Question of Examination

Q/ If we have a system contain two modules having a CFR related in redundance and these modules are independent identical Derive :-

① the MTTF of the system

② Compute the reliability of the system at 720 hr if $\lambda(t) = 0.00034$

Sol:

∴ The modulals are iid and the system is redundant

$$R_s(t) = 2e^{-\lambda t} - e^{-2\lambda t}$$

$$MTTF_s = \int_0^\infty R_s(t)\, dt$$

$$= \int_0^\infty (2e^{-\lambda t} - e^{-2\lambda t})\, dt$$

$$= \int_0^\infty -\lambda e^{-\lambda t}\, dt + \frac{1}{2\lambda}\int_0^\infty 2\lambda e^{-\lambda t}\, dt$$

$$= \frac{-2}{\lambda}$$

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$$-\frac{2}{\lambda}\left[e^{-\lambda t}\right]_0^\infty + \frac{1}{2\lambda}\left[e^{-2\lambda t}\right]_0^\infty$$

$$\Rightarrow MTTF_s = \left[\frac{2}{\lambda} - \frac{1}{2\lambda}\right]$$

$$R_s(t) = 2e^{-0.00034 \times 720} - e^{-2(0.00034)(720)}$$

Q2/ If $R(t) = \sqrt{0.001t}$ $\quad t \geq 0$

then compute (1) p.d.f

(2) median time to failure

(3) hazard function

Sol:

$$\therefore R(t) = \sqrt{0.001t}$$

(1) $$f(t) = \frac{-dR(t)}{dt} \qquad -\frac{1}{2}$$

$$= -\frac{0.001}{2}(0.001t)$$

$$f(t) = -\frac{0.001}{2\sqrt{0.001t}}$$

(2) median time to failure

$$R(t) = (0.001t)^{1/2}$$

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$$0.5 = (0.001t)^{1/2}$$

$$\sqrt{0.5} = 0.001\,t$$

$$\Rightarrow t_{med} = \frac{\sqrt{0.5}}{0.001}$$

(3) $$\lambda(t) = \frac{f(t)}{R(t)}$$

$$= \frac{-0.001/2\sqrt{0.001t}}{\sqrt{0.001t}}$$

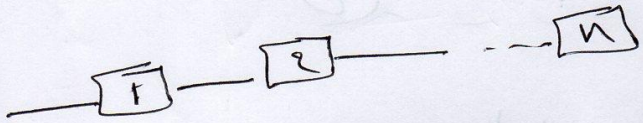
$$\lambda(t) = \frac{-0.001}{2}$$

Lecture 13

4 Reliability of systems

4.1 Serial configuration

Component with a serial system, in series all component must function for the system to function.



let E_1 : The event of component 1 does not fail

E_2 : The event of component 2 does not fail

$P(E_1) = R_1$, $P(E_2) = R_2$

$$R_s = P_r(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$$
$$= R_1 \cdot R_2 \quad -- (1)$$

assuming that 2 components are independent

Generalizing to n mutually independent components in series

$$R_s(t) = R_1(t) * R_2(t) * \cdots R_n(t) \leq \min\{R_1(t), R_2(t), \ldots R_n(t)\}$$

If each component has a CFR (exponential distribution) of λ_i, then the system reliability

$$R_s(t) = \prod_{i=1}^{n} R_i(t) \quad \text{---} (2)$$

$$= \prod_{i=1}^{n} \exp(-\lambda_i t)$$

$$= \exp\left(\sum_{i=1}^{n} -\lambda_i t\right)$$

$$\Rightarrow R_s(t) = \exp(-\lambda_s t) \quad \text{---} (3)$$

where $\lambda_s = \sum_{i=1}^{n} \lambda_i$

If component failure are governed by Weibull distribution