1- Operations on Functions

Since functions and partial functions are special types of binary relations, all operations defined on binary relations can be applied to functions. The most interesting operations, however, are composition and inversion.

1.2 Composition Functions

The composition of a function is an operation where two functions say f and g generate a new function say h in such a way that h(x) = g(f(x)). It means here function g is applied to the function of x. So, basically, a function is applied to the result of another function.

1.2.1 Definition of Composition Functions

Let f: A → B and g: B → C be two functions. Then the composition of **f** and **g**, denoted by g ∘ f, is defined as the function g ∘ f: A → C given by g ∘ f (x) = g (f (x)), ∀ x ∈ A.

The below figure shows the representation of composite functions.

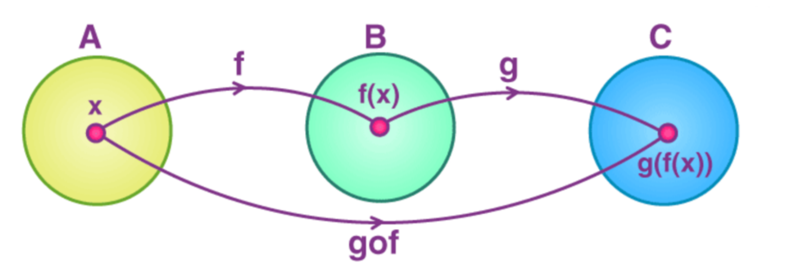

Figure 1. Shows the representation of composite functions.

Note: The order of function is an important thing while dealing with the composition of functions since (f ∘ g) (x) is not equal to (g ∘ f) (x).

Example 1. If f(x) = 2x +1 and g(x) = -x^2, then find (g∘f) (x) for x = 2.

Solution.

f(x) = 2x + 1

g(x) = -x^2

To find: g(f(x))

g(f(x)) = g(2x + 1) = -(2x + 1) 2

Now put x =2 to get;

g(f(2)) = -(2.2+1)2

= -(4+1)2

=-(5)2

=-25

Example 2. If there are three functions, such as f(x) = x, g(x) = 2x and h(x) = 3x. Then find the composition of these functions such as [f ∘ (g ∘ h)] (x) for x = -1.

Solution.

f(x) = x

g(x) = 2x

h(x) = 3x

To find: [f ∘ (g ∘ h)] (x)

[f ∘ (g ∘ h)] (x) = f ∘ (g(h(x)))
= f ∘ g(3x)

= f(2(3x))

= f(6x)

= 6x

If x = -1, then;

[f ∘ (g ∘ h)] (-1) = 6(-1) = -6

1.3 Inverses of Functions

An **inverse function** or an anti function is defined as a function, which can reverse into another function. In simple words, if any function "f" takes x to y then, the inverse of "f" will take y to x. If the function is denoted by 'f' or 'F', then the inverse function is denoted by f^{-1} or F^{-1}. One should not confuse (-1) with exponent or reciprocal here. **Note**, bijection functions are only functions inversed.

Example 3. If there is function: f(x) = 2x + 3, and x = 4. Find if the function is inversed or not.

Solution.

We have,

$f(4) = 2 \times 4 + 3$

$f(4) = 11$

Now, let's apply for reverse on 11.

$f^{-1}(11) = (11 – 3) / 2$

$f^{-1}(11) = 4$

Magically we get 4 again.

Therefore, $f^{-1}(f(4)) = 4$ and function is inverse function.

The function:	f(x)	=	2x+3
Put "y" for "f(x)":	y	=	2x+3
Subtract 3 from both sides:	y-3	=	2x
Divide both sides by 2:	(y-3)/2	=	x
Swap sides:	x	=	(y-3)/2
Solution (put "$f^{-1}(y)$" for "x") :	$f^{-1}(y)$	=	(y-3)/2