(Lecture Seven)

• Methods for measuring population growth rate

Methods for measuring the population growth rate require the availability of two or more population censuses, through which a straight or curved line can be drawn on the basis of which estimates of the total population in the required period can be arrived at. To estimate the population number, there are several formulas:

1. Population growth based on the numerical formula (numerical sequence)

This method assumes that population growth occurs on the basis of a straight line, meaning that the increase is in the form of a fixed quantity, and that the required change in population size can be expressed through the difference between the two censuses. This is an assumption far from reality, but it is possible to use this method to estimate Population size for periods close to the census due to its simplicity. However, if it is used to estimate population size for years far from the census, it will lead to deviations in the estimates.

The formula for the numerical sequence is:

$$P_n = P_o + (n - 1)\, r$$

Where:

P_n: Population number in the last census

P_o: Population number in the previous census

n: The number of years between the two censuses, including the first census year

r: The basis of the numerical sequence (the constant increment)

Example:

The population of one country in 1999 was 17618500, and in 2009 the population was 21115320. The population in 2014 was estimated using the numerical sequence.

Sol.

$P_n = P_{2009}$ =21115320 , $P_o = P_{1999}$ =17618500 ,

The formula for the numerical sequence is:

$$P_{2009} = P_{1999} + (n-1)\ r$$

21115320 = 17618500 +(11-1) r

3496820 =10 r → r = 349682

Now, we use r to calculate P_{2014} as follows:

$$P_{2014} = P_{2009} + (6-1) * \mathrm{r}$$

P_{2014} = 21115320 + 5 * 349682

P_{2014} = 22863730

2. **Population growth based on the geometric formula (geometric sequence)**

In this method, we assume that the population change is in a compound manner, as the annual change during the first year (in the case of an increase) is added to the original, forming a first increase in the second year, and so on. Thus, the size of the population increases at the beginning of each period, causing the population increase in a gradual manner, and the geometric sequence formula It will be as follows:

$$P_n = P_o * r^{(n-1)}$$

Where:

P_n: Population number in the last census

P_o: Population number in the previous census

n: The number of years between the two censuses, including the first census year

r: The Annual rate of change .

The annual increase percentage is as follows:

$$z = (r - 1) * 1000$$

Note:

- In order to facilitate calculations, the logarithm of the geometric sequence formula is used, as follows:

$$\log P_n = \log P_o + (n - 1) \log r$$

- If it is required to find the annual rate of change, it can be found from the following formula:

$$\log r = \frac{\log P_n - \log P_o}{(n - 1)}$$

- If it is required to find the number of years in which the population doubles, it can be found according to the following formula

$$n = \frac{\log(P_n * r) - \log P_o}{\log r}$$

Example:

In the previous example, the country's population was estimated in 2014, according to the geometric progression.

Sol.

$P_n = P_{2009} = 21115320$, $P_o = P_{1999} = 17618500$

the annual rate of change r is:

$$\log r = \frac{\log P_n - \log P_o}{(n - 1)}$$

$$\log r = \frac{\log P_{2009} - \log P_{1999}}{(11-1)}$$

$$\log r = \frac{\log 21115320 - \log 17618500}{(11-1)}$$

لحساب r باستخدام الحاسبة يمكن من خلال الزر 10^x للناتج الذي يتم حسابه من الطرف الاخر من الصيغة

$\log r = 0.00786287 \rightarrow r = 1.0182698$

Now, we calculate P_{2014} by the formula :

$$\log P_n = \log P_o + (n-1)\log r$$

$$\log P_{2014} = \log P_{2009} + (6-1)\log 1.0182698$$

$$\log P_{2014} = \log 21115320 + (6-1)\log 1.0182698$$

$$\log P_{2014} = 7.324597667 + (5 * 0.007862863587)$$

$$\log P_{2014} = 7.363912032 \rightarrow P_{2014} = 23115965$$

Homework:

1. Calculate the annual increase percentage?
2. What is the population growth rate so that the population of this country will reach 30000000 in 2016?

3. Population growth based on the exponential formula

We mentioned that the geometric formula dictates that the growth rate represents an annual addition to the population each year, while the reality of life shows that the growth process is continuous and not a regular increase calculated each year separately. Therefore, the exponential formula came, which is characterized by the nature of continuous, complex, and uninterrupted changes, and therefore it is Most commonly used to estimate population growth.

The formula of the Lacy model is as follows:

$$P_n = P_o\, e^{nr}$$

Where

P_n: Population number in the last census

P_o: Population number in the previous census

n: The difference between the first census year and the second census year, excluding (باستثناء) the first census year

r: Annual increase rate or prevailing growth rate (population growth rate)

e: The basis of the natural logarithm .

Note:

1. $e = 2.71828$ and $\log e = 0.4343$

2. To facilitate calculations, we take the logarithm of both sides of the equation to become as follows:

$$\log P_n = \log P_o + n * r \log(e)$$

- If it is required to find the annual rate of change, it can be found from the following formula:

$$r = \frac{\log P_n - \log P_o}{n * \log(e)}$$

Example:

The population in one country in 2007 reached 12000497, assuming that the prevailing growth rate during the study period r =3.25% .

Using the exponential model, find:

1. Estimated population in 2017
2. What is the population growth rate required for the population to reach 30000000 in 2030?

Sol.

$P_{2007} = 12000497$, $r = 0.0325$, $P_{2030} = 30000000$

1. $\log P_{2017} = \log P_{2007} + n * r \ \log(e)$

$$\log P_{2017} = \log 12000497 + 10 * 0.0325 \ \log(2.71828)$$

$$\log P_{2017} = 7.079199233 + 10 * 0.0325 * 0.4343$$

$$\log P_{2017} = 7.079199233 + 0.141411475 = 7.220346733$$

$$P_{2017} = 16609124.21 \approx 16609124$$

2.

$$r = \frac{\log P_n - \log P_o}{n * \log(e)}$$

$$r = \frac{\log P_{2030} - \log P_{2007}}{23 * 0.4343}$$

$$r = \frac{\log 30000000 - \log 12000497}{23 * 0.4343} = 0.03983$$

The population growth rate required for the population to reach 30000000 in 2030 is 39.8% or $\approx 40\%$