1- What 's a proposition?

A proposition is a declarative statement that's either **TURE** or **FALSE** (but **not both**). However, here some examples of propositions and not propositions statements shown in the below table: -

Table 3.1: Some examples of Propositional/Not Propositional statements

Propositions	Not propositions
3 + 2 =32	3 + 2
UoMCS104 is Ahmed's favorite class.	Bring me coffee!
There is other life in the universe.	Do you like Cake?
1 + 0 =1	x + 1= 2

2- Tautology

A formula of propositional logic is a tautology **if the formula itself is always true**, regardless of Which valuation is used for propositional variables.

Example 1. Construct a truth table to show that $((p \land q) \rightarrow p)$ is a tautology.

Solution.

The truth table for $((p \land q) \rightarrow p)$

p	q	$p \land q$	$((p \land q) \rightarrow p)$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Science all entries under $((p \land q) \rightarrow p)$ are T, the formula is a **tautology.**

Example 2. Construct a truth table to show that $(p \rightarrow q) \leftrightarrow (\neg p \lor q)$ is tautology.

Solution.

p	q	$p \rightarrow q$	$\neg p$	$\neg p \lor q$	$(p \rightarrow q) \leftrightarrow (\neg p \lor q)$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Science all entries under $(p \rightarrow q) \leftrightarrow (\neg p \lor q)$ are T, the formula is a **tautology.**

Example 3. Construct a truth table to show that $(\neg p \rightarrow \neg q) \rightarrow \neg p$ is tautology.

Solution.

p	q	$\neg p$	$\neg q$	$\neg p \rightarrow \neg q$	$(\neg p \rightarrow \neg q) \rightarrow \neg p$
T	T	F	F	T	F
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

Since the last row has rows with ***false***, the formula is **not tautology**.

3- Predicates

A property or relationship between objects is called a **predicate**. A description of a predicate in logic is called a **formula**. **Figure 3.1** illustrate the formula: -

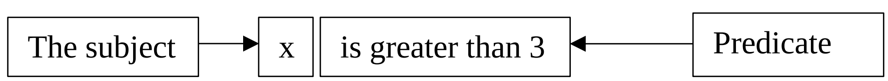

Figure 3.1. Predicate structure

We can denote the statement "x is $greater$ $than$ 3" by $\boldsymbol{P}$ $\boldsymbol{x}$, where $\boldsymbol{P}$ denotes the predicate "is $greater$ $than$ 3" and $\boldsymbol{x}$ is the variable.

The statement $\boldsymbol{P}(\boldsymbol{x})$ is also said to be the value of the **propositional function** $\boldsymbol{P}$ at $\boldsymbol{x}$. Once a value has been assigned to the variable $\boldsymbol{x}$, the statement $\boldsymbol{P}(\boldsymbol{x})$ becomes a proposition and has a truth value.

Example 4. Let P(x) denote the statement "x > 3". What are the truth values of P(4) and P(2)?

Solution.

We obtain the statement P (4) by setting x = 4 in the statement "x > 3." Hence, P (4), which is the statement "4 > 3," is **true**.

However, P (2), which is the statement "2 > 3," is **false**.

Example 5. Let Q (x, y) denote the statement "x = y + 3" what are the truth values of the propositions Q (1,2) and Q (3,0)?

Solution.

Q (1,2) ⟶ 1 = 1 + 3 ⟶ 1 = 4 **False** (F)

Q (3, 0) ⟶ 3 = 0 + 3 ⟶ 3 = 3 **True** (T)

Example 6.

1- Let P(x) denote the statement "x <= 4". What are the truth values?

a) P(0)
b) P(4)
c) P(6)

Solution.

a) T
b) T
c) F

2- Let P(x) be the statement "the word x contains the letter (a). What are the truth values?

a) P(orange)
b) P(Lemon)
c) P(true)
d) P(false)

Solution.

a) T
b) F
c) F
d) T