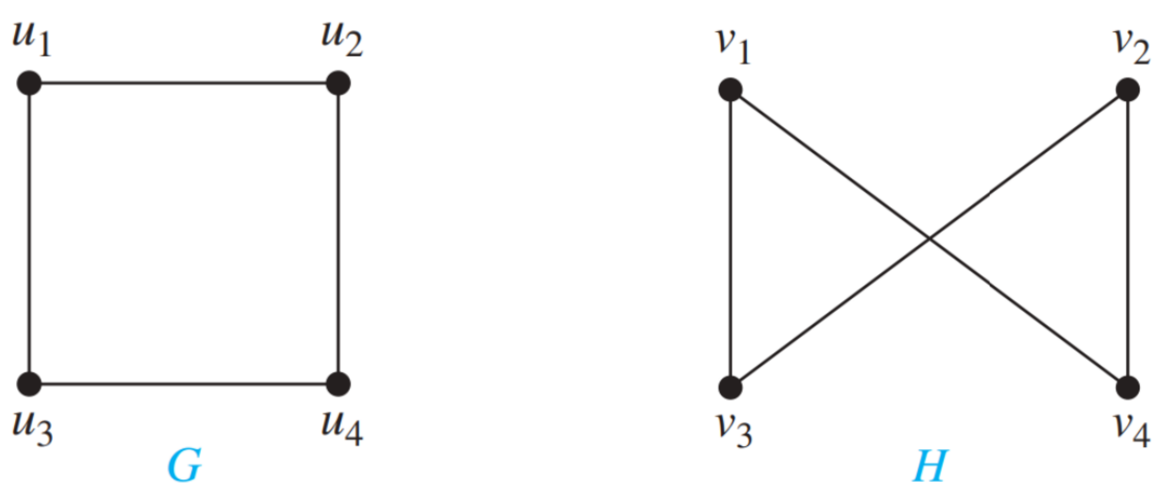
2- Isomorphism of Graphs

- The word isomorphism comes from the Greek roots **isos** for "equal" and **morphe** for "form."
- Graphs with the same structure.
- When two simple graphs are isomorphic, there is a one-to-one correspondence between vertices of the two graphs that preserves the adjacency relationship.

2.1 Definition

The simple graphs $G_1 = (V1, E1)$ and $G2 = (V2, E2)$ are isomorphic if there exists a one-to-one and onto function f from $V1$ to $V2$ with the property that a and b are adjacent in G_1 if and only if $f(a)$ and $f(b)$ are adjacent in G_2, for all a and b in V_1. Such a function f is called an isomorphism. Two simple graphs that are not isomorphic are called nonisomorphic.

Example 8. Show that the graphs G = (V,E) and H = (W, F) are isomorphic.



Solution.

The function f with $f(u1) = v1$, $f(u2) = v4$, $f(u3) = v3$, and $f(u4) = v2$ is a one-to-one correspondence between V and W.

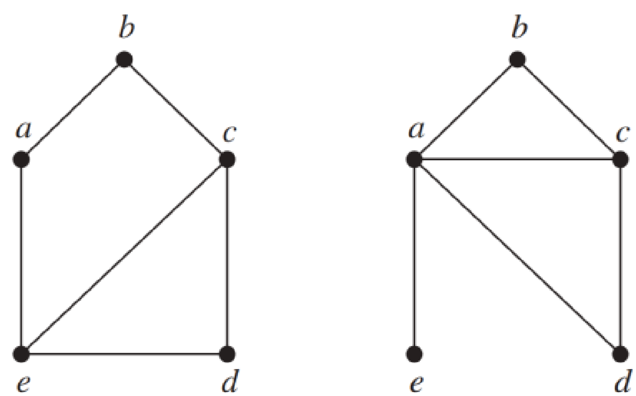
$$\begin{array}{c|cccc} G & u_1 & u_2 & u_3 & u_4 \\ \hline u_1 & 0 & 1 & 1 & 0 \\ u_2 & 1 & 0 & 0 & 1 \\ u_3 & 1 & 0 & 0 & 1 \\ u_4 & 0 & 1 & 1 & 0 \end{array} = \begin{array}{c|cccc} H & v_1 & v_4 & v_3 & v_2 \\ \hline v_1 & 0 & 1 & 1 & 0 \\ v_4 & 1 & 0 & 0 & 1 \\ v_3 & 1 & 0 & 0 & 1 \\ v_2 & 0 & 1 & 1 & 0 \end{array}$$

2.2 Graph Invariant

A property preserved by isomorphism of graphs is called a graph invariant.

1. Same number of vertices,
2. Same number of edges,
3. Same degrees of the vertices,
4. Adjacency test, …

Example 9. Determine whether the graphs $G = (V, E)$ and $H = (W, F)$ are isomorphic or not.

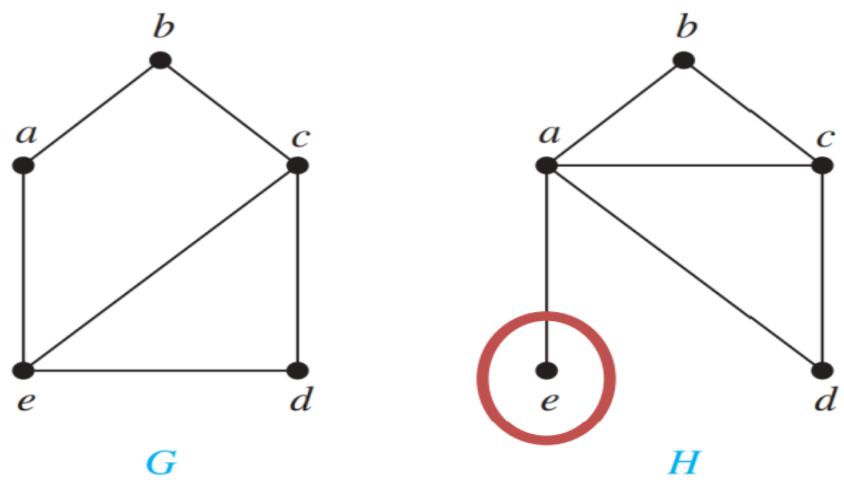


G H

Solution.

1. Same number of vertices (Yes)
2. Same number of edges (Yes)
3. Same degrees of the vertices (No) [G = {2, 2, 3,2,3}, H = {3, 2, 3, 2, 1}

For instance, ***H*** has a vertex of degree one, namely, e, whereas **G** has no vertices of degree one. It follows that **G** and **H** are **not isomorphic**.


b
a
c
e
d
G
b
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H