1- Introduction to Propositional Logic

The simplest variant of formal logic is propositional logic. Its basic object is a simple, declarative sentence, called a proposition. Propositional logic is concerned with combining sentences, such as "The world is round" and "Columbus was right" to form "If the world is round, then Columbus was right."

A proposition is something that is either true or false; it is not both. (The cover of this book is pink) is a proposition. "***Napoleon spent at least one day of his life in Paris***" and "***Either the butler did it with a bottle or the colonel did it with a lead pipe***" are also propositions. On the other hand, "Justice," "The Queen's birthday," "Whoever is the stronger," and "Why is the world almost round?" are neither true nor false and, therefore, are not propositions.

In formal notation, the letters p, q, r, and s (plus those letters subscripted with natural numbers, such as p_1, q_2, and r_{127}) are used to stand for, or to denote, propositions. Such a variable is called a proposition letter. We consider **proposition letters** to be essentially the same as Boolean (logical) variables in a programming language. **T** and **F** are **propositional constants-that** is, propositions with fixed truth values of **TRUE** and **FALSE**, respectively.

Propositional logic is concerned with certain ways in which simple sentences can be combined into more complex sentences. Several standard operations are used on propositions to form other propositions. Such an operation is called a propositional connective. The common propositional connectives are shown in **Table 1.1.**

Proposition is the basic building block of logic. It is defined as a declarative sentence that is either **True** or **False**, but not both. The Truth Value of a proposition is **True** (denoted as **T**) if it is a true statement, and **False** (denoted as **F**) if it is a false statement.

Table 1.1: Logic symbols (Propositional Connectives).

Connective	Symbol Name	Sample Use	Common Translation
$\neg$	Not	$\neg$ p	" Not p"
$\wedge$	And	$q \wedge p$	"p and q"
$\vee$	Or	$q \vee p$	"p or q (or both)"
$\rightarrow$	Implies	$q \rightarrow p$	"If p, then q" or "p implies q"
$\leftrightarrow$	Equivalent	$q \leftrightarrow p$	"p if and only if q," or "p is equivalent to q"

Example 1. Let p denote "Henry eats burger" and q denote "Catherine eats pizza."

a) The proposition $\neg \boldsymbol{p}$ is read "Henry does not eat burger."
b) The proposition $\boldsymbol{p} \wedge \boldsymbol{q}$ is read "Henry eats burger, and Catherine eats pizza."
c) The proposition $\boldsymbol{p} \vee \boldsymbol{q}$ is read "If Henry eats burger, then Catherine eats pizza."
d) The proposition $\boldsymbol{p} \rightarrow \boldsymbol{q}$ is read "Henry eats burger if and only if Catherine eats pizza."
e) The proposition $\neg(\boldsymbol{p}) \wedge \neg(\boldsymbol{q})$ is read "Henry does not eat burger, or Catherine does not eat pizza."
f) The proposition $\boldsymbol{p} \leftrightarrow (\neg \boldsymbol{q})$ is read "Henry eats burger if and only if Catherine does not eat pizza."

Example 2. Let ***p*** denote "Henry eats burger," ***q*** denote "Catherine eats pizza," and ***r*** denote "I'll eat my hat."

a) Write a proposition that reads "If Henry eats burger but Catherine does not eat pizza, then I'll eat my hat."
b) Write a proposition that reads "Either Henry eats burger or Catherine eats pizza, but not both.

Solution.

a) $(p \wedge \neg q) \rightarrow r$. Since (***and)*** and but usually both get translated as $\wedge$, the difference between the two English words is usually an issue not of what is the case but, rather, of what we would have expected to be the case.

b) $(p \vee q) \wedge \neg(p \wedge q)$. This proposition is "logically equivalent to" the proposition in Example 1 (f), meaning that $p \leftrightarrow (\neg q)$ is an equally good answer. We shall discuss logical equivalence in the next section.

Table 1.2 is read as follows: For any proposition ***p***, if ***p*** is ***T***, then $\neg p$ is ***F***, and if ***p*** is ***F***, then $\neg p$ is ***T***. This assignment of truth values agrees with the common usage of the word ***not***. Truth tables for the other propositional connectives are shown in **Table 1.3**.

Table 1.2: Truth Table for Negation.

Truth table for $\neg$	
p	$\neg p$
T	F
F	T

Table 1.3. Truth Tables for Logical Connectives.

Truth Table for $\wedge$			Truth Table for $\vee$		
p	q	$p \wedge q$	p	q	$p \vee q$
T	T	T	T	T	T
T	F	F	T	F	T
F	T	F	F	T	T
F	F	F	F	F	F
Truth Table for $\rightarrow$			Truth Table for $\leftrightarrow$		
p	q	$p \rightarrow q$	p	q	$p \leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	T	F
F	F	T	F	F	T