

# Logical Design

## Lecture 8: Combinational Circuits

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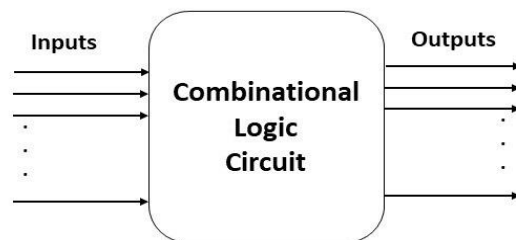
*2023-2024*

### *Digital circuits*

Digital circuits can be categorized into two types: Combinational circuits and sequential circuits.

#### *Combinational circuit*

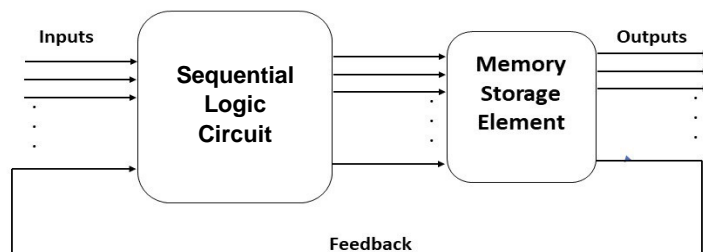
- Output of combinational circuit based on the combination of present input only.



- No feedback
- No memory
- Ex: encoder, decoder, multiplexer, demultiplexer, adder, and subtractor.

#### *Sequential circuit*

- ❖ The output of a sequential circuit depends on the current input and a previous output.



- ❖ Feedback is present
- ❖ Memory is present
- ❖ Ex: Flipflop, Registers and Counters

## Combinational Circuits

Connecting logical gates together to produce a specified output with no storage involved. Combinational circuits are divided into three main parts:

### 1. Arithmetic and logical function

Adder, Subtractor, and Comparator.

### 2. Code convertors

Binary, Gray, and BCD

### 3. Data transmission

Encoder, Decoder, Multiplexer, and Demultiplexer.

## Design a combinational circuit

**Example:** Design a combinational circuit with three variables that will produce a logic 1 output when more than one input variable is at logic 1.

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

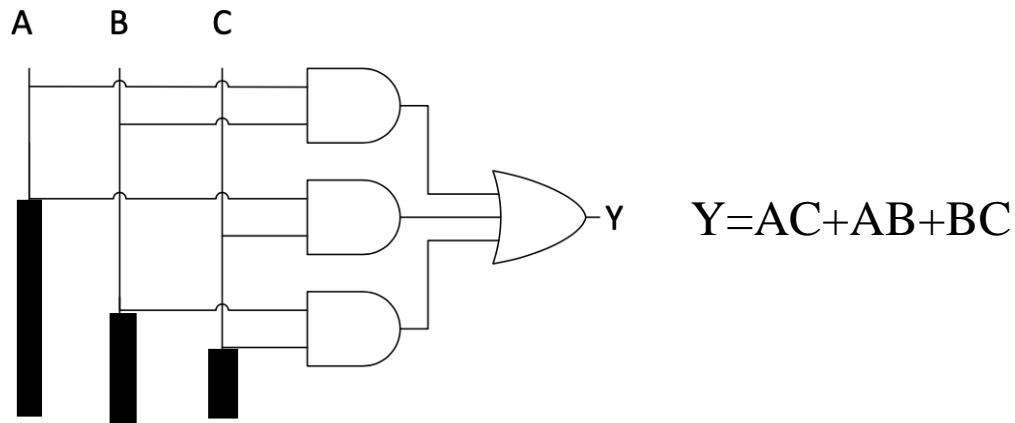
$$Y = \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$$

After simplification as follows:

	B'C'	B'C	BC	BC'
A'	0	1	3	2
A	4	5	7	6

The Karnaugh map shows the following groupings:

- Group 1: m3 (011) and m7 (111) - circled in black.
- Group 2: m5 (010) and m7 (111) - circled in red.
- Group 3: m6 (110) and m7 (111) - circled in red.
- Group 4: m3 (011), m5 (010), m6 (110), and m7 (111) - circled in red.



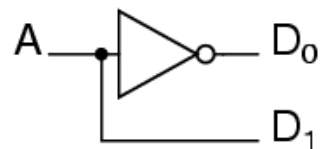
## Decoder

A **decoder** is a combinational logic circuit that is used to change the code into a set of signals. It is called a decoder because it does the reverse of encoding. decoder which takes an  $n$ -digit binary number and decodes it into  $2^n$  data lines. The simplest is the 1-to-2 line decoder. The truth table is:

A	$D_1$	$D_0$
0	0	1
1	1	0



**A** is the address and **D** is the data-line.  **$D_0$**  is NOT A and  **$D_1$**  is A. The circuit looks like



## 2-to-4 Line Decoder

The truth table is:

$A_1$	$A_0$	$D_3$	$D_2$	$D_1$	$D_0$
0	0	0	0	0	1
0	1	0	0	1	0
1	0	0	1	0	0
1	1	1	0	0	0

The equations are:

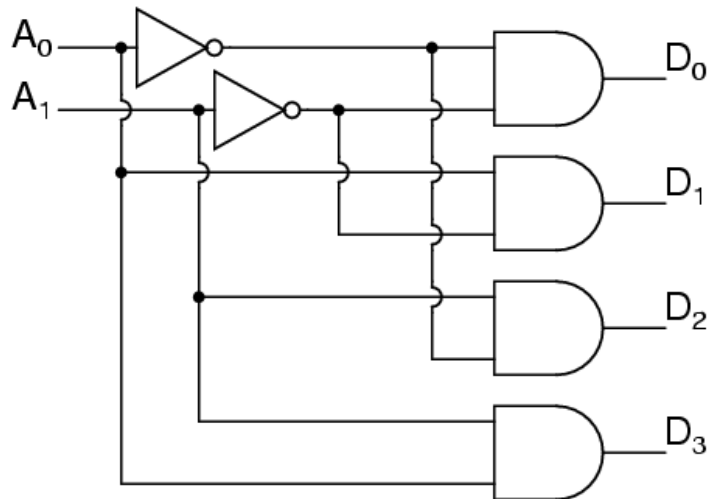
$$D_0 = \overline{A_1} \cdot \overline{A_0}$$

$$D_1 = \overline{A_1} \cdot A_0$$

$$D_2 = A_1 \cdot \overline{A_0}$$

$$D_3 = A_1 \cdot A_0$$

The 2-to-4 decoder logic circuit is



### 3 to 8 Decoder

- A 3 to 8 decoder has three inputs (A, B, C) and eight outputs (D0 to D7).
- Based on the 3 inputs **one** of the eight outputs is selected.
- The truth table for 3 to 8 decoder is shown in the below table.

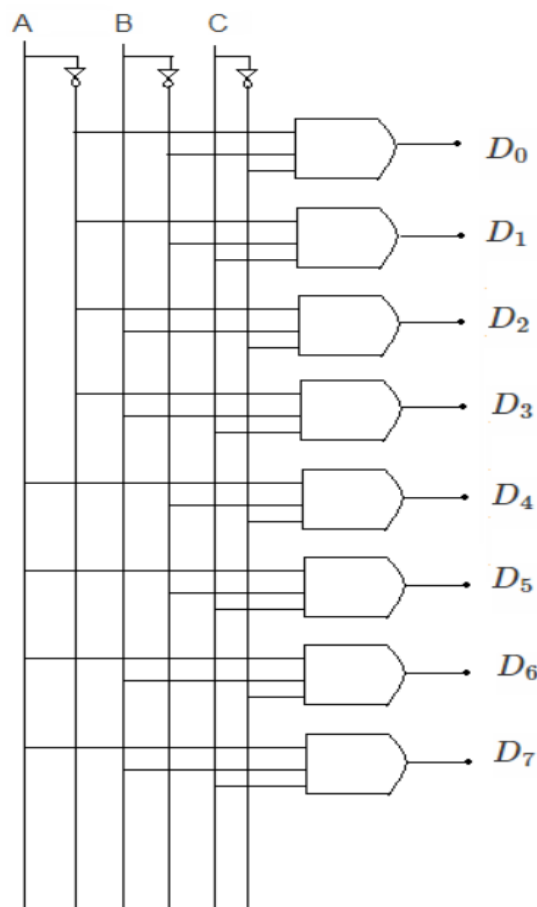
Truth table of 3 to 8 decoder:

A	B	C	D0	D1	D2	D3	D4	D5	D6	D7
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

- From the truth table, it is seen that only one of eight outputs (D0 to D7) is selected based on three select inputs.
- From the truth table, the logic expressions for outputs can be written as follows:

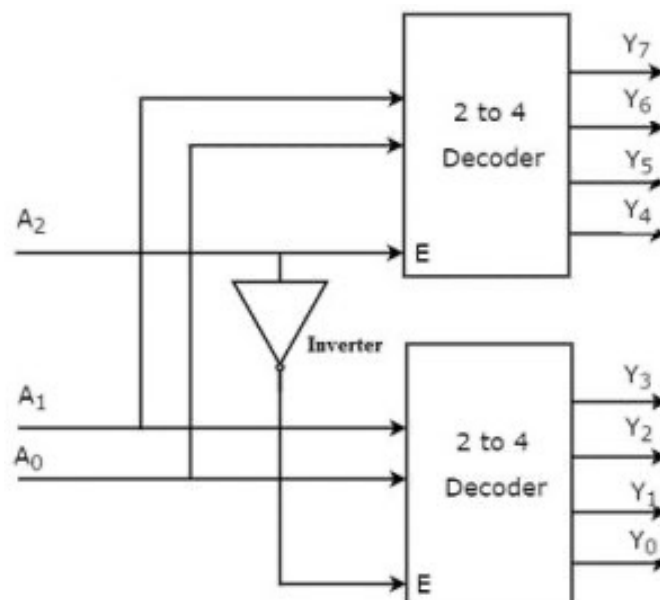
$$\begin{aligned}
 D_0 &= \bar{A}\bar{B}\bar{C}, & D_1 &= \bar{A}\bar{B}C, & D_2 &= \bar{A}B\bar{C}, \\
 D_3 &= \bar{A}BC, & D_4 &= A\bar{B}\bar{C}, & D_5 &= A\bar{B}C, \\
 D_6 &= AB\bar{C}, & D_7 &= ABC
 \end{aligned}$$

- Using the above expressions, the circuit of a 3 to 8 decoder can be implemented using three NOT gates and eight 3-input AND gates as shown in the following figure.



## ***Building a 3-to-8 decoders using 2-to-4 decoders***

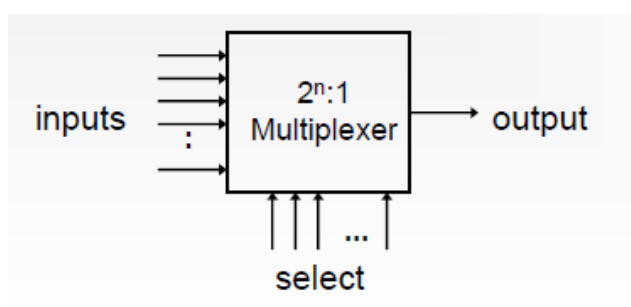
The implementation of this 3 line to 8 line decoder can be done using two 2 lines to 4 line decoders. We have discussed above that 2 to 4 line decoder includes two inputs and four outputs. So, in 3 lines to 8 line decoder, it includes three inputs like  $A_2$ ,  $A_1$  &  $A_0$  and 8 outputs from  $Y_7 - Y_0$ . In addition to input pins, the decoder has a ***Enable Pin***.



## ***Multiplexers (MUX)***

The multiplexer is a combinational logic circuit designed to switch one of several input lines to a single common output line.

It steers one of  $2^n$  inputs to a single output line, using  $n$  selection lines. Also known as a ***data selector***.



Routes one of its N data inputs to its one output, based on the binary value of select inputs:

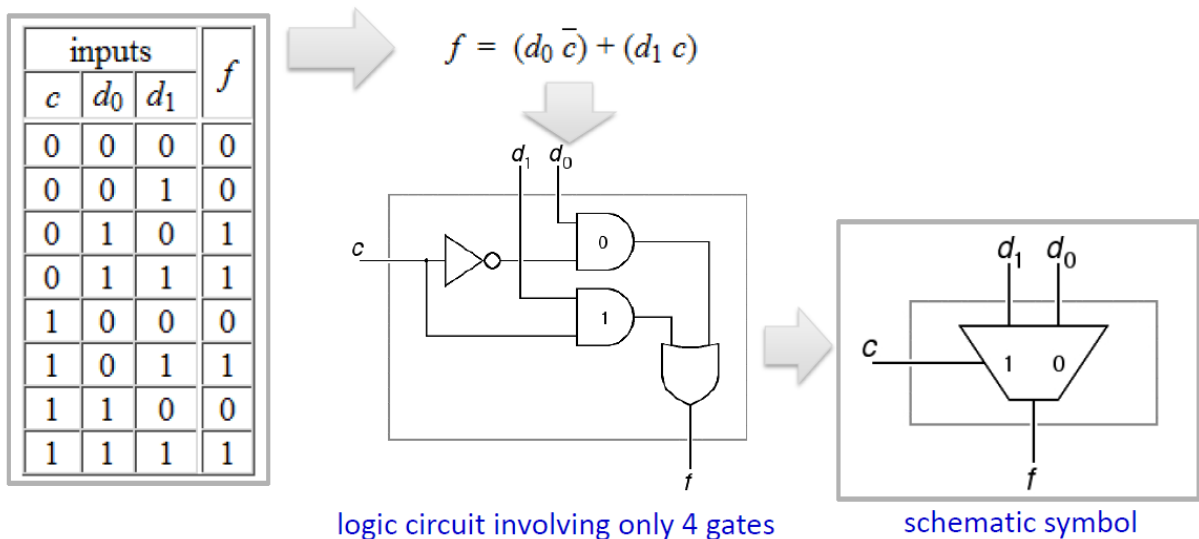
- ✓ 4 input mux needs 2 select inputs to indicate which input to route through.
- ✓ 8 input mux needs 3 select inputs.
- ✓ N inputs  $\rightarrow \log_2(N)$  selects.

### ***MUX Advantages:***

1. Reduce the number of wires.
2. Reduce circuit complexity and cost.
3. The multiplexer is a very useful electronic circuit that has uses in many different applications such as signal routing, data communications, and data bus control applications.
4. Helps share a single communication line among a number of devices.
5. At any time, only one source and one destination can use the communication line.

### ***2-input multiplexor***

The truth table for a multiplexor with **2** data inputs  **$d_0$**  and  **$d_1$**  and one control input  **$c$**  is as follows:

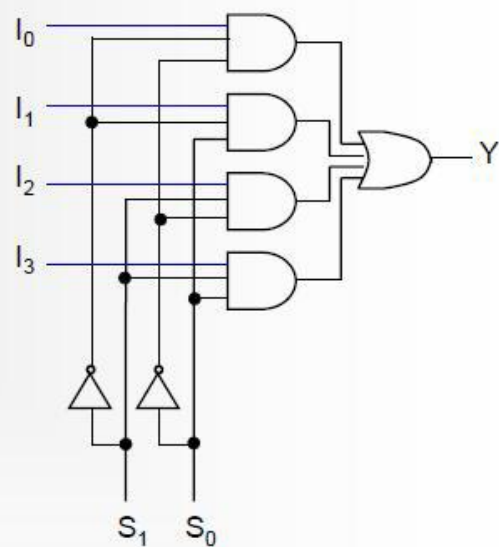
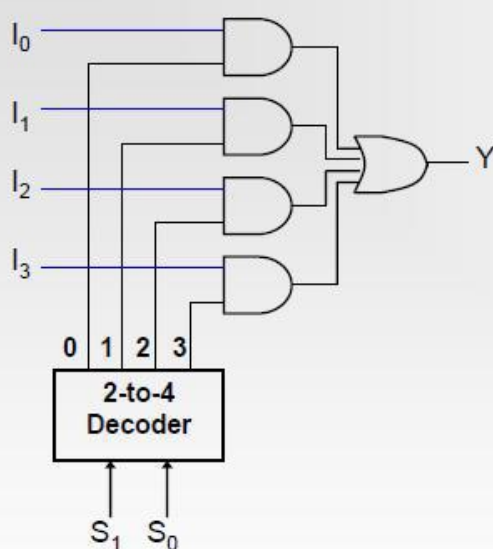
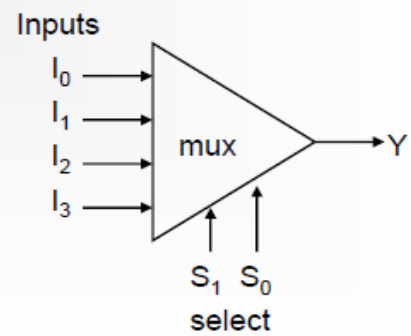
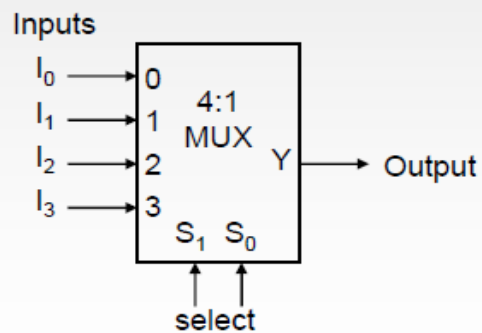


## 4-input multiplexor

- Truth table for a 4-to-1 multiplexer:

$I_0$	$I_1$	$I_2$	$I_3$	$S_1$	$S_0$	$Y$
$d_0$	$d_1$	$d_2$	$d_3$	0	0	$d_0$
$d_0$	$d_1$	$d_2$	$d_3$	0	1	$d_1$
$d_0$	$d_1$	$d_2$	$d_3$	1	0	$d_2$
$d_0$	$d_1$	$d_2$	$d_3$	1	1	$d_3$

$S_1$	$S_0$	$Y$
0	0	$I_0$
0	1	$I_1$
1	0	$I_2$
1	1	$I_3$

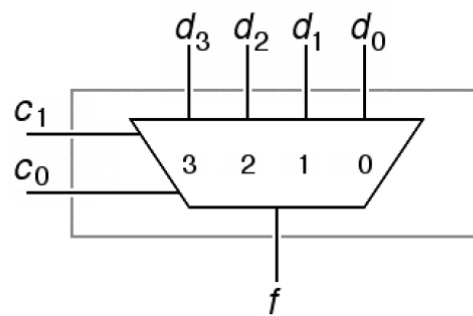
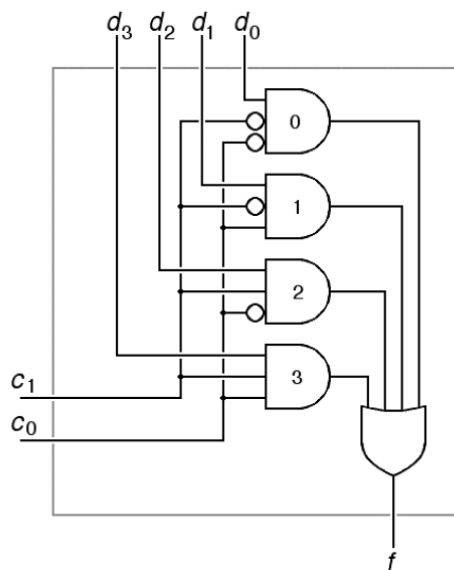


Four-to-one multiplexer design.



## Gate-level design for a 4-input multiplexor

$$f = (d_0 c'_1 c'_0) + (d_1 c'_1 c_0) + (d_2 c_1 c'_0) + (d_3 c_1 c_0)$$



schematic symbol

## Demultiplexers (Demux)

The **demultiplexer** is a combinational logic circuit designed to switch one common input line to one of several separate output lines.

The **demultiplexer** takes one single input data line and then switches it to any one of a number of individual output lines one at a time. The **demultiplexer** converts a serial data signal at the input to parallel data at its output lines as shown in the figure.

