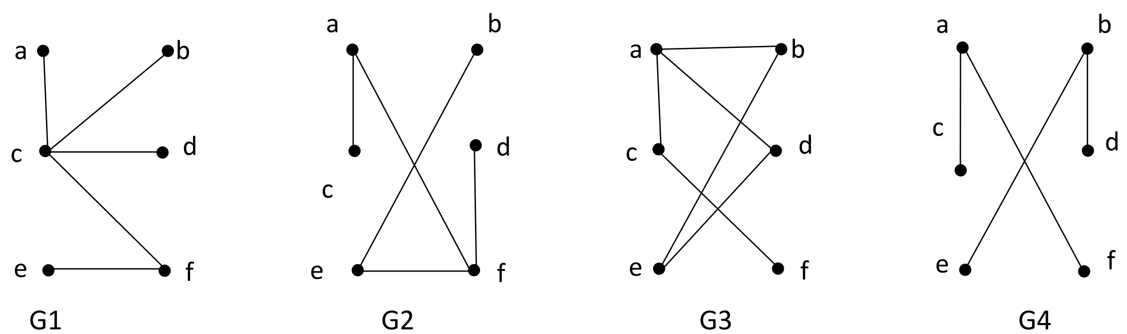
1- Trees

Definition: A tree is a connected undirected graph with no simple circuits.

Example 4. Which of these graphs are trees?



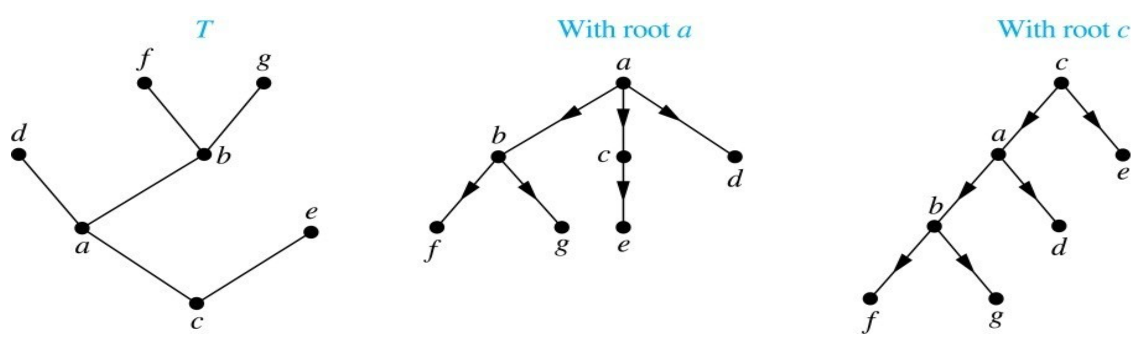
Solution.

G1 and G2 are trees-both are connected and have no simple circuits. Because e,b,a,d,e is a simple circuit, G3 are not tree.

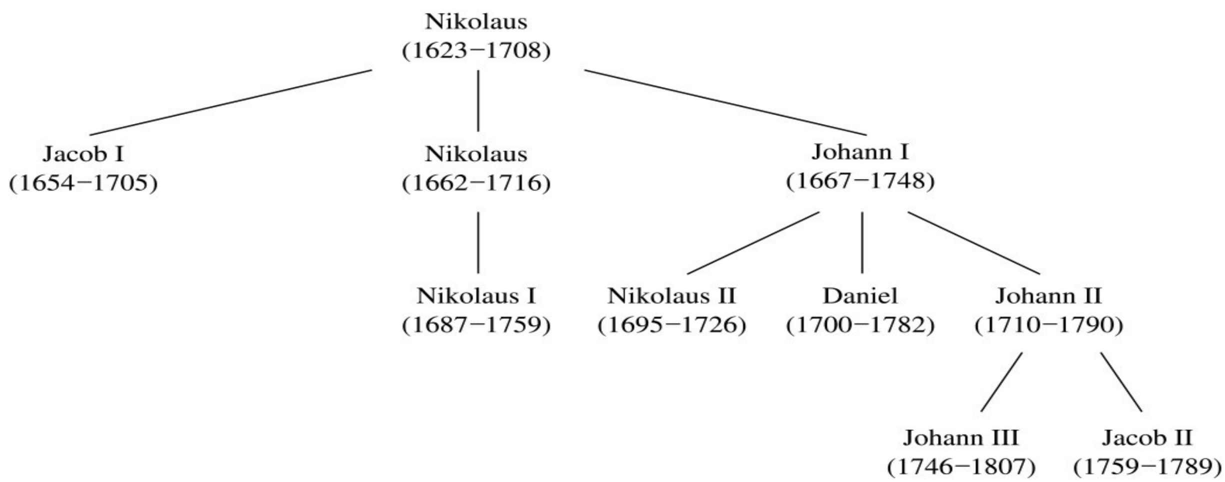
1.1 RootedTrees

Definition: A *rooted tree* is a tree in which one vertexhas been designated as the *root* and every edge is directed away from the root.

An unrooted tree is converted into different rootedtrees when different vertices are chosen as the root.



1.1.1 Rooted Tree Terminology

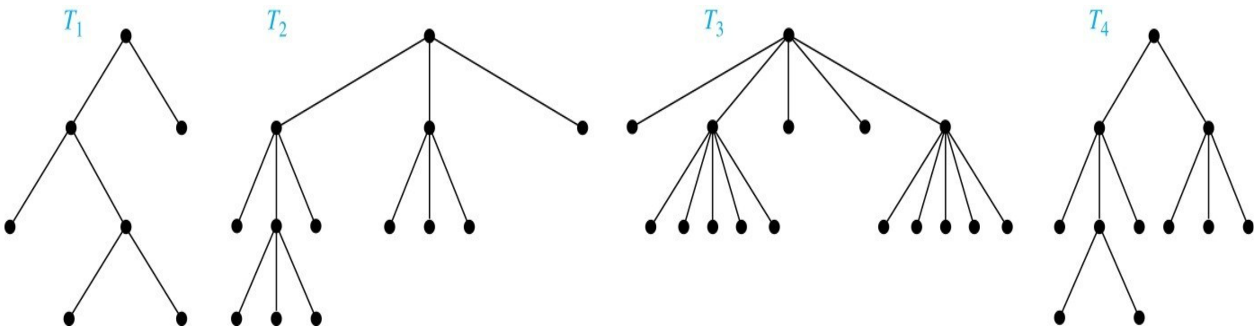


- If v is a vertex of a rooted tree other than the root, the ***parent*** of v is the unique vertex u such that there is a directed edge from u to v. When u is a parent of v, v is called a ***child*** of u. Vertices with the same parent are called ***siblings***.
- The ancestors of a vertex are the vertices in the path from the root to this vertex, excluding the vertex itself and including the root. The **descendants** of a vertex **v** are those vertices that have **v** as an **ancestor**.
- A vertex of a rooted tree with no children is called a ***leaf***. Vertices that have children are called ***internal vertices***.
- If a is a vertex in a tree, the ***subtree*** with a as its root is the subgraph of the tree consisting of a and its descendants and all edges incident to these descendants.

1.2 m-ary Rooted Trees

Definition: A rooted tree is called an *m-ary tree* if every internal vertex has no more than m children. The tree is called a *full m-ary tree* if every internal vertex has exactly m children. An m-ary tree with $m = 2$ is called a *binary* tree.

Example 5. Are the following rooted trees full *m*-ary trees for some positive integer *m*?



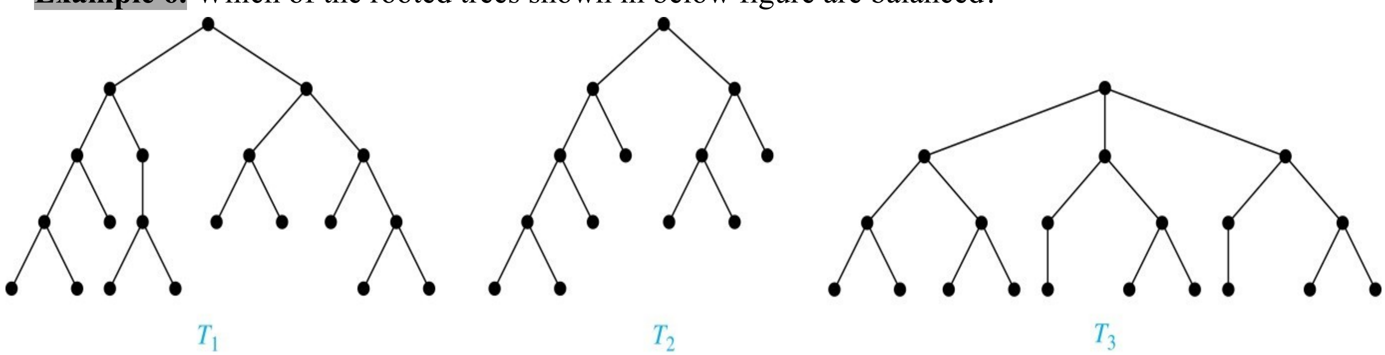
Solution.

T_1 is a full binary tree because each of its internal vertices has two children. T_2 is a full 3-ary tree because each of its internal vertices has three children. In T_3 each internal vertex has five children, so T_3 is a full 5-ary tree. T_4 is not a full m-ary tree for any m because some of its internal vertices have two children and others have three children.

1.3 Balanced m-ary Trees

Definition: A rooted *m*-ary tree of height h is *balanced* if all leaves are at levels h or $h-1$.

Example 6. Which of the rooted trees shown in below figure are balanced?



Solution.

T_1 is balanced, because all its leaves are at levels 3 and 4. However, T_2 is not balanced, because it has leaves at levels 2, 3, and 4. Finally, T_3 is balanced, because all its leaves are at level 3.