Lecture 2

ex! Given the following probability density function for the random variable compressor. what is the reliability for 100-hr operating life ~~and~~ and prob! of failure?

$$f(t) = \begin{cases} \dfrac{0.001}{(0.001t+1)^2} & t \geq 0 \\ 0 & o.w \end{cases}$$

Sol!

$$R(t) = \int_t^\infty f(s)\,ds$$

$$= \int_t^\infty \frac{0.001}{(0.001s+1)^2}\,ds$$

$$\Rightarrow R(t) = \int_t^\infty 0.001\,(0.001s+1)^{-2}\,ds$$

$$= \frac{-1}{(0.001s+1)}\Big|_t^\infty$$

$$\Rightarrow R(t) = 0 + \frac{1}{0.001t+1}$$

$$F(t) = 1 - R(t)$$

$$= 1 - \frac{1}{0.001t+1} \Rightarrow F(t) = \frac{0.001t}{0.001t+1}$$

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$$R(100) = \frac{1}{0.001(100)+1} = \frac{1}{0.1+1} = 0.909$$

1.3 A design life is defined to be the time to failure t_R that correspond to a specified reliability R. That is $R(t_R) = R$

To find the design life to example

① if $R(t) = 0.95$

$$R(t) = \frac{1}{0.001t+1} \quad 0.95$$

$$0.95(0.001t+1) = 1$$

$$0.95 * 0.001t = 1 - 0.95$$

$$0.00095\,t_r = 0.05$$

$$t_r = \frac{0.05}{0.00095}$$

$$\Rightarrow t_r = 52.631 \approx 52.6 \text{ hr}$$

To find the prob'. of a failure occuring within some interval of time $[a,b]$ may be found using any of the three prob'. function

$$P_r(a \le T \le b) = F(b) - F(a)$$

$$= R(a) - R(b)$$

$$= \int_a^b f(t)\,dt$$

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ex: From example (1) To find

$Pr(a \le T \le b) = Pr(10 \le T \le 100)$

$= R(10) - R(100)$ ——— (6)

$= \frac{1}{0.001(10)+1} - \frac{1}{0.001 \times 100 + 1}$

$= \frac{1}{0.01+1} - \frac{1}{0.1+1} = 0.081$

1.4/ Mean time to failure معدل العمر الزمني للعمل

The mean time to failure (MTTF) is defined by (متوقع)

$$MTTF = E(t) = \int_0^\infty t\, f(t)\, dt$$

From eq. $f(t) = -\frac{dR(t)}{dt}$

$$\Rightarrow E(t) = \int_0^\infty t \left(-\frac{dR(t)}{dt}\right) dt$$

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$u = t \quad ; \quad dv = -\frac{dR(t)}{dt}$

$du = dt \quad ; \quad v = -R(t)$

$$\Rightarrow E(t) = -tR(t)\Big|_0^\infty + \int_0^\infty R(t)\, dt$$

$$= 0 + \int_0^\infty R(t)\, dt$$

$$\therefore E(t) = \int_0^\infty R(t)\, dt \quad ——— (7)$$

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(2)

ex! Consider the probability density function

$$f(t) = \begin{cases} 0.002\, e^{-0.002t} & t \geq 0 \\ 0 & o.w \end{cases}$$

with t in hours. Then compute R(t) & MTTF

$$R(t) = \int_t^{\infty} -0.002\, e^{-0.002s}\, ds$$

$$= e^{-0.002s} \Big|_t^{\infty}$$

$$= 0 + e^{-0.002t}$$

$$\Rightarrow R(t) = e^{-0.002t}$$

(2) $$MTTF = \int_0^{\infty} R(t)\, dt$$

$$= \int_0^{\infty} e^{-0.002t}\, dt$$

$$= \frac{-1}{0.002} \int_0^{\infty} -0.002\, e^{-0.002t}\, dt$$

$$= \frac{-1}{0.002} \left[e^{-0.002t} \right]_0^{\infty}$$

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$$E(t) = \frac{e^{-0.002}}{-0.002}\Big|_{0}^{\infty}$$

$$E(t) = \frac{1}{0.002}$$

$$E(t) = 500 \text{ hr}$$

H.W

① The time-to failure density function for system is

$$f(t) = 0.01 \qquad 0 \le t \le 100 \text{ days}$$

Find:

① R(t)
② hazard function
③ The mean time to failure

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