Ex Determine the following Sequences Convergs or Diverge ?

1. $a_n = \dfrac{3+5n^2}{1+n^2} = \dfrac{10n}{2n} = 5 \Rightarrow$ Converge

2. $a_n = \dfrac{3n}{4+\sqrt{n}} \Rightarrow \lim\limits_{n\to\infty} \dfrac{3n}{4+\sqrt{n}} = \dfrac{\infty}{\infty}$

$$= \lim_{n\to\infty} \frac{3n}{\sqrt{n}} = 3\sqrt{n} \Rightarrow \text{Diverge}$$

3. $a_n = \dfrac{4+3n^2}{1-n^3} \Rightarrow \lim\limits_{n\to\infty} \dfrac{4+3n^2}{1-n^3} = \dfrac{\infty}{\infty}$

$$= \lim_{n\to\infty} \frac{3n^2}{-n^3} = \frac{-3}{n} = \frac{-3}{\infty} = 0$$

Converge

4. $a_n = \dfrac{\ln n}{n^2} = \dfrac{\infty}{\infty} = \dfrac{\frac{1}{n}}{2n} = \dfrac{1}{\infty} \Rightarrow$ Converge

5. $a_n = n^2 e^{-n} \Rightarrow \lim\limits_{n\to\infty} n^2 e^{-n} = \infty(0)$!!

$$= \lim_{n\to\infty} \frac{n^2}{e^n} = \frac{\infty}{\infty} = \frac{\text{عدد}}{\infty} = 0$$

Converge

6. $a_n = \dfrac{\ln n}{2n} \Rightarrow \lim\limits_{n\to\infty} \dfrac{\ln(n)}{2n} = \dfrac{\infty}{\infty}$

$$= \frac{\frac{1}{n}}{2} = \frac{1}{2n} = 0 \Rightarrow \text{Converge}$$

7. $a_n = \dfrac{(-1)^n + n}{n+4}$

even $\Rightarrow \lim\limits_{n\to\infty} \dfrac{n}{n+4} = \dfrac{\infty}{\infty} \Rightarrow \lim\limits_{n\to\infty} \dfrac{n}{n} = 1$

Dose not exist

odd $\Rightarrow \lim\limits_{n\to\infty} \dfrac{-n}{n+4} = \dfrac{-\infty}{\infty} \Rightarrow \lim\limits_{n\to\infty} \dfrac{-n}{n} = -1$

Diverge

8. $a_n = \dfrac{\cos^n(\pi)}{n^2+3} = \dfrac{(\cos(\pi))^n}{n^2+3} = \dfrac{(-1)^n}{n^2+3}$

even $\Rightarrow \lim\limits_{n\to\infty} \dfrac{1}{n^2+3} = \dfrac{1}{\infty} = 0$

$\Rightarrow$ Converge

odd $\Rightarrow \lim\limits_{n\to\infty} \dfrac{-1}{n^2+3} = \dfrac{-1}{\infty} = 0$

9. $a_n = (1 - \dfrac{3}{n})^{2n}$

Rule :

$$\lim_{n\to\infty} (1 + \frac{a}{n})^n = e^a$$

$$\lim_{n\to\infty} (1 + \frac{-3*2}{2n})^{2n}$$

$$= \lim_{n\to\infty} (1 + \frac{-6}{2n})^{2n} = e^{-6} \Rightarrow \text{Converge}$$

10. $a_n = \left(\frac{n}{n+1}\right)^n$

$$\lim_{n \to \infty} \left(\frac{n}{n(1+\frac{1}{n})}\right)^n$$

$$= \frac{1}{\lim_{n \to \infty} \frac{n}{(1+\frac{1}{n})^n}} = \frac{1}{e^1} = e^{-1} \Rightarrow \text{Converge}$$

11. $a_n = \sqrt{n^2+n} - n$

$$\lim_{n \to \infty} \underbrace{\sqrt{n^2+n} - n}_{\infty - \infty} * \frac{\sqrt{n^2+n} + n}{\sqrt{n^2+n} + n}$$

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$$= \frac{n^2+n-n^2}{\sqrt{n^2+n} - n} = \frac{\infty}{\infty} = \frac{n}{\sqrt{n^2+n}}$$

$$\Rightarrow \lim_{n \to \infty} \frac{n}{2n} = \frac{1}{2} \Rightarrow \text{Converge}$$

Series

$$S_n = \sum_{n=1}^{\infty} a_{(n)}$$

$$S_1 = a_1$$
$$S_2 = a_1 + a_2$$
$$S_3 = a_1 + a_2 + a_3$$
$$\vdots$$
$$S_n = a_1 + a_2 + a_3 + \dots + a_n$$

Ex 1. $S_n = \sum_{n=1}^{\infty} a_n = 1+2+3+4+\dots$

2. $S_n = \sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$

3. $S_n = \sum_{n=0}^{\infty} n^2 = 0+1+4+9+16+25$

$= 55 \Rightarrow$ finite series