1- Types of Quantification

Before we start explaining the types of quantification, we will illustrate all types of number sets, as shown in below **Figure 1**: -

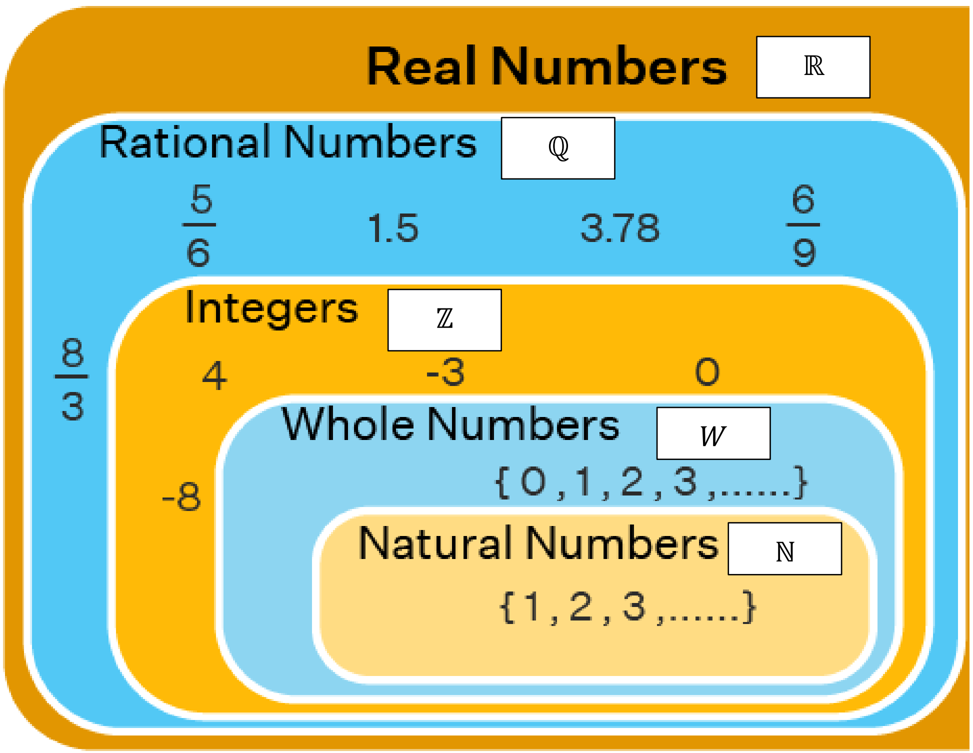


Figure 1. List of all kinds of numbers.

1.1 Restricted Quantifiers

These quantifiers can be further modified or restricted by **conditions**, leading to the concept of restricted quantifiers. We can limit the **Domain** of the quantifier by modifying the notation a bit.

Example 1. What is the truth value of the quantification of $\forall x < 0\ (x^2 > 0)$. Where the domain consists of all real numbers?

Solution.

The meaning of the above statement: the square root of a negative real number is positive.

1.2 Nested Quantifiers

Example 2. Let x and y be the real numbers and P(x, y) denotes " x + y = 0 ." Find the truth values of

a) $\forall x\, \forall y\, P(x, y)$
b) $\forall x\, \exists y\, P(x, y)$
c) $\exists x\, \forall y\, P(x, y)$
d) $\exists x\, \exists y\, P(x, y)$

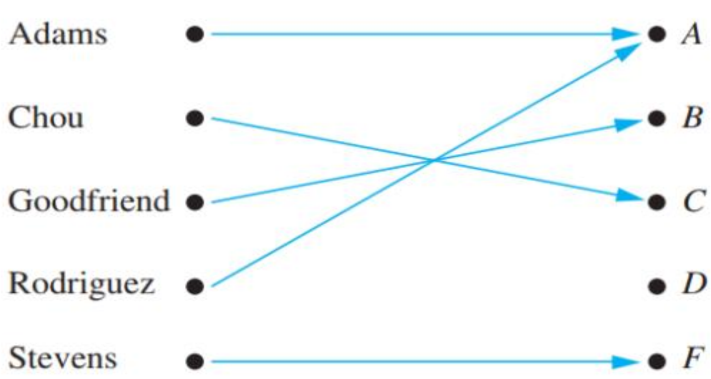
Solution.

Domain: All real numbers.

a) $\forall x\, \forall y\, P(x, y) \equiv \forall x\, \forall y\, (x + y = 0)$
   False and can't be true.
b) $\forall x\, \exists y\, P(x, y) \equiv \forall x\, \exists y\, (x + y = 0)$
   Ture
c) $\exists x\, \forall y\, P(x, y) \equiv \exists x\, \forall y\, (x + y = 0)$
   False
d) $\exists x\, \exists y\, P(x, y)$
   Ture.

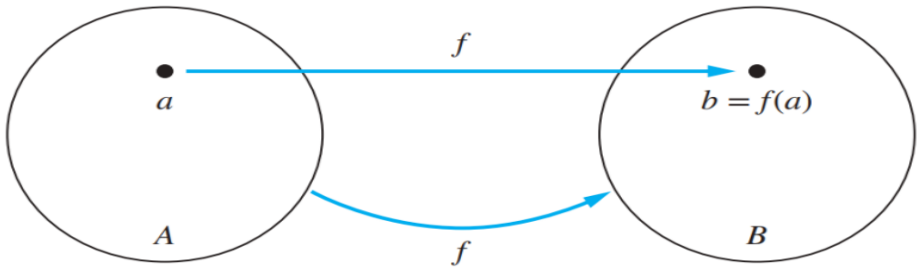
2- Functions

Functions are an important part of discrete mathematics. A function assigns exactly one element of a set to each element of the other set. Functions are the rules that assign one input to one output. The function can be represented as f: A ⇢ B. A is called the domain of the function and B is called the codomain function.



Assignment of grades in a discrete mathematics class.

The Function $f: A \to B$



The function f maps A to B.

The Function $f: A \to B$

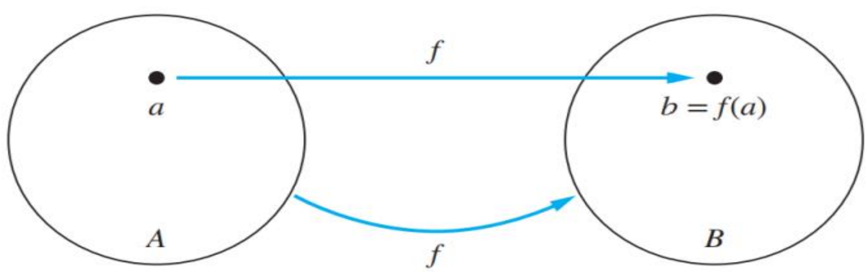
Domain: A

Co-Domain: B

$f(a) = b$

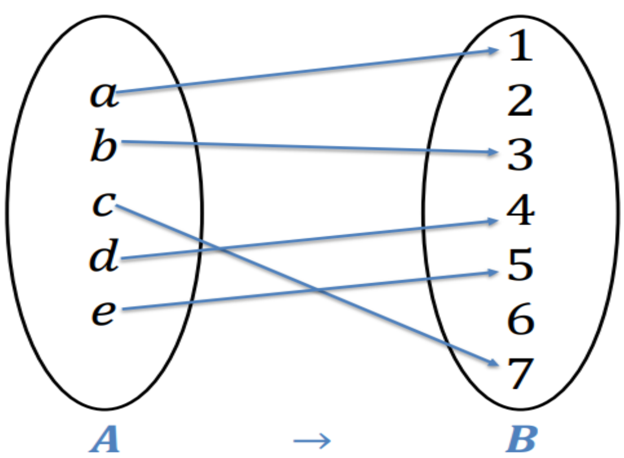
b is the ***image*** of a
a is a ***preimage*** of b

The **range**, or image, of f is the ***set of all images*** of elements of A.



The function f maps A to B.

The Function $f: A \to B$



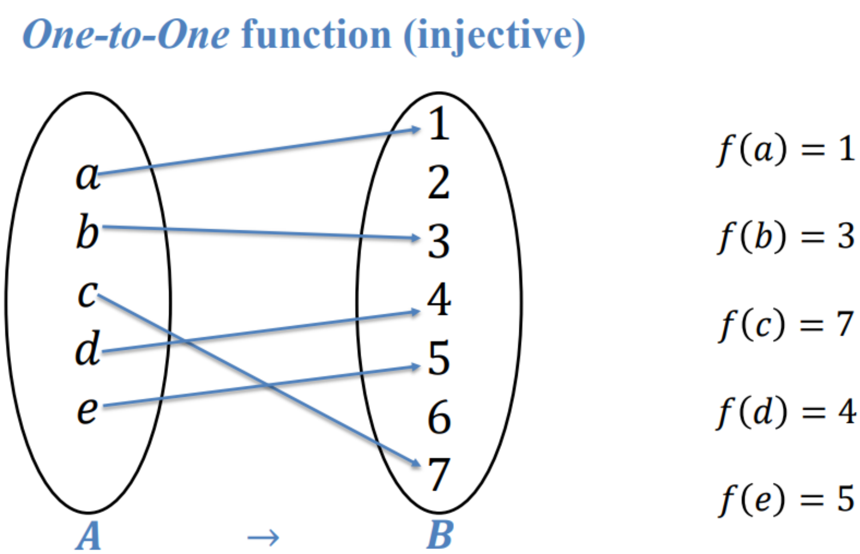
Domain = $\{a, b, c, d, e\}$

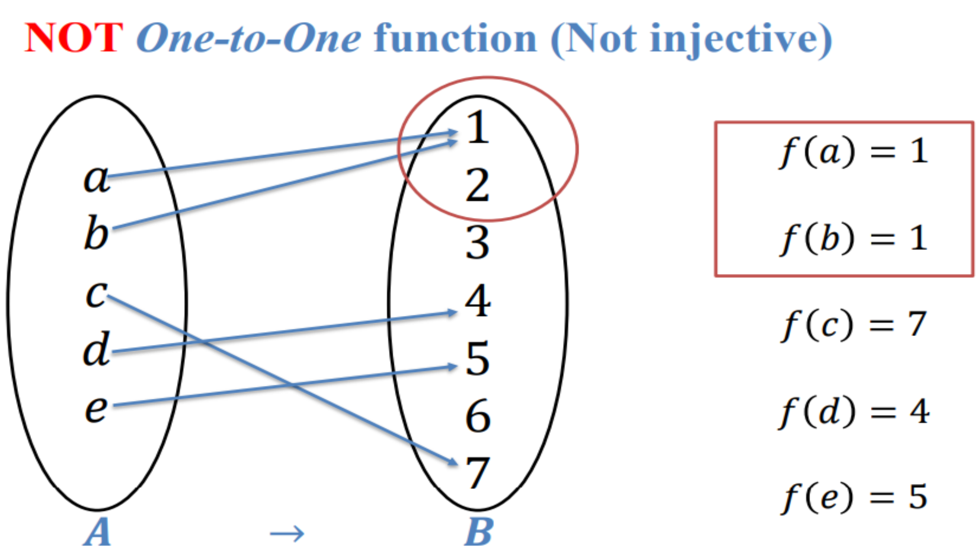
Co-Domain = $\{1,2,3,4,5,6,7\}$

Range = $\{1,3,4,5,7\}$

2.1 One-to-One function (injective)

A function f is said to be one-to-one, or injective, **if and only if** f(a) = f(b) implies that a = b for all a and b in the domain of f.

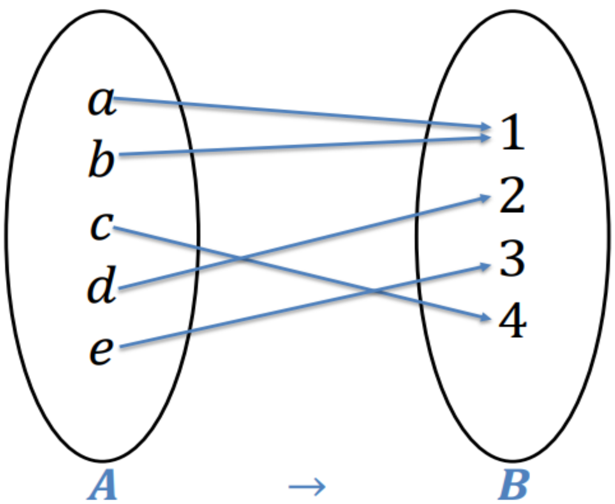




2.2 onto function (surjective)

A function f from A to B is called onto, or surjective, **if and only if** for every element b ∈ B there is an element a ∈ A with f(a) = b.

onto **function (surjective)**



$f(a) = 1$

$f(b) = 1$

$f(c) = 4$

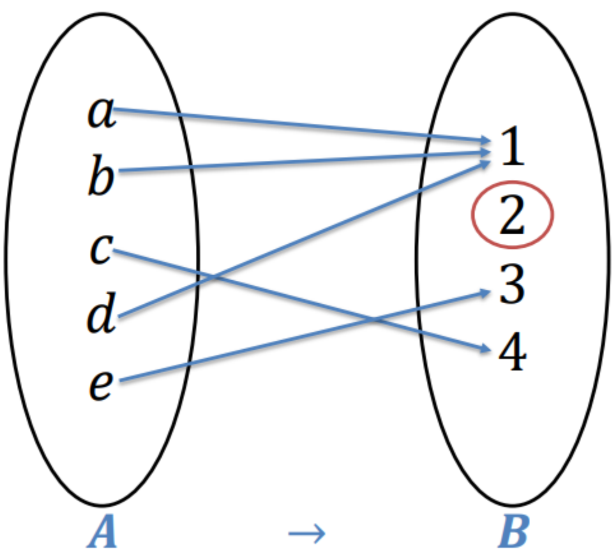
$f(d) = 2$

$f(e) = 3$

Co-Domain = {1,2,3,4}

Range = {1,2,3,4}

NOT *onto* **function (Not surjective)**



$f(a) = 1$

$f(b) = 1$

$f(c) = 4$

$f(d) = 1$

$f(e) = 3$

Co-Domain = {1,2,3,4}

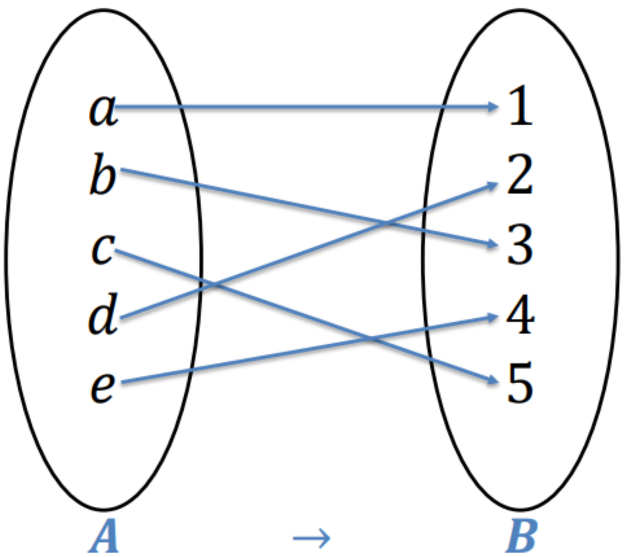
Range = {1,3,4}

2.3 One-to-one correspondence (bijection)

The function f is a one-to-one correspondence, or a bijection, if it is both **one-to-one** and **onto**.

One-to-one correspondence **(bijection)**

$|A| = |B|$



$f(a) = 1$

$f(b) = 3$

$f(c) = 5$

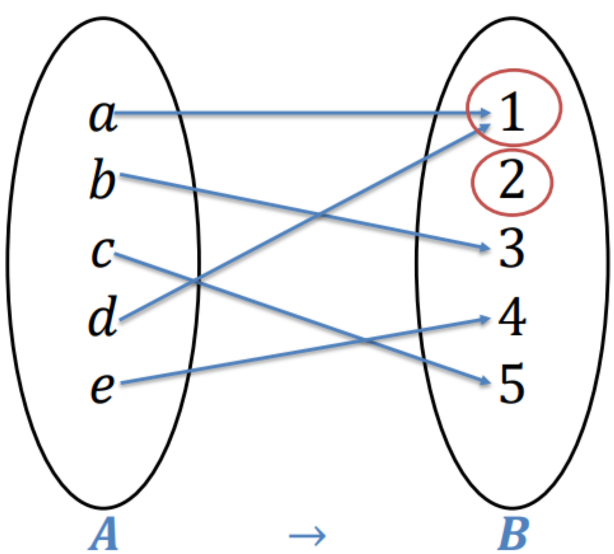
$f(d) = 2$

$f(e) = 4$

Co-Domain = {1,2,3,4,5}

Range = {1,2,3,4,5}

NOT ***One-to-one correspondence*** **(Not bijection)**



$f(a) = 1$

$f(b) = 3$ **NOT one-to-one**

$f(c) = 5$ **NOT onto**

$f(d) = 1$

$f(e) = 4$

Co-Domain = {1,2,3,4,5}

Range = {1,3,4,5}