# Logical Design

## Lecture 6: Boolean Algebra

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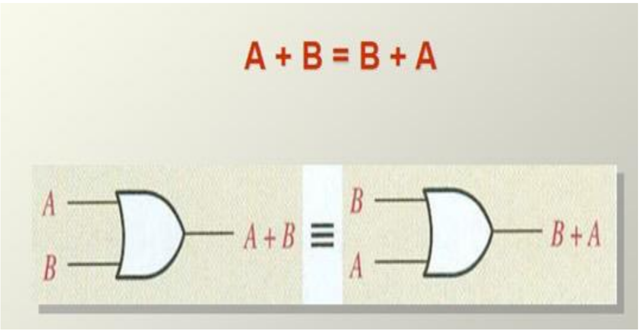
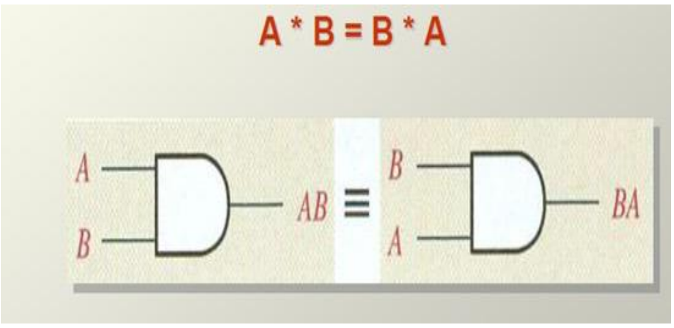
***2023-2024***

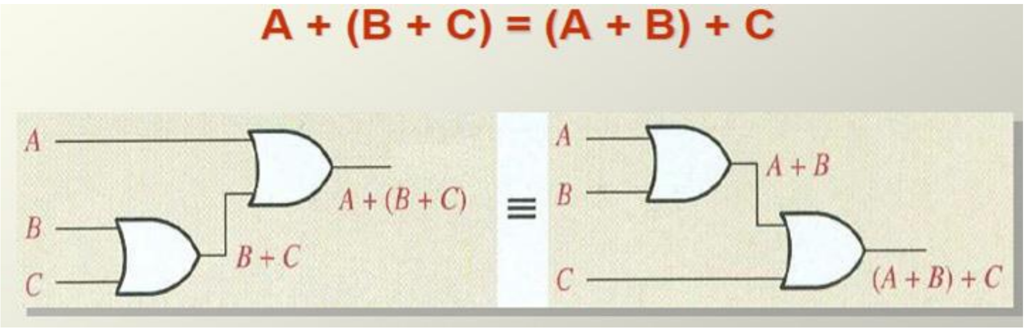
# *Laws of Boolean Algebra*

## 1- Commutative Law

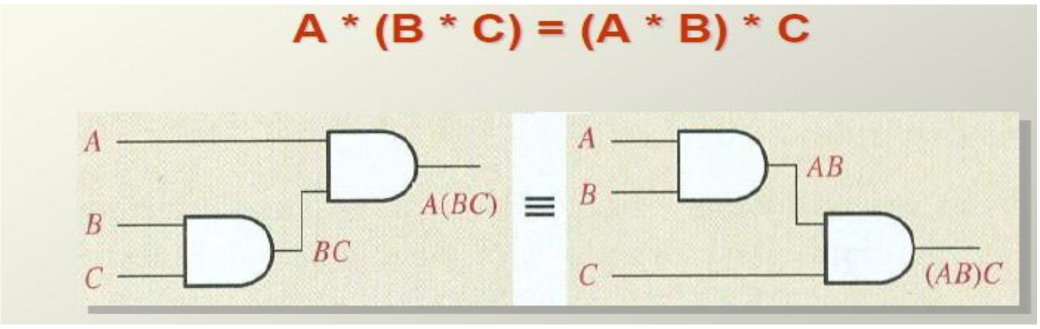


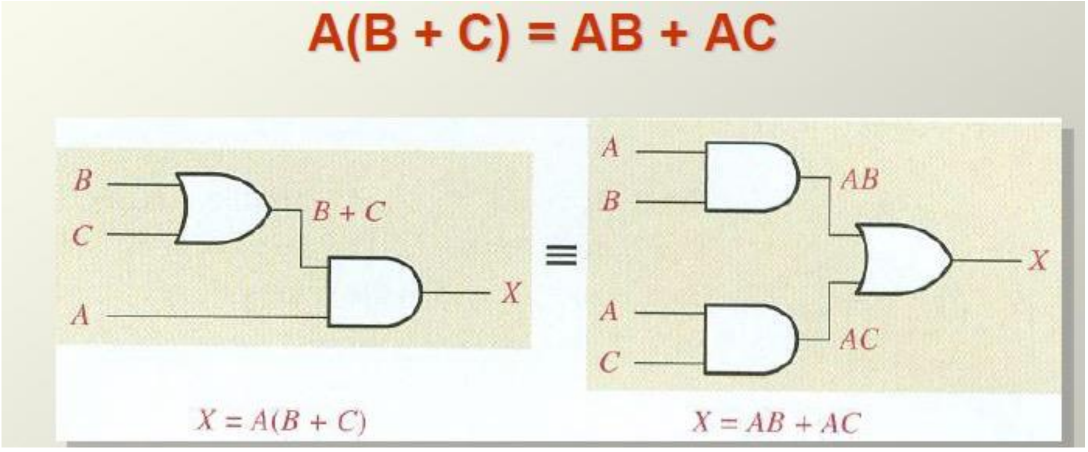

## 2- Associative Law ( Addition )




## 3- Associative Law ( Multiplication )



**4- Distributive Law**

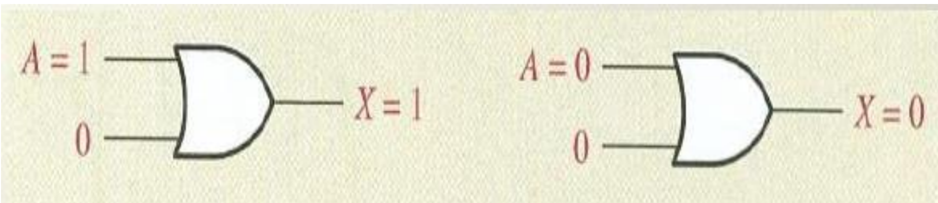



## Rules of Boolean Algebra

***<u>Rule1:</u>***

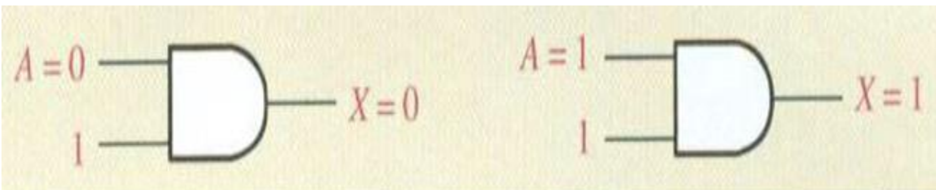
### a- Identity (Addition)

$$X=A+0=A$$




### b- Identity (Multiplication)

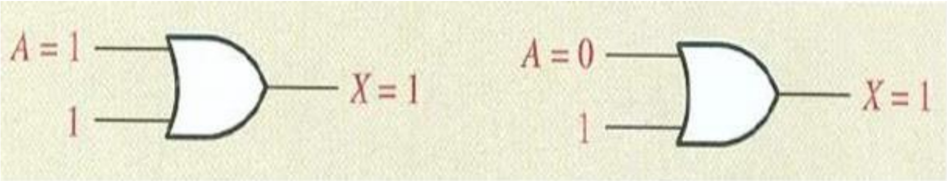
$$X=A*1=A$$



***Rule2:***

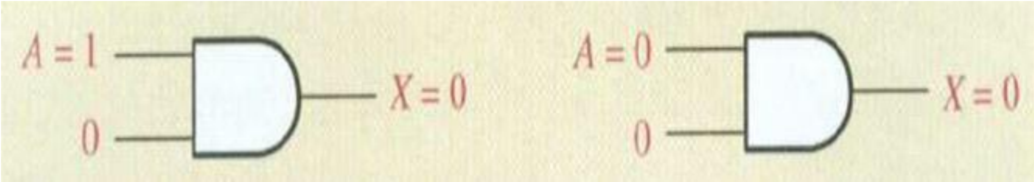
**a- NULL Rule (Addition)**

$$X=A+1=1$$




**b- NULL Rule (Multiplication)**

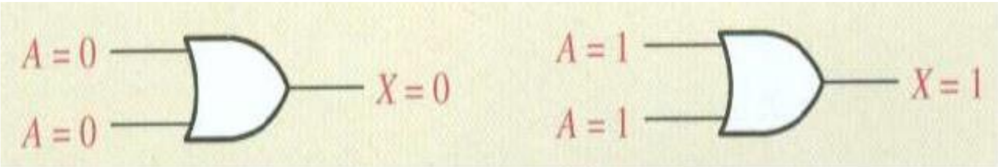
$$X=A*0=0$$




***Rule3:***

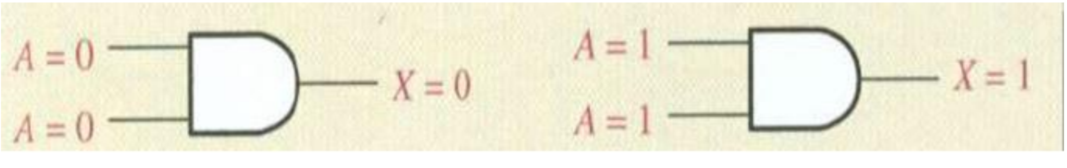
**a- Idempotent (Addition)**

$$X=A+A=A$$




**b- Idempotent (Multiplication)**

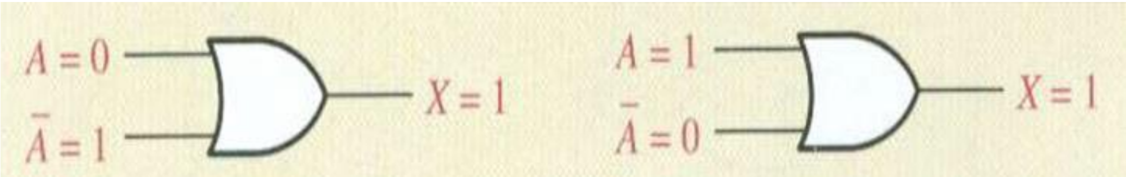
$$X= A*A=A$$



***Rule4:***

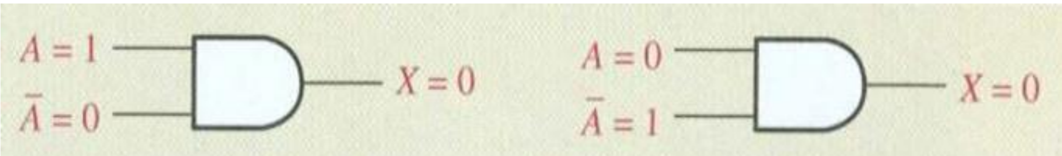
**a- Complement (Addition)**

$$X=A+\overline{A}=1$$



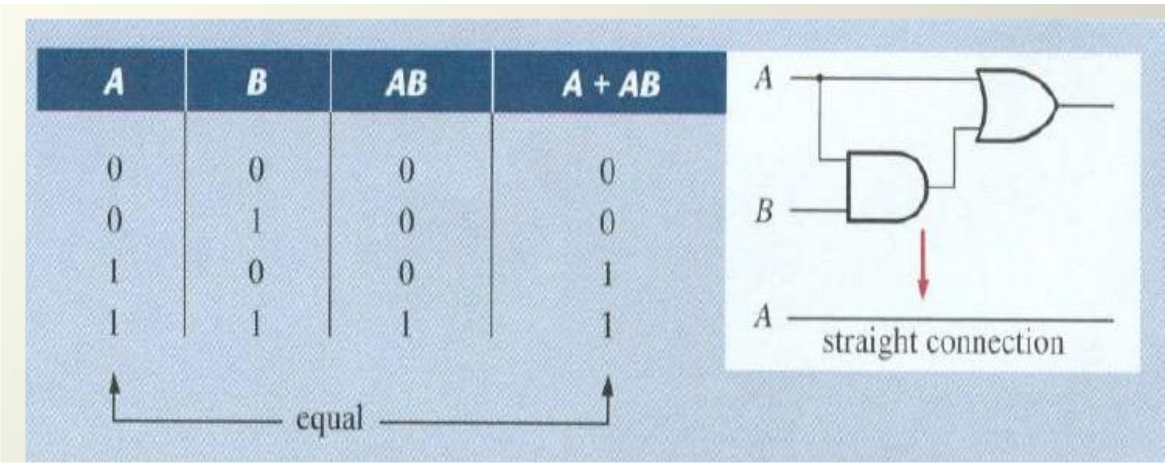

**b- Complement (Multiplication)**

$$X= A*\overline{A}=0$$




***Rule4:***

**OR Absorption: A + (A • B) = A**



| A | B | AB | A + AB |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 |



***The proof:***

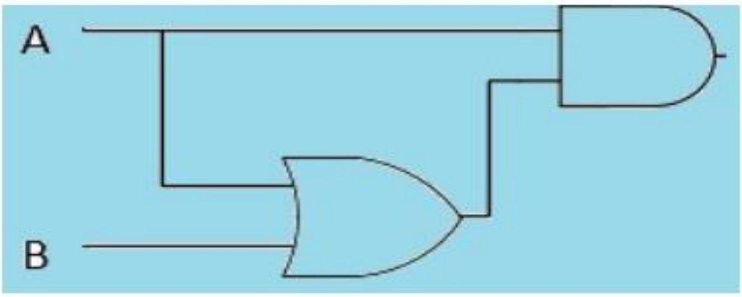
Left side = A + AB

= A (1+B)
=A·1
=A

## ***Rule5:***

## AND Absorption: A • (A + B) = A

| A | B | A+B | A.(A+B) |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 |




***The proof:***

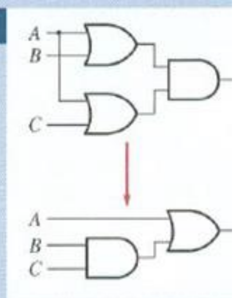
Left side = A(A+B)
= A·A+AB
= A + AB
= A(1 + B)
=A·1
=A

## ***Rule6:***

## (A + B)(A + C) = A + BC

| A | B | C | A + B | A + C | (A + B)(A + C) | BC | A + BC |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

equal



***The proof:***

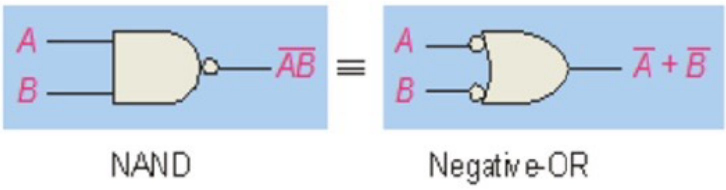
Left side = (A+B)(A+C)
= AA + AC + AB + BC
= A + AC + AB + BC
= A(1 + C + B) + BC
= A · 1 + BC
= A + BC = Right side

## DeMorgan's Theorem (First Theorem)

The complement of a product of variables is equal to the sum of complemented variables.

$$\overline{AB} = \bar{A} + \bar{B}$$

Applying DeMorgan's First Theorem to gates:



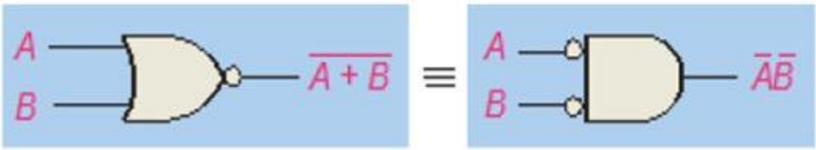

| A | B | ~ A | ~ B | ~ AB | ~ A + ~B |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 | 0 |

## DeMorgan's Theorem (Second Theorem)

The complement of a sum of variables is equal to the product of complemented variables.

$$\overline{A + B} = \bar{A} \bullet \bar{B}$$

Applying DeMorgan's Second Theorem to gates:




NOR　　　　Negative-AND

| A | B | ~ A | ~ B | ~ (A+B) | ~ A . ~B |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 |

***Example 1*** : Apply DeMorgan's theorem to simplify the following expression.

$X=\overline{\bar{c}+D}$

To apply DeMorgan's theorem to the above expression, we can break the over-bar that covers both terms and change the sign between them as follows:

$X=\bar{\bar{C}}\bullet \bar{D}$

$X=C\bullet \bar{D}$

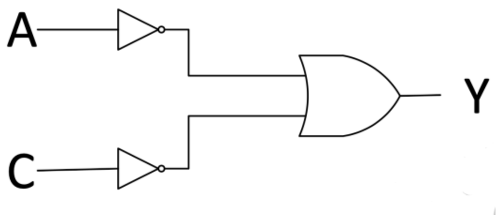
***Example 2:*** Simplify the following expression, and then draw the logic circuit after simplification.

$Y=\overline{A+B}+\overline{ABC}+\overline{A\bar{B}}$

$=\bar{A}\,\bar{B}+\bar{A}+\bar{B}+\bar{C}+\bar{A}+B$

$=\bar{A}(\bar{B}+1+1)+(B+\bar{B})+\bar{C}$

$=\bar{A}+\bar{C}$



***__Example 3:__*** Simplify the following expression.

$Y = (A+B)(A+\bar{B})$

$= AA + A\bar{B} + AB + B\bar{B}$

$= A + A(\bar{B} + B)$

$= A + A$

$= A$

OR:

$Y = AA + A\bar{B} + AB + B\bar{B}$

$= AA + A\bar{B} + AB$

$= A + A\bar{B} + AB$

$= A(1 + \bar{B} + B)$

$= A \bullet 1$

$= A$

***__H.W: Simplify the following expression.__***

1- $Y = ABC + A\bar{B}C + AB\bar{C}$

2- $Y = ABC + A\,\bar{B}\,(\overline{\bar{A}\,\bar{C}})$

***__Example4:__*** Determine if the following equation is valid (Derived from the truth table)

$$\bar{X}1\ \bar{X}3 + X2\ X3 + X1\ \bar{X}2 = \bar{X}1\ X2 + X1\ X3 + \bar{X}2\ \bar{X}3$$

**Left-Hand Side (LHS)**

| Row number | $x_1$ | $x_2$ | $x_3$ | $\bar{x}_1\bar{x}_3$ | $x_2x_3$ | $x_1\bar{x}_2$ | $f$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| 3 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| 4 | 1 | 0 | 0 | 0 | 0 | 1 | 1 |
| 5 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 6 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 7 | 1 | 1 | 1 | 0 | 1 | 0 | 1 |

**Right-Hand Side (RHS)**

| Row number | $x_1$ | $x_2$ | $x_3$ | $\bar{x}_1x_2$ | $x_1x_3$ | $\bar{x}_2\bar{x}_3$ | $f$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| 3 | 0 | 1 | 1 | 1 | 0 | 0 | 1 |
| 4 | 1 | 0 | 0 | 0 | 0 | 1 | 1 |
| 5 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 6 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 7 | 1 | 1 | 1 | 0 | 1 | 0 | 1 |

***__H.W:__*** For the given equation, give the ***truth table***, then ***simplify*** and ***draw*** the circuit ***before and after*** simplification.

$Y = \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C} + ABC$