Lecture 4

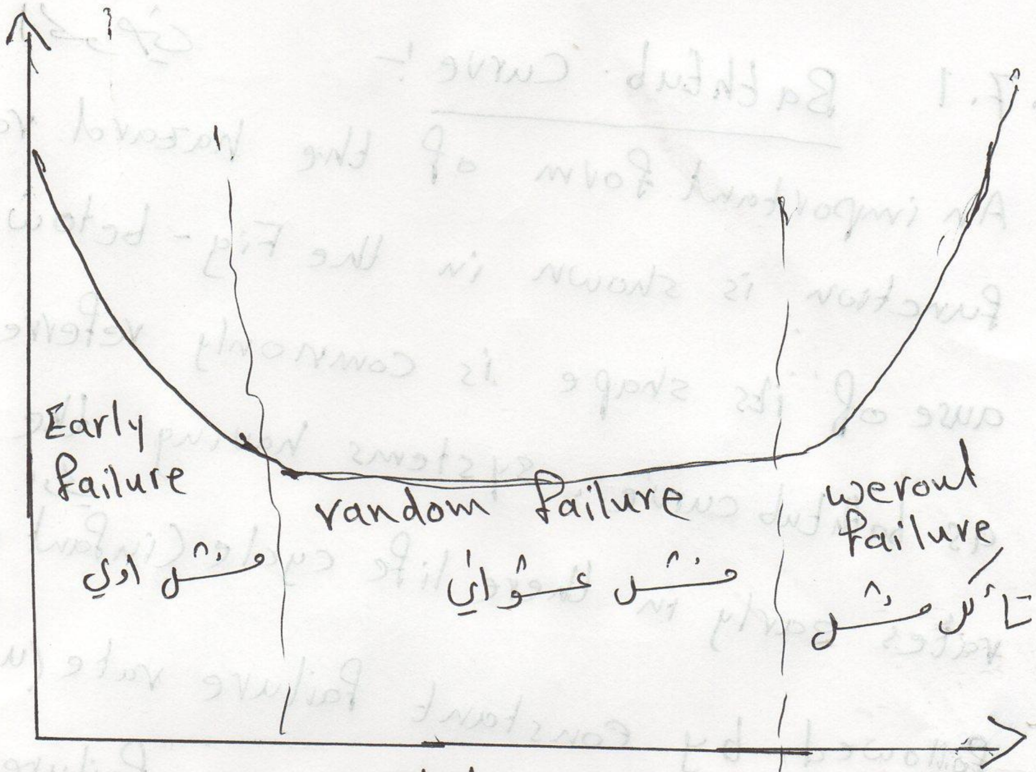
المنحنى الحوضي

1.7.1 Bathtub curve :-

An important form of the hazard rate function is shown in the Fig - below. Bec-ause of its shape is commonly referred to as bathtub curve. systems having the hazard rates early in there life cycle (infant mortality) فناء اولي followed by constant failure rate (useful life) followed by increasing failure rate (wearout) تآكل. This curve may be obtained as composite of several failure distribution.

ان صيغة داله المخاطرة موضحة بالشكل ادناه ليدعى بالشكل
الحوضي نسبة الى شكله حيث ان دالة المخاطرة قد تكون في بداية
العمر الزمني يطلق عليه (الفناء الاولي) تتبع بان تكون دالة المخاطرة
ثابتة يطلق عليها (العمر الانتاجي) وتتبع اخيرا بدالة مخاطرة متزايدة

مع الزمن (التآكل).. هذا الشكل مركب من عدة توزيعات
فشل



The battube curve

1.8 Conditional reliability المعولية الشرطية

A conditional reliability is useful in describing the reliability of a Component or system following a burn-in (الاحتراق) period (فترة). we define conditional reliability as the reliability of a system given that it has operated for time T_0

$$R(t \mid T_0) = P_r(T > T_0 + t \mid T > T_0)$$

$$= \frac{P_r(T > T_0 + t)}{P_r(T > T_0)} = \frac{R(T_0 + t)}{R(T_0)}$$

$$\Longrightarrow R(t \mid T_0) = \frac{\exp\left(-\int_0^{T_0+t} \lambda(t')\,dt'\right)}{\exp\left(-\int_0^{T_0} \lambda(t')\,dt'\right)}$$

$$\therefore \; R(t \mid T_0) = \exp\left(-\int_{T_0}^{t+T_0} \lambda(t')\,dt'\right)$$

<u>ex/.</u> If $\lambda(t) = \frac{0.5}{1000}\left(\frac{t}{1000}\right)^{-0.5}$ t in years, $t > 0$

which is decreasing failure rate
أي له دالة خطورة متناقصة

Find: ① $R(t)$ ② design life

<u>sol/.</u> $R(t) = \exp\left(-\int_0^t \lambda(t')\,dt'\right)$

$$R(t) = \exp\left(-\int_0^t \frac{0.5}{1000}\left(\frac{t'}{1000}\right)^{-0.5} dt'\right)$$

$$= \exp\left(\frac{-\cancel{0.5}}{(\cancel{0.5})}\left(\frac{t'}{1000}\right)^{0.5}\Bigg|_0^t\right)$$

$$\boxed{R(t) = \exp\left(-\left(\frac{t}{1000}\right)^{0.5} - 0\right)}$$

② design life if $R(t) = 0.90$

$$0.90 = \exp\left(-\left(\frac{t}{1000}\right)^{0.5}\right)$$

نأخذ ln للطرفين لإيجاد قيمة t_R

$$-\ln 0.90 = \left(\frac{t}{1000}\right)^{0.5}$$

نربع الطرفين لإيجاد قيمة t

$$1000(-\ln 0.9) = t$$

$$\Longrightarrow t_R = 11.1 \text{ Year}$$

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ex!2

The probability density function for the time to failure in years of drive train on Regional ~~tran~~ transit Authority bus is given by

$$f(t) = 0.2 - 0.02\, t \qquad 0 \leq t \leq 10$$

اذا كانت دالة الكثافة الاحتمالية لزمن الفشل مقاسا بالسنوات من نظام الدفع على حافلة هيئة النقل الاقليمية

comput:-

① R(t) ② show that hazard rate fn. is increasing

② MTTF معدل العمر الزمني للفشل

③ median time to failure

Sol:

① $R(t) = P_r(T \geq t)$

$$= \int_t^{10} (0.2 - 0.02t')\,dt'$$

$$= \left[0.2t' - 0.02\frac{t'^2}{2}\right]_t^{10}$$

$$R(t) = 0.2(10) - 0.02\frac{(\cancel{100})^{50}}{\cancel{2}} - \left(0.2t - 0.02\frac{t^2}{2}\right)$$

$$= 2 - 1 - 0.2t + 0.02\frac{t^2}{2}$$

$$R(t) = 1 - 0.2t + 0.02\frac{t^2}{2}$$

$$R(t) = 1 - 0.2t + 0.01t^2$$

② $$MTTF = \int_0^{10} R(t)\,dt$$

$$= \int_0^{10} (1 - 0.2t + 0.01t^2)\,dt$$

$$= t - 0.2\frac{t^2}{2} + 0.01\frac{t^3}{3}\Big|_0^{10}$$

$$MTTF = 10 - 0.2\frac{(100)}{2} + \frac{0.01}{3}(1000) - 0$$

$$MTTF = \cancel{10} - \cancel{10} + 3.333$$

$$MTTF = 3.333$$

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3 median time to Failure

عندما يطلب إيجاد الوسيط نعوض بدل $R(t)$ بـ 0.5

$$\because R(t) = 1 - 0.2t + 0.01\,t^2$$

نعوض $R(t)=0.5$

$$0.5 = 1 - 0.2t + 0.01\,t^2$$

$$-0.5 = -0.2t + 0.01\,t^2 \qquad \times -1$$

$$0.5 = 0.2t + 0.01\,t^2$$

$$= -0.01t^2 + 0.2t - 0.5 = 0$$

تكملة الحل واجب