Lecture 10

ex: A flash light contain two batteries each having an MTTF of 5 operating hours (assume CFR)

a- what i the reliability of this system if two batteries in redundant

b what is the MTTF

sol:

a $R_s(t) = 1-(1-R_1)(1-R_2)$

$$= 1-(1-e^{-\lambda t})(1-e^{-\lambda t})$$

because $\lambda_1 = \lambda_2 = \frac{1}{5}$

$$\Rightarrow R_s(t) = 1-(1-e^{-\frac{1}{5}t})(1-e^{-\frac{1}{5}t})$$

$$R_s(t) = 2e^{-\frac{1}{5}t} - e^{-\frac{2}{5}t}$$

b $MTTF = \frac{1}{n\lambda}$

$$= \frac{1}{2\lambda}$$

$MTTF = \frac{1}{2\frac{1}{5}} = \frac{5}{2} = 2.5$ operating hours

2

ex: Consider two redundant component having constant but different failure rates

Find the reliability at 1000 hr of a two component independent redundent system if $\lambda_1(t) = 0.000356$ and $\lambda_2(t) = 0.00156$ per hour

$$R_s(t) = 1 - (1 - e^{-\lambda_1(t)})(1 - e^{-\lambda_2(t)})$$

$$\cancel{= 1 - (1 - e^{-0.000356t})(1 - e}$$

$$R_s(t) = 1 - (1 - e^{-0.000356t})(1 - e^{-0.00156t})$$

$$R_s(1000) = 1 - (1 - e^{-0.00056(1000)})(1 - e^{-0.00156(1000)}$$

Lecture 10:

ch: 3 Time-dependent failure model

3.1 The Weibull distribution

One of the most useful probability dist^n. in reliability is the Weibull

The Weibull failure distribution may be used to model both increasing and decreasing failure rates

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It is characterized by a hazard rates function of the form

$$\lambda(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1}$$

which is power fn. (دالة قوى). The function $\lambda(t)$ is increasing for $\theta > 0$ and $\beta > 0$ and decreasing for $\theta > 0$ and $\beta < 0$

نشتق

To derive the reliability fn. from hazard

معادلة

fn using the equation of hazard

$$R(t) = \exp\left(-\int_0^t \lambda(t')\,dt'\right)$$

$$R(t) = \exp\left(-\int_0^t \frac{\beta}{\theta}\left(\frac{t'}{\theta}\right)^{\beta-1} dt'\right)$$

$$= \exp\left(-\frac{\cancel{\beta}}{\cancel{\beta}}\left(\frac{t'}{\theta}\right)^{\beta}\Big|_0^t\right)$$

$$= \exp\left(-\left(\frac{t}{\theta}\right)^{\beta}\right)$$

$$f(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} e^{-\left(\frac{t}{\theta}\right)^{\beta}} \qquad t \geq 0$$

$$= 0 \qquad \text{o.w.}$$

β : is the shape parameter معلمة الشكل

when $\beta < 1$ the pdf is similare in shape to the exponential dist^n.

عندما $\beta<1$ فان شكل دالة الكثافة الاحتمالية يشبه توزيع الاسي

نوعا ما

when $\beta \geq 3$ the pdf is somewhat

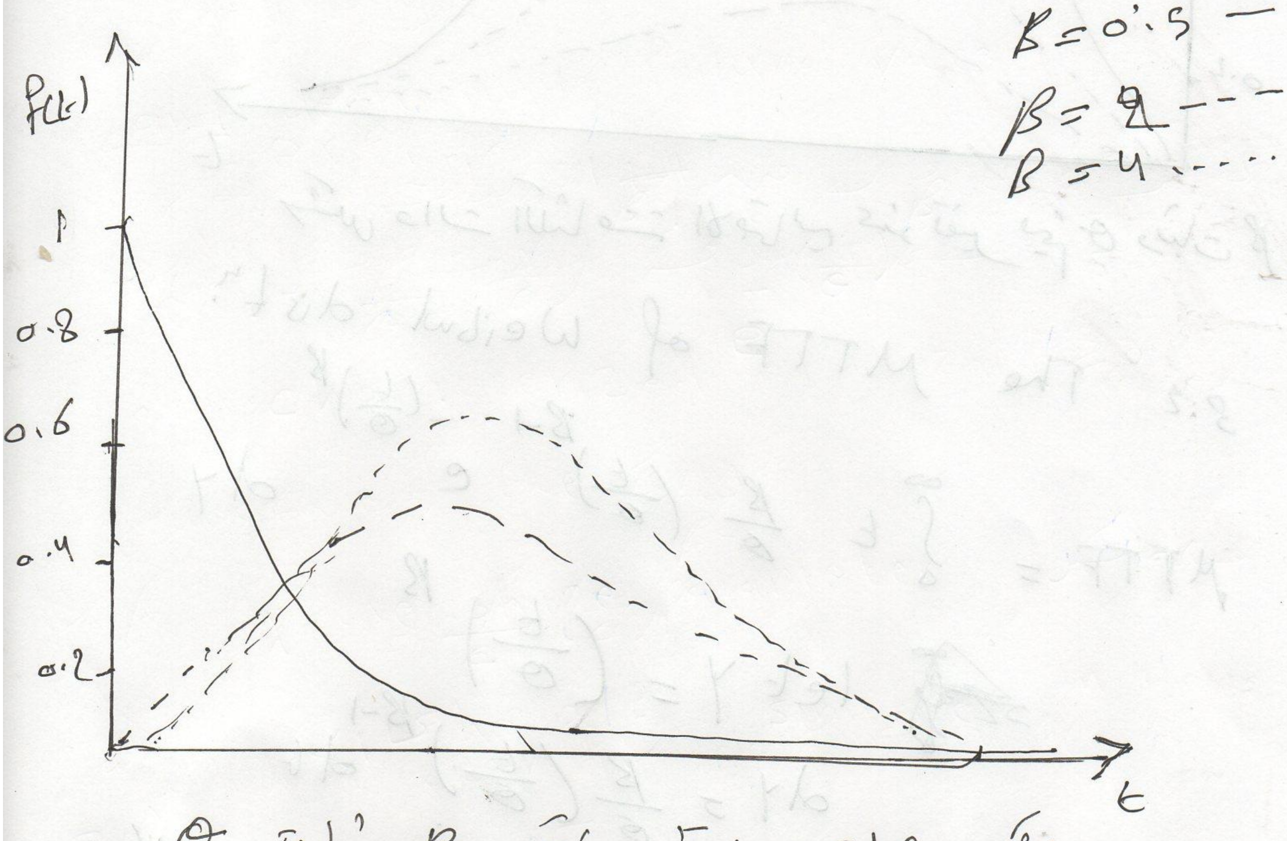
متماثل

symmetrical like normal dist^n.

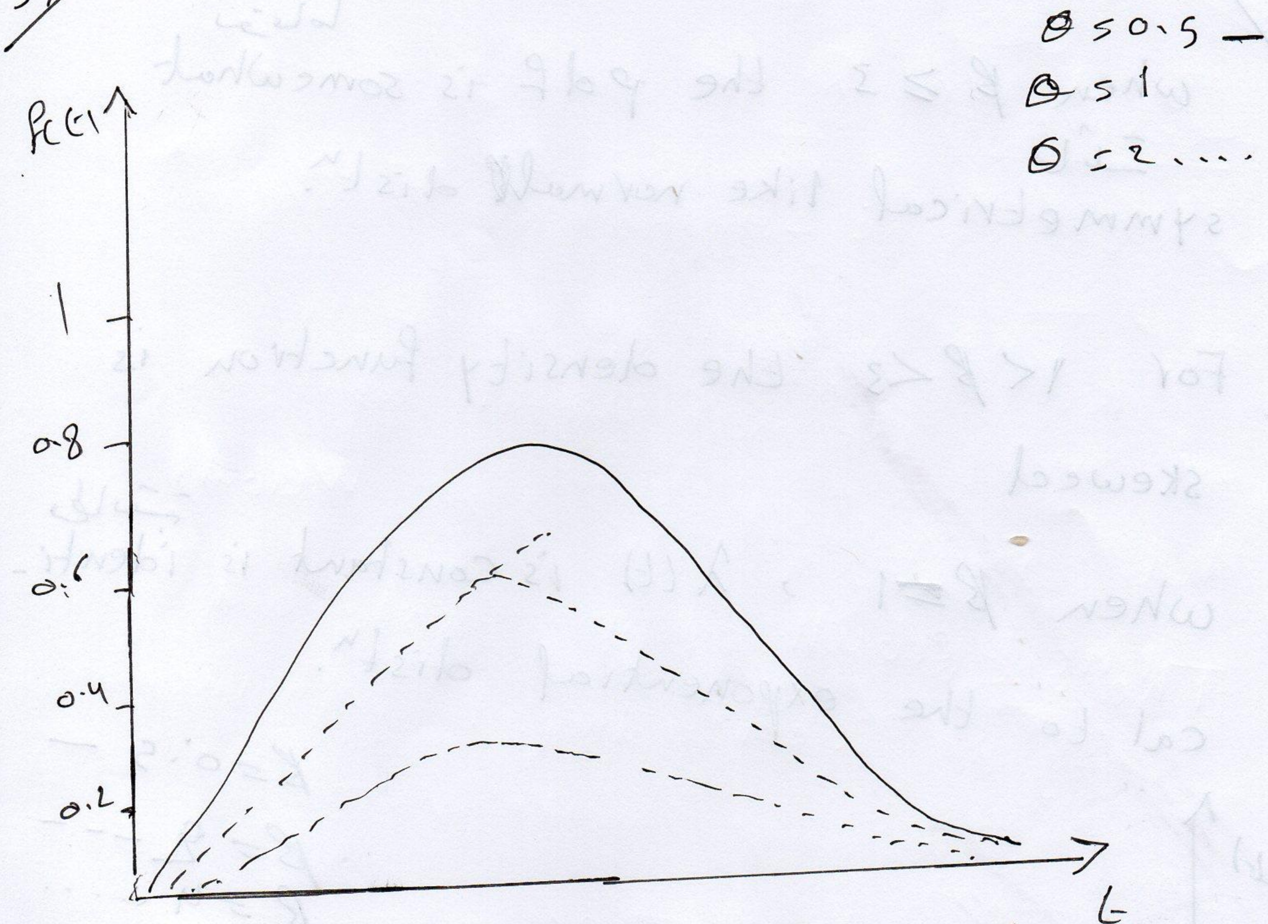
For $1 < \beta < 3$ the density function is skewed

مطابقة

when $\beta = 1$, $\lambda(t)$ is constant is identical to the exponential dist^n.



الشكل pdf عند تغير قيمة β وثبات θ



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8.2 The MTTF of Weibul dist^n

$$MTTF = \int_0^\infty t \, \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} e^{-\left(\frac{t}{\theta}\right)^{\beta}} \, dy$$

$$\text{let } y = \left(\frac{t}{\theta}\right)^{\beta}$$

$$dy = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} dt$$

$$= \int_0^\infty t \, \cancel{\frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta+1}} \, e^{-y} \, \frac{1}{\cancel{\frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1}}} \, dy$$

$$MTTF = \int_0^\infty t \, e^{-y} \, dy$$

$$\therefore \; y = \left(\frac{t}{\theta}\right)^{\beta}$$

لنستخرج t نرفع الاسس للطرفين للاس $1/\beta$

$$y^{1/\beta} = \left(\left(\frac{t}{\theta}\right)^{\cancel{\beta}}\right)^{1/\cancel{\beta}}$$

$$\Longrightarrow t = \theta \, y^{1/\beta}$$ نعوض

$$MTTF = \theta \int_0^\infty y^{(1/\beta + 1) - 1} \, e^{-y} \, dy$$

$$MTTF = \theta \, \Gamma(1 + 1/\beta)$$

$$\sigma^2 = \theta^2 \left\{ \Gamma\left(1 + \frac{2}{\beta}\right) - \left[\Gamma(1 + 1/\beta)\right]^2 \right\}$$

Note! There is no direct relationship between MTTF of $\lambda(t)$ in Weibull dist^n.