Time Series Analysis and Forecasting

A time series was defined by Box *et al.* (2008) as a sequence of observations generated sequentially over time and the adjacent observations are dependent. Time series analysis can be classified into two domains, time and frequency based on discrete or continuous nature of time series dataset (Shumway and Stoffer, 2010; Wei, 2006). Wind speed data often takes into consideration as nonlinear and non-stationary time series data (Donner and Barbosa, 2008; Lange and Focken, 2006; Livingstone and Imboden, 1993; Pourmousavi-Kani and Ardehali, 2011). Many wind speed forecasting methods have been used in the previous studies. Box and Jenkins (1976) proposed their novel approach that was followed in all preceding studies in time series modelling and forecasting.

The stationarity of time series regression

The stationarity is a fundamental concept in time series indicating unaffected joint probability by shifting the time of all observations backward or forward by *k* time period. In another word, changing the time origin does not influence the time series properties. This concept can be expressed as in Equation (1).

$$F(Z_1, Z_2, \ldots, Z_{t-1}, Z_t) = F(Z_{1+k}, Z_{2+k}, \ldots, Z_{t-1+k}, Z_{t+k}) \quad (1)$$

where Z_t is time series variable and *k* is any integer value.

Strictly stationary cannot be applied easily. Supposing a less restriction of stationary conditions was investigated. The wide sense of stationary appears clearly in weakly stationary that is implicated strictly stationary. Weakly stationary can be called second order stationarity, covariance stationary, and mean and variance stationary. It implicates the stabilization of mean, variance, and autocovariance through the time for any length of time series (Chan, 2002; Kitagawa, 2010; Palma, 2007; Yaffee and McGee, 2000). The stationarity of time series can be inspected by changing the mean or the variance over time. Therefore the stationary time series must satisfy the following conditions.

i. Stable mean over time that can be expressed as follows.

$$E(Z_t) = \mu_t = \mu_{t+k} = \mu = \frac{1}{n}\sum_{t=1}^{n} Z_t \qquad (2)$$

where μ is fixed mean, and μ_t and μ_{t+k} are the means of the variables Z_t and Z_{t+k} respectively.

ii. Stable variance over time that can be expressed as follows.

$$E(Z_t - \mu)^2 = Var(Z_t) = \sigma^2_{Z_t} = \sigma^2_{Z_{t+k}} = \sigma^2_Z = \frac{1}{n}\sum_{t=1}^{n}(Z_t - \mu)^2 \qquad (3)$$

where σ^2_Z is fixed variance, and $\sigma^2_{Z_t}$ and $\sigma^2_{Z_{t+k}}$ are the variances of the variables Z_t and Z_{t+k} respectively.

iii. Stable auto-covariance over time that just depends on time-lag. It can be expressed as follows.

$$E[(Z_t - \mu)(Z_{t+k} - \mu)] = Cov(Z_t, Z_{t+k}) = \gamma(k) = Cov(Z_t, Z_{t-k}) = \gamma(-k) = \gamma \qquad (4)$$

where γ is fixed covariance, $\gamma(k)$ and $\gamma(-k)$ are the covariance of the variables Z_t and Z_{t+k} and the covariance of the variables Z_t and Z_{t-k}, respectively, and $\gamma(k) = \gamma(-k)$ because changing the direction of time-lags does not influence the covariance.

First step in time series analysis is plotting time series data via line drawing. Time series plot can check the properties of data that include the status of time series stationarity. The stationarity may be affected due to many reasons such as the seasonal effects, the cycle effects, the outliers or others.

The stationarity status can also be detected by using time series data plot, ACF, and PACF figures. To clarify the ACF, the auto-covariance must be clarified firstly. The covariance and the correlation between two time-lag variables for the same time series dataset are called auto-covariance and autocorrelation respectively. The auto-covariance between Z_t and Z_{t-k} with k time-lags is estimated as follows.

$$\gamma(k) = \frac{1}{n}\sum_{t=k+1}^{n}(Z_t - \mu)(Z_{t-k} - \mu)\,. \tag{5}$$

and

$$\gamma(0) = \frac{1}{n}\sum_{t=1}^{n}(Z_t - \mu)^2 = \sigma_Z^2\,. \tag{6}$$

Based on Equations (5) and (6), the autocorrelation between Z_t and Z_{t-k} can be estimated as follows.

$$\rho(k) = \frac{\gamma(k)}{\gamma(0)} = \frac{\sum_{t=k+1}^{n}(Z_t - \mu)(Z_{t-k} - \mu)}{\sum_{t=1}^{n}(Z_t - \mu)^2}\,. \tag{7}$$

Box and Jenkins (1976) introduced a partial autocorrelation as a novel tool for better characterization of model’s orders. The concept of partial autocorrelation involves the conditional correlation between Z_t and Z_{t-k} only, while the variables are fixed with no effects. This conditional correlation $Corr\left(Z_t, Z_{t-k} \mid Z_{t-1}, Z_{t-2}, \ldots, Z_{t-k+1}\right)$ represents a partial autocorrelation that can be explained by focusing on the coefficient ϕ_{kk} at the last of each model of the following sequence autoregressive system.

$$
\begin{aligned}
Z_t &= \phi_{11} Z_{t-1} + a_t \\
Z_t &= \phi_{21} Z_{t-1} + \phi_{22} Z_{t-2} + a_t \\
&\vdots \\
Z_t &= \phi_{k1} Z_{t-1} + \phi_{k2} Z_{t-2} + \cdots + \phi_{kk} Z_{t-k} + a_t
\end{aligned} \quad . \tag{8}
$$

Using Yule-Walker equations and Cramer's rule help to estimate the coefficients of partial autocorrelation such as $\phi_{11} = \rho_1$, and $\phi_{22} = (\rho_2 - \rho_1^2) / (1 - \rho_1^2)$, and it can be written in general form such as follows.

$$
\phi_{kk} = \frac{\begin{bmatrix} 1 & \rho_1 & \cdots & \rho_{k-2} & \rho_1 \\ \rho_1 & 1 & \cdots & \rho_{k-3} & \rho_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \rho_{k-2} & \rho_{k-3} & \cdots & 1 & \rho_{k-1} \\ \rho_{k-1} & \rho_{k-2} & \cdots & \rho_1 & \rho_k \end{bmatrix}}{\begin{bmatrix} 1 & \rho_1 & \cdots & \rho_{k-2} & \rho_{k-1} \\ \rho_1 & 1 & \cdots & \rho_{k-3} & \rho_{k-2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \rho_{k-2} & \rho_{k-3} & \cdots & 1 & \rho_1 \\ \rho_{k-1} & \rho_{k-2} & \cdots & \rho_1 & 1 \end{bmatrix}} . \tag{9}
$$

After estimating, the confidence limits can be computed for significant level 5% by $\pm 1.96 \times SE(\hat{\rho}_k)$ for an autocorrelation and $\pm 1.96 \times SE(\hat{\phi}_{kk})$ for a partial autocorrelation (Liu, 2006; Wei, 2006) where

$$
SE(\hat{\rho}_k) = \sqrt{\frac{1}{n}(1 + \hat{\rho}_1 + \cdots + \hat{\rho}_{k-1})} \tag{10}
$$

and

$$
SE(\hat{\phi}_{kk}) \simeq \sqrt{\frac{1}{n}} . \tag{11}
$$

The plotted sequences of autocorrelations and partial autocorrelations are called as ACF and PACF respectively.

Very slow dying out pattern in ACF or PACF may indicate that there are some troubles in the stationarity characteristic. Using time series plot along with ACF and PACF figures may give better analysis and reasoning.

Using time series plot, ACF, and PACF, the stationarity status can be checked as mentioned previously. Unstable variance causes nonstationary time series data. In this case, power transformation is applied on time series data to achieve the stationarity. By applying the suitable power transformation on the original time series Z_t, the new transformed time series W_t is obtained such as follows.

$$W_t = (Z_t^{\lambda} - 1)/\lambda . \qquad (12)$$

The type of power transformation depends on the choice of the suitable power λ. If $\lambda = 0.5$ or $\lambda = -0.5$, the transformation is a square root or a reciprocal square root respectively. The natural log transformation is used when $\lambda \to 0$. If $\lambda = -1$ the transformation is just an inverse transformation, while there is no transformation when $\lambda = 1$. to improve the linearity in forecasting and the normality approximation, power transformations are often employed (Shumway and Stoffer, 2011; Wei, 2006).

Unstable mean is another reason for nonstationary time series. In this case, time series data need to be pre-processed by differencing to accomplish stationarity. A first-order differencing or regular differencing can be denoted by $(1-B)$, while a seasonal differencing is denoted by $(1-B^s)$. The stationarity may request dth-order differencing for regular and Dth-order differencing for seasonal cases that is denoted by $(1-B)^d$ and $(1-B^s)^D$ respectively. B is the backshift operator. The new time series W_t can be written such as follows.

$$W_t = (1-B)Z_t = Z_t - (B)Z_t = Z_t - Z_{t-1} \qquad (13)$$

or

$$W_t = (1-B^s)Z_t = Z_t - (B^s)Z_t = Z_t - Z_{t-s} . \qquad (14)$$