Lecture 6:

Chapter 2

Constant failure rate

2.1 Introduction

There are several probability models useful in describing a failure process. These models are based upon exponential, Weibull, normal and log-normal prob'. dist'.

هنالك العديد من النماذج الاحتمالية المهمة في وصف تحليلات الفشل. هذه النماذج مبنية على توزيعات الأسي وبيل و الطبيعي و اللوغاريتم الطبيعي.

There are often refered to as theortical dist'. since they are derived mathematically and not empirically. The exponential dist'. is one of the most important reliability dist'.

عادة ما يطلق على هذه التوزيعات بالتوزيعات الرياضية لانها تشتق رياضيا وليست تجريبيا. ويعتبر التوزيع الأسي اهم هذه التوزيعات في الموثوقية

Many systems exhibit constant failure rates, and exponenbrial is in many respects the ~~exponentral~~ simplest distn. to analyze. If the failure rates of all failure modes of components are constant and independent, then the overall rates of the component is also constant

العديد من الانظمة تسلك سلوك معدلات فشل ثابتة ويعتبر التوزيع الآسي او البسيط هو التوزيع

2.2 The exponential distribution.

One of the most common failure distribution in reliability engineering, is the exponential distn or constant failure rate (CFR). Failure due to completely random or chance ~~#~~ event

واحد من اهم توزيعات الفشل في الهندسية الموثوقية هو التوزيع الآسي او معدل الفشل الثابت (CFR) will follow distn.

يعزو المثل الحاصل به الى العشوائية .

It should domainate during the useful life of a system or component.

It is also one of eaist distribution to analyze statistically.

صوابط التوزيعات في التحليل الاحصائي

For example good parameter estimateros and exact confidence interval can be easly computed under different testing conditions

فعلى سبيل المثال نستطيع استخدام تقدير طريقة التوزيع بشكل جيد وايجاد حدود الثقة لها بشكل كامل

If the r.v T represent failure time having prob. ~~distn~~ density function

$$f(t) = \lambda e^{-\lambda t} \qquad t \geq 0, \ \lambda > 0$$

$$= 0 \qquad \text{o.w}$$

37

$$F(t) = P(T \le t)$$

$$= -\int_0^t -\lambda e^{-\lambda t'} dt'$$

$$= -e^{-\lambda t'} \Big|_0^t$$

$$F(t) = -e^{-\lambda t} + e^0$$

$$F(t) = 1 - e^{-\lambda t}$$

$$R(t) = 1 - F(t)$$

$$= 1 - (1 - e^{-\lambda t})$$

$$= 1 - 1 + e^{-\lambda t}$$

$$\Rightarrow R(t) = e^{-\lambda t}$$

$$\lambda(t) = \frac{f(t)}{R(t)} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda$$

$$MTTF = \int_0^\infty R(t)\, dt$$

$$= \frac{-1}{\lambda} \int_0^\infty -\lambda e^{-\lambda t} dt = -\frac{1}{\lambda} e^{-\lambda t} \Big|_0^\infty$$

$$= 0 + \frac{1}{\lambda}$$

$$MTTF = \frac{1}{\lambda}$$

38/

$$\sigma^2 = \int_0^\infty (t-\mu)\,\lambda e^{-\lambda t}\,dt$$

$$= \int_0^\infty \left(t-\frac{1}{\lambda}\right)\lambda e^{-\lambda t}\,dt$$

$$= \int_0^\infty \left(t^2 - 2\frac{t}{\lambda} + \lambda^2\right)\lambda e^{-\lambda t}\,dt$$

$$= \int_0^\infty \lambda t^2 e^{-\lambda t}\,dt - \frac{2}{\lambda}\int t\lambda e^{-\lambda t}\,dt + \lambda^2 \int \lambda e^{-\lambda t}\,dt$$

$$\int_0^\infty t^2 \lambda e^{-\lambda t}\,dt$$

by using the integral by part

let $u = t^2$, $du = 2t\,dt$

$dv = \lambda e^{-\lambda t}$

$v = -e^{-\lambda t}$

$$-t^2 e^{-\lambda t}\Big|_0^\infty + 2\int_0^\infty t e^{-\lambda t}\,dt$$

$$0 + 2\int_0^\infty t e^{-\lambda t}\,dt$$

so $\int t e^{-\lambda t}\,dt$ by the part

$u = t$, $du = dt$

$dv = e^{-\lambda t}$

$v = \frac{-1}{\lambda} e^{-\lambda t}$

$$-\frac{1}{\lambda} t e^{-\lambda t}\Big|_0^\infty + 2\int_0^\infty \frac{1}{\lambda} e^{-\lambda t}\,dt$$

$$0 + \frac{2}{\lambda^2}\int_0^\infty -\lambda e^{-\lambda t}\,dt$$

$$= -\frac{2}{\lambda^2}\left[e^{-\lambda t} \right]_0^\infty$$

$$= -\frac{2}{\lambda^2}[0 - 1]$$

$$\int_0^\infty t^2 e^{-\lambda t} dt = \frac{2}{\lambda^2}$$

$$\Rightarrow \sigma^2 = \int_0^\infty t^2 \lambda e^{-\lambda t} - 2\int_0^\infty t e^{-\lambda t} dt + \frac{1}{\lambda^2}\int_0^\infty e^{-\lambda t} dt$$

$$\sigma^2 = \frac{2}{\lambda^2} - \frac{2}{\lambda^2} + \frac{1}{\lambda^2}$$

standard deviation (S.D) is $\frac{1}{\lambda} = MTTF$

This is an interesting result since it implies that the variability increases as the reliability (MTTF) increases

Note! MTTF is reciporcal of the failure rate

معدل العمر الزمني للعنصر هو مقلوب معدل العنصر