

H.W: Convert

$(93)_{10} = (?)_{16}$

$(431)_{10} = (?)_{16}$

Binary, Octal, and Hexadecimal Conversion

1- Binary to Octal Conversion

There are ***two approaches*** to convert a binary number to its equivalent octal number:

a- First, we convert the given binary to decimal, **and after that** convert the decimal number to an equivalent octal number.

Example 1: Convert $(1010101)_2$ to octal

First, we convert the given binary to decimal

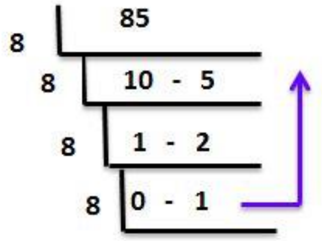
$(1 * 2^6) + (0 * 2^5) + (1 * 2^4) + (0 * 2^3) + (1 * 2^2) + (0 * 2^1) + (1 * 2^0) =$

64 + 0 + 16 + 0 + 4 + 0 + 1 =

64 + 21 =

0101012= 85 (Decimal form)

Then, convert the decimal number to an equivalent octal number.



Therefore, the equivalent octal number is $(125)_8$

b- Digit Grouping

- ✓ Divide the digits into groups of ***three*** starting from the right.
- ✓ Convert each group of three binary digits to one octal digit using the method of binary to decimal conversion.

Example: $101111010110_2 = ?_8$

101 111 010 110

5 7 2 6

$101111010110_2 = 5726_8$

2- Binary to Hexadecimal Conversion

There are ***two approaches*** to convert a binary number to its equivalent Hexadecimal number:

a- First, we convert the given binary to decimal, **and after that** convert the decimal number to an equivalent Hexadecimal number.

Example − Convert binary number 1101010 into a hexadecimal number.

First, convert this into a decimal number:

$= (1101010)_2 = 1x2^6 + 1x2^5 + 0x2^4 + 1x2^3 + 0x2^2 + 1x2^1 + 0x2^0$

= 64+32+0+8+0+2+0

$= (106)_{10}$

Then, convert it into a hexadecimal number

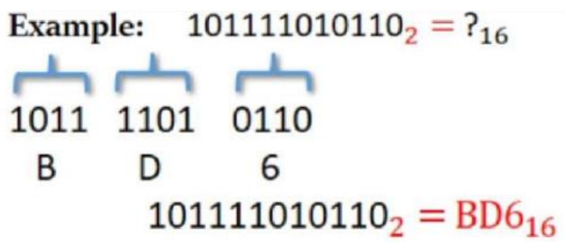
106/16 = 6 Remainder →10

6/16 = 0 Remainder → 6

Therefore, the equivalent hexadecimal number is $(6A)_{16}$

b- Digit Grouping

- ✓ Divide the digits into groups of ***four*** starting from the right.
- ✓ Convert each group of four binary digits to one hexadecimal digit using the method of binary-to-decimal conversion.



3- Octal to Binary Number Conversion

There are ***two approaches*** to convert the Octal number to its equivalent Binary number:

a- First, we convert a given octal to a decimal, **and after that** convert the decimal number to an equivalent binary number.

Example: Convert $(41)_8$ to a binary number.

First, convert the given octal number into its equivalent decimal number

$(41)_8 = (4 * 8^1) + (1 * 8^0)$

= 4 * 8 + 1 * 1

= 32+1

= 33(Decimal number)

Then, convert the decimal number to an equivalent binary number

Decimal Number divided by 2	Quotient	Remainder
33 divided by 2	16	1
16 divided by 2	8	0
8 divided by 2	4	0
4 divided by 2	2	0
2 divided by 2	1	0
1 divided by 2	0	1

Therefore, the equivalent binary number is $(100001)_2$.

b- Digit Conversion

- ✓ Convert each octal digit to a 3-digit binary number.
- ✓ Merge all the resulting binary groups into a single binary number.

Example: $275166_8 = ?_2$

2	7	5	1	6	6
010	111	101	001	110	110

$275166_8 = 010111101001110110_2$

4- Hexadecimal to Binary Conversion

There are ***two approaches*** to convert the Hexadecimal number to its equivalent Binary number:

a- First, we convert the given hexadecimal to decimal, **and after that** convert the decimal number to an equivalent binary number.

Example: Convert $(A2B)_{16}$ to an equivalent binary number.

First, convert the given hexadecimal to the equivalent decimal number.

$(A2B)_{16} = (A \times 16^2) + (2 \times 16^1) + (B \times 16^0)$

$= (A \times 256) + (2 \times 16) + (B \times 1)$

$= (10 \times 256) + 32 + 11$

$= 2560 + 43$

$= 2603$ (Decimal number)

Then, convert the decimal number to an equivalent binary number

2 | 2603
2 | 1301 -- 1
2 | 650 -- 1
2 | 325 -- 0
2 | 162 -- 1
2 | 81 -- 0
2 | 40 -- 1
2 | 20 -- 0
2 | 10 -- 0
2 | 5 -- 0
2 | 2 -- 1
2 | 1 -- 0
2 | 0 -- 1

The binary number obtained is $(101000101011)_2$

Hence, $(A2B)_{16} = (101000101011)_2$

b- Digit Conversion

- ✓ Convert each hexadecimal digit to a 4-digit binary number.
- ✓ Merge all the resulting binary groups into a single binary number.

Example: $E7C1F6_{16} = ?_2$

E	7	C	1	F	6
1110	0111	1100	0001	1111	0110

$E7C1F6_{16} = 111001111100000111110110_2$