4- Quantification

Quantifiers are words that refer to quantities such as" **some**" or" **all**" and tell for how many elements a given predicate is true. **Figure 3.2** shows the types of quantifiers and **table 3.2** explains the **some** and **all**.

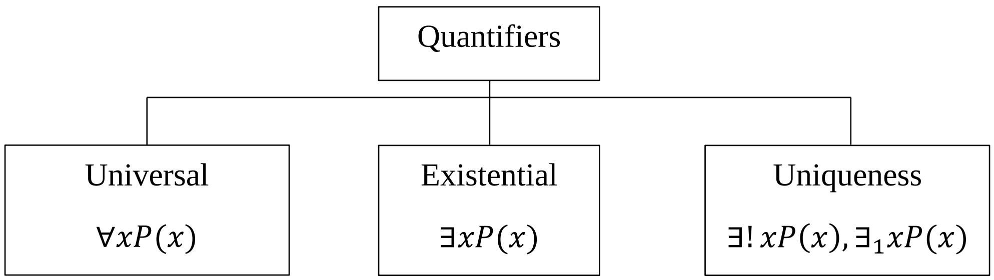

Figure 3.2. Expresses the extent to which a predicate is true over a range of elements.

Table 3.2. Quantifiers expression

Statement	When True?	When False?
$\forall x P(x)$	P(x) is true for every x.	There is an x for which P(x) is false
$\exists x P(x)$	There is an x for which P(x) is true	P(x) is false for every x.

Example 7. Let P(x) be the statement “x + 1 > x”. what is the truth value of the quantification $\forall x\ P(x)$, where the domain consists of all real numbers?

Solution.

Because P(x) is true for all real numbers x, the quantification $\forall x\ P(x)$ is **true**.

Example 8. Let Q(x) be the statement “x < 2” what is the truth value of the quantification $\forall x\ Q(x)$, where the domain consists of all real numbers?

Solution.

Q(x) is not true for all real number x, because, for instance, Q(3) is false. That is, x = 3 is a counter example for the statement $\forall x\ Q(x)$. Thus $\forall x\ Q(x)$ is **false**.

Example 9. Let P(x) denote the statement "$x > 3$". What is the truth value of the quantification $\exists x P(x)$, where the domain consists of all real numbers?

Solution.

Because "$x > 3$" is sometimes true (for instance, when $x = 4$) the existential quantification of P(x), which is $\exists x P(x)$, **true**.

Example 10. What is the truth value of $\exists x P(x)$, where P(x) is the statement "$x^2 > 10$" and the universe of discourse consists of the positive integers not exceeding 4?

Solution.

Because the domain is $\{1,2,3,4\}$, the proposition $\exists x P(x)$ is the same as the disjunction P(1) V P(2) V P(3) V P(4). Because P(4), which is the statement "$4^2 > 10$", is true, it follows that $\exists x P(x)$ is **true**.