Lecture II:

ex: A compressor experience wearout الاندثار
with a linear hazard fn. خطي

$$\lambda(t) = \frac{2}{1000}\left(\frac{t}{1000}\right)$$

so $\beta = 2$, $\theta = 1000$

① compute $R(t)$
② comput design life if $R(t) = .99$
③ MTTF

sol:

$$R(t) = \exp\left(-\int_0^t \lambda(t')\,dt'\right)$$

$$= \exp\left(-\int_0^t \frac{2}{1000}\left(\frac{t'}{1000}\right) dt'\right)$$

$$= \exp\left(-\frac{2}{2}\left(\frac{t'}{1000}\right)^2 \Big|_0^t\right)$$

$$R(t) = \exp\left(-\left(\frac{t}{1000}\right)^2\right)$$

② $$R(t) = e^{-\left(\frac{t}{1000}\right)^2}$$

$$0.99 = e^{-\left(\frac{t}{1000}\right)^2}$$

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$$(-\ln 0.99)^{1/2} = \left(\left(\frac{t}{1000}\right)^{2}\right)^{1/2}$$

$$\Rightarrow t = \sqrt{-\ln 0.99} \times 1000$$

$$t = 100.25$$

3 $\quad MTTF = \theta\,\Gamma(1 + 1/\beta)$

$$\cancel{= 1000\,\Gamma\,88}$$

$$MTTF = 1000\,\Gamma(1 + 1/2)$$

Note! In general

$$t_r = \theta(-\ln R)^{1/\beta} \quad \text{design life}$$

of Weibull dist^n.

3.2 Failure modes! أساليب الفشل (بالتوالي)

For a system comprized of n serially related components or having n independent failure modes. each having an independent Weibull dist^n. with shape parameter β and scale parameter θ_i , the system failure rate function can be determined

في نظام متكون من n مركبة مرتبطة على التوالي او لديه n
اسلوب فشل مستقل كل مركبة تتوزع توزيع ويبل بمعلمتي
شكل β وقياس θ_i

دالة المخاطرة للنظام هي الآتي
مجموع دوال المخاطرة لكل مركبة

$$\lambda(t) = \sum_{i=1}^{n} \lambda_i(t)$$

$$\because \lambda_i(t) = \frac{\beta}{\theta_i}\left(\frac{t}{\theta_i}\right)^{\beta-1} \quad \text{--- (2)}$$

$$\Rightarrow \lambda(t) = \beta\, t^{\beta-1}\left[\sum_{i=1}^{n}\left(\frac{1}{\theta_i}\right)^{\beta}\right] \quad \text{--- (3)}$$

However, this is hazard rate for a weibull dist^n. having a shape parameter β and characteristic life

$$\left[\sum_{i=1}^{n}\left(\frac{1}{\theta}\right)^{\beta}\right]^{-1/\beta}$$

3.2.1 Identical Weibull Components

If a system of n serially (توالي) related and independent component have identical (متطابقة) hazard rate functions.

∴ For each component hazard rate

$$\lambda_i(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1}$$

For system $\lambda(t) = \sum_{i=1}^{n} \lambda_i(t)$

$$\lambda(t) = \sum_{i=1}^{n} \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} = \frac{n\beta}{\theta^{\beta}} \cdot t^{\beta-1}$$

and the reliability of system

$$R(t) = e^{-n\left(\frac{t}{\theta}\right)^{\beta}}$$

ex: A jet engine (محرك طائرة) consist of five modules (مقياس) each of which ~~has~~ was found have a Weibull dist^n. with shape parameter $\beta = 1.5$, Their scale parameters are (in operating cycle)

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3600, 7200, 5850, 4780 and 9300. Find:

(1) MTTF (2) median time to failure

<u>Sol:</u> The engine has a weibull failure dist^n. with $\beta = 1.5$ al the scale parameter

$$\theta = \left[\sum_{i=1}^{5} \left(\frac{1}{\theta_i} \right)^{\beta} \right]^{-1/\beta}$$

$$\theta = \left[\left(\frac{1}{3600} \right)^{1.5} + \left(\frac{1}{7200} \right)^{1.5} + \left(\frac{1}{4780} \right)^{1.5} + \left(\frac{1}{9300} \right)^{1.5} + \left(\frac{1}{5850} \right)^{1.5} \right]^{-1/1.5}$$

$$\theta = 1842.7$$

$$MTTF = \theta \, \Gamma(1 + 1/\beta)$$

$$= 1842.7 \, \Gamma\left(1 + \frac{1}{1.5}\right)$$

$$MTTF = 1842.7 \, \Gamma\left(\frac{5}{2}\right)$$

$$\because R(t) = e^{-\left(\frac{t}{\theta}\right)^{\beta}}$$

$$t_{med} = \theta \, (-\ln R)^{1/\beta}$$

$$t_{med} = 1842.7 \, (-\ln 0.5)^{1/1.5}$$

$$t_{med} = 1443.2$$

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Lecture 11

H.W

① For asystem having a weibull distribution with shape parameter of 1.4 and a scale parameter of 550 days, find the following:

1- R(100) days   2- MTTF

3- design life for reliability 0.90

4- medain time to failure

② A power supply مجهز متررة consists of three rectifiers in series. Each rectifier has a Weibull failure distribution with $\beta = 2.1$. They have different characteris ~~lifett~~ lifetimes given by 12000 hr, 18000 and 21500 hr

Find

① MTTF ② the design life of power supply corresponding to a reliability of 0.90